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Probing the MSSM Higgs Sector at an e^-e^- Collider

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Abstract

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The theoretical structure of the Higgs sector of the Minimal Supersymmetric Standard Model (MSSM) is briefly described. An outline of Higgs phenomenology at future lepton colliders is presented, and some opportunities for probing the physics of the MSSM Higgs sector at an e^-e^- collider are considered.

1. Review of the MSSM Higgs Sector

The Minimal Supersymmetric extension of the Standard Model (MSSM) contains the Standard Model particle spectrum and the corresponding supersymmetric partners.[1,2,3] In addition, the MSSM must possess two Higgs doublets in order to give masses to up and down type fermions in a manner consistent with supersymmetry (and to avoid gauge anomalies introduced by the fermionic superpartners of the Higgs bosons). In particular, the MSSM Higgs sector is a CP-conserving two-Higgs-doublet model, which can be parameterized at tree-level in terms of two Higgs sector parameters. This structure arises due to constraints imposed by supersymmetry that fix the Higgs quartic couplings in terms of electroweak gauge coupling constants.

Let Φ_1 and Φ_2 denote two complex $Y = 1$, $SU(2)_L$ doublet scalar fields. The most general gauge invariant CP-conserving scalar potential is given by

$$\mathcal{V} = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}]$$

$$+ \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

$$+ \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\}, \tag{1}$$

where all parameters introduced above are real. Supersymmetry imposes the following constraints on the tree-level values of the quartic couplings λ_i:

$$\lambda_1 = \lambda_2 = \tfrac{1}{4}(g^2 + g'^2), \qquad \lambda_3 = \tfrac{1}{4}(g^2 - g'^2),$$
$$\lambda_4 = -\tfrac{1}{2}g^2, \qquad \lambda_5 = \lambda_6 = \lambda_7 = 0. \tag{2}$$

The tree-level Higgs masses and mixing can now be computed in the usual manner.[4] The scalar fields will develop non-zero vacuum expectation values if the mass matrix

m_{ij}^2 has at least one negative eigenvalue. Imposing CP invariance and $U(1)_{\rm EM}$ gauge symmetry, the minimum of the potential is

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \tag{3}$$

where the v_i are real. It is convenient to introduce the following notation:

$$v^2 \equiv v_1^2 + v_2^2 = \frac{4m_W^2}{g^2} = (246 \text{ GeV})^2, \qquad \tan \beta \equiv \frac{v_2}{v_1}. \tag{4}$$

Of the original eight scalar degrees of freedom, three Goldstone bosons ($G^\pm$ and G^0) are absorbed and become the longitudinal components of the $W^\pm$ and Z. The remaining five physical Higgs particles are: two CP-even scalars (h^0 and H^0, with $m_{h^0} \leq m_{H^0}$), one CP-odd scalar (A^0) and a charged Higgs pair ($H^\pm$). The squared-mass parameters m_{11}^2 and m_{22}^2 can be eliminated by minimizing the scalar potential. The resulting squared masses for the CP-odd and charged Higgs states are

$$m_{A^0}^2 = m_{12}^2 (\tan \beta + \cot \beta),$$
$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2, \tag{5}$$

and the tree-level neutral CP-even mass matrix is given by

$$\mathcal{M}_0^2 = \begin{pmatrix} m_{A^0}^2 \sin^2 \beta + m_Z^2 \cos^2 \beta & -(m_{A^0}^2 + m_Z^2) \sin \beta \cos \beta \\ -(m_{A^0}^2 + m_Z^2) \sin \beta \cos \beta & m_{A^0}^2 \cos^2 \beta + m_Z^2 \sin^2 \beta \end{pmatrix}. \tag{6}$$

The CP-even Higgs mass eigenstates are linear combinations of the real components of the neutral Higgs fields

$$H^0 = \;\; (\sqrt{2} \, \text{Re} \, \Phi_1^0 - v_1) \cos \alpha + (\sqrt{2} \, \text{Re} \, \Phi_2^0 - v_2) \sin \alpha$$
$$h^0 = -(\sqrt{2} \, \text{Re} \, \Phi_1^0 - v_1) \sin \alpha + (\sqrt{2} \, \text{Re} \, \Phi_2^0 - v_2) \cos \alpha, \tag{7}$$

where α is the angle that diagonalizes the squared-mass matrix. The eigenvalues of $\mathcal{M}_0^2$ are the squared masses of the two CP-even Higgs scalars

$$m_{H^0, h^0}^2 = \tfrac{1}{2} \left(m_{A^0}^2 + m_Z^2 \pm \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{A^0}^2 \cos^2 2\beta} \right). \tag{8}$$

and the CP-even Higgs mixing angle α is determined by

$$\cos 2\alpha = -\cos 2\beta \left(\frac{m_{A^0}^2 - m_Z^2}{m_{H^0}^2 - m_{h^0}^2} \right), \qquad \sin 2\alpha = -\sin 2\beta \left(\frac{m_{H^0}^2 + m_{h^0}^2}{m_{H^0}^2 - m_{h^0}^2} \right). \tag{9}$$

From these results, it is easy to obtain:

$$\cos^2(\beta - \alpha) = \frac{m_{h^0}^2 (m_Z^2 - m_{h^0}^2)}{m_{A^0}^2 (m_{H^0}^2 - m_{h^0}^2)}. \tag{10}$$

Thus, in the MSSM, two parameters (conveniently chosen to be m_{A^0} and $\tan\beta$) suffice to fix all other tree-level Higgs sector parameters.

Eq. (8) implies the following tree-level sum rule:

$$m_{h^0}^2 + m_{H^0}^2 = m_{A^0}^2 + m_Z^2 \,. \tag{11}$$

In addition, a number of important mass inequalities can be derived from the expressions for the tree-level Higgs masses obtained above:

$$m_{h^0} \leq m_{A^0} |\cos 2\beta| \leq m_{A^0} \,,$$
$$m_{h^0} \leq m_Z |\cos 2\beta| \leq m_Z \,,$$
$$m_{H^0} \geq (m_{A^0}^2 \sin^2 2\beta + m_Z^2)^{1/2} \geq m_Z \,,$$
$$m_{H^0} \geq (m_Z^2 \sin^2 2\beta + m_{A^0}^2)^{1/2} \geq m_{A^0} \,,$$
$$m_{H^\pm} \geq m_W \,. \tag{12}$$

Of particular note is the tree-level upper bound of the mass of the lightest CP-even Higgs boson, $m_{h^0} \leq m_Z |\cos 2\beta|$. If this prediction were exact, it would imply that the Higgs boson must be discovered at the LEP-2 collider during its 1998 run, assuming the expected center-of-mass energy of 190 GeV is achieved with an integrated luminosity of 150 pb^{-1}. Absence of a Higgs boson lighter than m_Z would then (apparently) rule out the MSSM. However, when radiative corrections are included, the light Higgs mass upper bound may increase significantly. In the one-loop leading logarithmic approximation (assuming $m_{A^0} \gtrsim m_Z$),[5]

$$m_{h^0}^2 \lesssim m_Z^2 \cos^2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \ln\left(\frac{M_{\tilde{t}}^2}{m_t^2}\right) \,, \tag{13}$$

where $M_{\tilde{t}}$ is the (approximate) common mass of the top-squarks. Observe that the Higgs mass upper bound is very sensitive to the value of the top mass and depends logarithmically on the top-squark masses. Although eq. (13) provides a rough guide to the Higgs mass upper bound, it is not sufficiently precise for LEP-2 phenomenology, whose Higgs mass reach depends delicately on the MSSM parameters. In addition, in order to perform precision Higgs measurements and make comparisons with theory, more accurate predictions for the Higgs sector masses are required. The formula for the full one-loop radiative corrected Higgs mass has been obtained in the literature, although it is very complicated since it depends in detail on the virtual contributions of the MSSM spectrum.[6] Moreover, if the supersymmetry breaking scale is larger than a few hundred GeV, then renormalization group (RG) methods are essential for summing up the effects of large logarithms and obtaining an accurate prediction.

The computation of the RG-improved one-loop corrections requires numerical integration of a coupled set of renormalization group equations.[7] (The dominant

two-loop next-to-leading logarithmic results are also known.[8]) Although this program has been carried out in the literature, the procedure is unwieldy and not easily amenable to large-scale Monte-Carlo analyses. In Refs. 9 and 10, a simple analytic procedure for accurately approximating m_{h^0} has been presented. This method can be easily implemented, and incorporate both the leading one-loop and two-loop effects and the RG-improvement. Also included are the leading effects at one loop of supersymmetric thresholds (the most important effects of this type arise from third-generation squark mixing). Here, I shall simply quote two specific bounds, assuming $m_t = 175$ GeV and $M_{\tilde{t}} \lesssim 1$ TeV: $m_{h^0} \lesssim 112$ GeV if top-squark mixing is negligible, while $m_{h^0} \lesssim 125$ GeV if top-squark mixing is "maximal". Maximal mixing corresponds to an off-diagonal squark squared-mass that produces the largest value of m_{h^0}. This mixing leads to an extremely large splitting of top-squark mass eigenstates. Current state-of-the-art calculations can obtain a mass bound for the light CP-even Higgs boson of the MSSM that is reliable to within a few GeV.

Radiative corrections also modify the tree-level prediction for the masses of H^0 and $H^{\pm}$. However, the corrections are always smaller than the corrections to m_{h^0} discussed above.[a] It should be noted that $m_{H^{\pm}} \geq m_W$ is also not a strict bound when the one-loop corrections are included, although this tree-level bound does hold approximately over most of MSSM parameter space (and can be significantly violated only when $m_{A^0} \lesssim m_Z$ and $\tan\beta$ is well below 1, a region of parameter space that is theoretically and phenomenologically disfavored).

Having reviewed the dependence of the MSSM Higgs masses on the model parameters, I now turn to the tree-level Higgs couplings. The Higgs couplings to gauge bosons follow from gauge invariance and are thus model independent. For example, the couplings of the two CP-even Higgs bosons to W and Z pairs are given by

$$g_{h^0 VV} = g_V m_V \sin(\beta - \alpha)\,,$$

$$g_{H^0 VV} = g_V m_V \cos(\beta - \alpha)\,, \tag{14}$$

where

$$g_V \equiv \begin{cases} g, & V = W\,, \\ g/\cos\theta_W, & V = Z\,. \end{cases} \tag{15}$$

There are no tree-level couplings of A^0 or $H^{\pm}$ to VV. Gauge invariance also determines the strength of the trilinear couplings of one gauge boson to two Higgs bosons. For example,

$$g_{h^0 A^0 Z} = \frac{g \cos(\beta - \alpha)}{2 \cos\theta_W}\,,$$

$$g_{H^0 A^0 Z} = \frac{-g \sin(\beta - \alpha)}{2 \cos\theta_W}\,. \tag{16}$$

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[a] More precisely, the corrections to m_{H^0} are small if $m_{A^0} \gtrsim m_Z$. Conversely, if $m_{A^0} \lesssim m_Z$, then the one-loop correction to $m_{h^0}^2$ is small, while the correction to $m_{H^0}^2$ is proportional to $g^2(m_t^4/m_W^2)\ln(M_{\tilde{t}}^2/m_t^2)$. In contrast, the leading logarithmically-enhanced correction to $m_{H^{\pm}}^2$ is proportional to $g^2 m_t^2 \ln(M_{\tilde{t}}^2/m_t^2)$, independent of the value of m_{A^0}.

In the examples shown above, some of the couplings can be suppressed if either $\sin(\beta - \alpha)$ or $\cos(\beta - \alpha)$ is very small. Note that all the vector boson–Higgs boson couplings cannot vanish simultaneously. From the expressions above, we see that the following tree-level sum rules must hold separately for $V = W$ and Z:

$$g_{H^0 VV}^2 + g_{h^0 VV}^2 = g_V^2 m_V^2\,,$$

$$g_{h^0 A^0 Z}^2 + g_{H^0 A^0 Z}^2 = \frac{g^2}{4\cos^2\theta_W}\,. \tag{17}$$

These results are a consequence of the tree-unitarity of the electroweak theory.[11] Moreover, if we focus on a given CP-even Higgs state, we note that its couplings to VV and $A^0 V$ cannot be simultaneously suppressed, since eqs. (14)–(16) imply that[12]

$$g_{HZZ}^2 + 4m_Z^2 g_{HA^0 Z}^2 = \frac{g^2 m_Z^2}{\cos^2\theta_W}\,, \tag{18}$$

for $H = h^0$ or H^0. Similar considerations also hold for the coupling of h^0 and H^0 to $W^\pm H^\mp$. We can summarize the above results by noting that the coupling of h^0 and H^0 to vector boson pairs or vector–scalar boson final states is proportional to either $\sin(\beta - \alpha)$ or $\cos(\beta - \alpha)$ as indicated below.[13,14]

$\cos(\beta - \alpha)$	$\sin(\beta - \alpha)$	
$H^0 W^+ W^-$	$h^0 W^+ W^-$	
$H^0 ZZ$	$h^0 ZZ$	
$ZA^0 h^0$	$ZA^0 H^0$	(19)
$W^\pm H^\mp h^0$	$W^\pm H^\mp H^0$	
$ZW^\pm H^\mp h^0$	$ZW^\pm H^\mp H^0$	
$\gamma W^\pm H^\mp h^0$	$\gamma W^\pm H^\mp H^0$	

Note in particular that *all* vertices in the theory that contain at least one vector boson and *exactly one* of the non-minimal Higgs boson states (H^0, A^0 or $H^\pm$) are proportional to $\cos(\beta - \alpha)$.

In contrast to the above results, none of the Higgs self-couplings and Higgs-fermion couplings vanish if either $\sin(\beta - \alpha) = 0$ or $\cos(\beta - \alpha) = 0$. The three-point and four-point Higgs self-couplings depend on the parameters of the two-Higgs-doublet potential [eq. (1)]. In the MSSM, the tree-level four-point self-couplings are fixed in terms of the electroweak gauge couplings by virtue of eq. (2). The three-point couplings are dimensionful, so they depend additionally on v and $\tan\beta$. A complete list of these couplings can be found in the appendices of Ref. 13.

Supersymmetry imposes a particular structure on the coupling of the Higgs bosons to the fermions. In particular, one Higgs doublet (before symmetry breaking) couples exclusively to down-type fermions and the other Higgs doublet couples exclusively to up-type fermions. As a result, the charged Higgs boson coupling to fermion pairs (with all particles pointing into the vertex) is given by

$$g_{H^- t\bar{b}} = \frac{g}{2\sqrt{2}m_W}\left[m_t \cot\beta\,(1 + \gamma_5) + m_b \tan\beta\,(1 - \gamma_5)\right], \tag{20}$$

where the notation of the 3rd family has been employed, and the effects of inter-generational mixing (which depends in the same way on the Cabibbo-Kobayashi-Maskawa mixing matrix as the usual weak charged current couplings) have been omitted. The neutral Higgs bosons couplings to $f\bar{f}$ are flavor diagonal, and are listed below using 3rd family notation, relative to the Standard Model value, $gm_f/2m_W$:

$$h^0 b\bar{b}: \quad -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)\,,$$

$$h^0 t\bar{t}: \quad \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)\,,$$

$$H^0 b\bar{b}: \quad \frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha)\,,$$

$$H^0 t\bar{t}: \quad \frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha)\,,$$

$$A^0 b\bar{b}: \quad \gamma_5 \tan\beta\,,$$

$$A^0 t\bar{t}: \quad \gamma_5 \cot\beta\,, \tag{21}$$

where the γ_5 indicates a pseudoscalar coupling.

All tree-level couplings discussed above are modified by radiative corrections. In a first approximation, one can consider the modifications to the couplings that arise through the renormalization of the CP-even Higgs mixing angle α. This simply requires a computation of the radiatively-corrected CP-even Higgs squared-mass matrix. By diagonalizing this matrix, one extracts the radiatively corrected value of α.[15] The parameter $\tan\beta$ is by assumption an input parameter.[b] The most significant implication of this procedure is the renormalization of $\cos(\beta-\alpha)$, shown in Fig. 1, which governs many of the couplings listed above. However, this approximation clearly misses some of the radiative corrections that arise from the radiatively corrected three-point (and four-point) vertex functions. In most cases, these corrections are not expected to be large, although a full set of Higgs coupling radiative corrections will be required to analyze future Higgs precision measurements.

2. The Decoupling Limit

The pattern of tree-level couplings exhibited in Section 1 can be understood in the context of the *decoupling limit* of the two-Higgs-doublet model.[16,17] First, consider the Standard Model Higgs boson ($h^0_{\rm SM}$). At tree-level, the Higgs quartic self-coupling is related to its mass: $\lambda = m^2_{h^0_{\rm SM}}/v^2$. This means that one cannot take $m_{h^0_{\rm SM}}$ arbitrarily large without the attendant growth in λ. That is, the heavy Higgs

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[b]This is a somewhat subtle issue, since one must define the input parameters of the theory through some suitable physical process. Perhaps the simplest procedure is to work with a minimally-subtracted definition for $\tan\beta$. Then, one can express all observables in terms of this definition. Eventually, a global fit to the observables of the Higgs and higgsino sectors can be used to experimentally determine the value of $\tan\beta$.
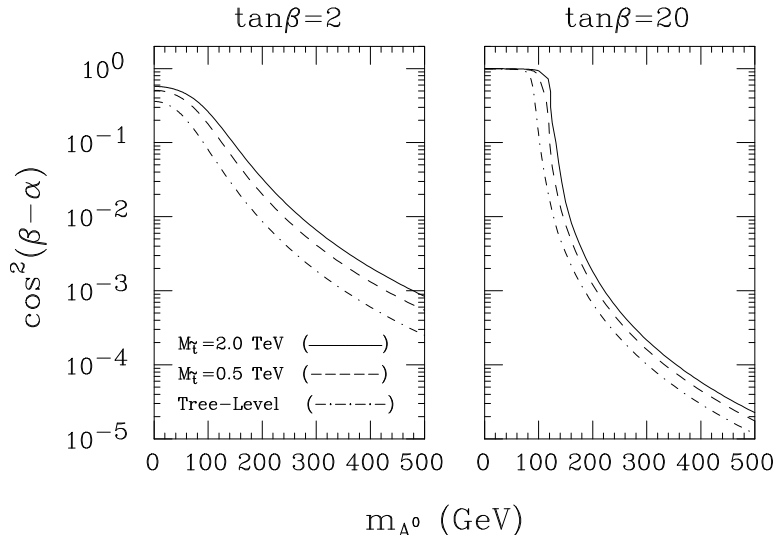
Fig. 1. The value of $\cos^2(\beta - \alpha)$ is shown as a function of m_{A^0} and $\tan\beta$. The tree-level prediction is depicted by the dot-dashed line. The solid and dashed lines include the effects of the renormalization-group-improved radiatively-corrected CP-even Higgs squared mass matrix for two different choices of the (common) top-squark mass, using the formulae of Ref. 9. The impact of the radiative corrections grows with increasing top-squark mass. Squark mixing effects are neglected; although they have a negligible effect on the results shown.

limit in the Standard Model exhibits non-decoupling. In models of a non-minimal Higgs sector, the situation is more complex. In some models, it is not possible to take any Higgs mass much larger than $\mathcal{O}(v)$ without finding at least one strong Higgs self-coupling. In models such as the MSSM, one finds that the non-minimal Higgs boson masses can be taken large at fixed Higgs self-couplings. More generally, such behavior can arise in models that possess one (or more) off-diagonal squared-mass parameters in addition to the diagonal scalar squared-masses. When the off-diagonal squared-mass parameters are taken large [keeping the dimensionless Higgs self-couplings fixed and $\lesssim \mathcal{O}(1)$], the heavy Higgs states decouple, while both light and heavy Higgs bosons remain weakly-coupled. In this decoupling limit, exactly one neutral CP-even Higgs scalar remains light, and its properties are precisely those of the (weakly-coupled) Standard Model Higgs boson. That is, $h^0 \simeq h^0_{\rm SM}$, with $m_{h^0} \sim \mathcal{O}(m_Z)$, and all other non-minimal Higgs states are significantly heavier than m_{h^0}. Squared-mass splittings of the heavy Higgs states are of $\mathcal{O}(m_Z^2)$, which means that all heavy Higgs states are approximately degenerate, with mass differences of order m_Z^2/m_{A^0} (where m_{A^0} is approximately equal to the common heavy Higgs mass scale). In contrast, if the non-minimal Higgs sector is weakly coupled but far from the decoupling limit, then h^0 is not separated in mass from the other Higgs states. In this case, the properties[c] of h^0 differ significantly from those of $h^0_{\rm SM}$.

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[c]The basic property of the Higgs coupling strength proportional to mass is maintained. But, the precise coupling strength patterns of h^0 will differ from those of $h^0_{\rm SM}$ in the non-decoupling limit.

In the MSSM, the decoupling limit corresponds to the limit of $m_{A^0} \gg m_Z$. In this case, the tree-level Higgs masses take on a particularly simple form

$$m_{h^0}^2 \simeq m_Z^2 \cos^2 2\beta \,,$$

$$m_{H^0}^2 \simeq m_{A^0}^2 + m_Z^2 \sin^2 2\beta \,,$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 \,,$$

$$\cos^2(\beta - \alpha) \simeq \frac{m_Z^4 \sin^2 4\beta}{4 m_{A^0}^4} \,. \tag{22}$$

A number of consequences are immediately apparent. First, $m_{A^0} \simeq m_{H^0} \simeq m_{H^\pm}$, up to corrections of $\mathcal{O}(m_Z^2/m_{A^0})$. Second, $\cos(\beta - \alpha) \simeq 0$ up to corrections of $\mathcal{O}(m_Z^2/m_{A^0}^2)$. One can also check that when $\cos(\beta - \alpha) = 0$, the h^0 couplings to vector boson pairs and fermion pairs, and the $(h^0)^3$ and $(h^0)^4$ self-couplings all reduce to the corresponding Standard Model couplings of h_{SM}^0. In general, the couplings of the non-minimal Higgs states do not vanish in the decoupling limit, with the notable exception of the couplings involving at least one vector boson and exactly one non-minimal Higgs boson [as noted below eq. (19)].

Should one expect $m_{A^0} \gg m_Z$ in the MSSM? Naively, one might expect the masses of all Higgs sector states to be roughly of $\mathcal{O}(v)$. However, somewhat heavier non-minimal Higgs states often arise in model building. In low-energy supersymmetric models, the mass scale of the non-minimal Higgs states is controlled by the soft-supersymmetry-breaking parameter m_{12}^2 [see eq. (5)], which could be as large as a few TeV. For example, in the minimal supergravity (mSUGRA) model, one finds that $m_{A^0} \gg m_Z$ over a large fraction of the mSUGRA parameter space.[1]

Although the radiative corrections to the Higgs masses can have a profound effect on the phenomenology, the overall size of such corrections is never larger than $\mathcal{O}(m_Z)$. As a result, the tree-level implications of decoupling for Higgs masses and couplings remain valid when radiative corrections are taken into account. This observation is supported by the results displayed in Fig. 1. Although the renormalized value of $\cos(\beta - \alpha)$ is somewhat enhanced above its tree-level value, it continues to vanish up to corrections of $\mathcal{O}(m_Z^2/m_{A^0}^2)$ in the decoupling limit.

It is not unlikely that the first Higgs state to be discovered will be experimentally indistinguishable from the Standard Model Higgs boson. This occurs in many theoretical models that exhibit the decoupling of heavy scalar states. As noted above, in the decoupling limit the lightest Higgs state, h^0, is a neutral CP-even scalar with properties nearly identical to those of the h_{SM}^0, while the other Higgs bosons of the non-minimal Higgs sector are heavy (compared to the Z) and are approximately mass-degenerate. Thus, discovery of $h^0 \simeq h_{\text{SM}}^0$ may shed little light on the dynamics underlying electroweak symmetry breaking. In particular, precision measurements are critical in order to distinguish between h^0 and h_{SM}^0. In addition, it is crucial to directly detect and explore the properties of the non-minimal Higgs

bosons. To accomplish these goals, future colliders of the highest possible energies and luminosities are essential.

3. Higgs Hunting at Future Lepton Colliders

Higgs hunting at future colliders will consist of three phases. Phase one is the initial Higgs boson search in which a Higgs signal is found and confirmed as evidence for new phenomena not described by Standard Model background. Phase two will address the question: should the signal be identified with Higgs physics? Finally, phase three will consist of a detailed probe of the Higgs sector and precise measurements of Higgs sector observables.

The potential for the discovery of the Higgs boson at future colliders is well documented.[18,19,20] Here, I shall briefly focus on the Higgs searches at a future (and hopefully, next) e^+e^- linear collider (NLC). The light CP-even Higgs boson is detected at the NLC via two processes. The first involves the extension of the LEP-2 search for

$$e^+e^- \to Zh^0 \tag{23}$$

to higher energies. In addition, a second process can also be significant: the (virtual) W^+W^- fusion process[d]

$$e^+e^- \to \nu\bar{\nu}W^*W^* \to \nu\bar{\nu}h^0. \tag{24}$$

The fusion cross section grows logarithmically with the center-of-mass energy and becomes the dominant Higgs production process at large $\sqrt{s}/m_{h^0}$. The NLC provides complete coverage of the MSSM Higgs sector parameter space once the center-of-mass energy is above 300 GeV. This conclusion is a consequence of the MSSM upper bound, $m_{h^0} \lesssim 125$ GeV, discussed in Section 1. Note that it is possible that the coupling of h^0 to vector boson pairs is suppressed if $\sin(\beta-\alpha) \sim 0$. In this case, h^0 will *not* be observable in the channels listed above. However according to Fig. 1, $\sin(\beta-\alpha) \ll 1$ occurs only for small values of m_{A^0} and large $\tan\beta$. In this case, we note that the Zh^0A^0 coupling, which is proportional to $\cos(\beta-\alpha)$, is unsuppressed. In this regard, the sum rule of eq. (18) [with $H = h^0$] is decisive. Specifically, the associated production

$$e^+e^- \to h^0A^0 \tag{25}$$

provides an addition discovery channel if $m_{A^0} \lesssim \sqrt{s}/2$.[e] If no Higgs signal is seen in either direct h^0Z or associated h^0A^0 production, then the MSSM can be unambiguously rule out at the NLC.

Let us suppose that h^0 is discovered. If its properties are significantly different from h^0_{SM}, then we know that the Higgs sector is far from the decoupling limit. This will be true only if the non-minimal Higgs sector states (A^0, H^0, and $H^\pm$) are all

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[d]The corresponding ZZ fusion process, $e^+e^- \to e^+e^-Z^*Z^* \to e^+e^-h^0$ is suppressed by about a factor of ten relative to the W^+W^- fusion process. Nevertheless, at large $\sqrt{s}/m_{h^0}$, the $ZZ \to h^0$ fusion rate compares favorably to that of $e^+e^- \to Zh^0$. As a result, the ZZ fusion process can be used in some cases to study Higgs properties.

[e]Although $e^+e^- \to h^0A^0$ is kinematically allowed when $\sqrt{s}/2 \lesssim m_{A^0} \leq \sqrt{s} - m_{h^0}$, the rate for this process is suppressed (due to the suppression of the ZA^0h^0 coupling in the decoupling limit).

rather light, with masses of $\mathcal{O}(m_Z)$. This would present a considerable opportunity for the NLC to probe the details of the MSSM Higgs sector. In particular, the processes $e^+e^- \to Zh^0$, ZH^0, h^0A^0, H^0A^0 and H^+H^- would all be observable at NLC running at or above its nominal center-of-mass energy of $\sqrt{s} = 500$ GeV. In contrast, if none of the non-minimal Higgs sector states are observed at the NLC, then the decoupling limit must be in effect.

The phenomenological consequences of the decoupling regime are both disappointing and challenging. In this case, h^0 (once discovered) will exhibit all the expected properties of h_{SM}^0. In order to confirm the existence of the non-minimal Higgs sector, one must either detect a deviation from Standard Model Higgs physics via precision measurements of h^0, or directly detect the non-minimal Higgs states and explore their properties. Clearly, the main obstacle for the discovery of non-minimal Higgs states at the NLC is the limit of the center-of-mass energy, which determines the upper limit of the Higgs boson discovery reach. In particular, because of the suppressed Higgs boson–gauge boson couplings in the decoupling limit, the production of a single heavy non-minimal Higgs state in association with either h^0 or Z is suppressed, even if the process is kinematically allowed [e.g., recall the footnote below eq. (25)]. The coupling of the Z to two heavy non-minimal Higgs states (all approximately equal in mass to m_{A^0}) is unsuppressed; however, the corresponding processes are kinematically forbidden for $\sqrt{s} \lesssim 2m_{A^0}$. Thus, the heavy Higgs states of the MSSM can be produced in sufficient number and detected only if $\sqrt{s} \gtrsim 2m_{A^0}$.[17] The discovery reach could in principle be extended by employing the $\gamma\gamma$ collider mode of the NLC. In the latter case, the heavy neutral Higgs states can be produced singly by s-channel resonance production. In this mode of operation, the search for $\gamma\gamma \to A^0$ and $\gamma\gamma \to H^0$ can somewhat improve the non-minimal Higgs mass discovery reach of the NLC.[21]

4. Higgs Boson Production at an e^-e^- Collider

Among all the NLC Higgs production mechanisms cited in Section 3, only ZZ-fusion is applicable for Higgs production at an e^-e^- collider. The leading single Higgs production mechanism at an e^-e^- collider is:

$$e^-e^- \to e^-e^-Z^*Z^* \to e^-e^-H \qquad (H = H^0, h^0), \tag{26}$$

where Z^* is a virtual Z boson.[f] Cross-sections for the production of h_{SM}^0 via ZZ-fusion have been obtained in Ref. 23. To obtain the corresponding cross sections for the production of h^0 $[H^0]$, the results of Ref. 23 should be multiplied by an overall factor of $\sin^2(\beta - \alpha)$ $[\cos^2(\beta - \alpha)]$. The ZZ-fusion rates are quite small; for $\sqrt{s} = 500$ GeV and $m_{h^0} = 100$ GeV [with, e.g., $\sin(\beta - \alpha) = 1$ in the decoupling limit], Ref. 23 quotes $\sigma(e^-e^- \to e^-e^-h^0) \simeq 9$ fb. The corresponding rates for $e^+e^- \to Zh^0$ at the NLC are an order of magnitude larger. For larger Higgs

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[f]A second process, $e^+e^- \to e^-e^-\gamma^*\gamma^* \to e^-e^-H$ ($H = H^0, h^0$, and A^0), can also produce the CP-odd Higgs boson (A^0).[22] However, this is a one-loop process and is thus suppressed in rate as compared to the ZZ-fusion production of the CP-even Higgs bosons.

masses, the ZZ-fusion rates become significantly smaller. However, as noted in the footnote preceding eq. (24), the rate for ZZ-fusion can become appreciable in size relative to $e^+e^- \to Zh^0$ for large $\sqrt{s}/m_{h^0}$. If the Higgs boson decays invisibly,[g] then both processes provide the possibility of indirect Higgs detection by measuring the invariant mass of the system that recoils against the Higgs boson.[24]

Let us now consider the possibility of double Higgs production at an e^-e^- collider. Double Higgs production can only occur via gauge boson fusion, and results in four body final states: $e^-e^- \to \ell\ell HH$, where H is a charged or neutral Higgs boson and ℓ is either an electron or ν_e. Cross-sections for these processes are known to be quite small. Nevertheless, if any of these processes could be detected, it could provide a very sensitive test of the underlying Higgs boson dynamics. In particular, such processes can probe some of the triple Higgs couplings, which would provide a direct measure of the Higgs potential. In the context of the MSSM, one could perhaps verify that the Higgs self-couplings are given by gauge couplings as discussed in Section 1. Unfortunately, there are only a few cases where the measurement of Higgs self-couplings at the NLC has been shown to be viable.[25] In particular, the four-body phase space leads to a substantial suppression that could only be overcome by machines of the highest possible energies and luminosities.

The corresponding calculations for double Higgs production at e^-e^- colliders have not yet been carried out systematically. An initial study was reported in Ref. 26. Here, I shall indicate the complete set of double Higgs processes that can be produced in e^-e^- collisions. Detailed calculations will not be provided here, although general expectations based on the decoupling limit will be indicated. The mechanism for double Higgs production is vector boson fusion (symbolically indicated by $VV \to HH$). Each electron can emit either a virtual W^-, Z or photon. In Table 1, I list the possible fusion mechanisms and the corresponding Higgs bosons that can appear in the final state.

The last column of Table 1 indicates whether the process is suppressed in the decoupling limit. The case of H^-H^- fusion is particularly noteworthy. As shown in Ref. 26, there are a number of contributions to the amplitude for $W^-W^- \to H^-H^-$, corresponding to the t-channel and u-channel exchange of h^0, H^0 and A^0. In the decoupling limit (where $m_{H^0} \simeq m_{A^0}$), the t-channel amplitude is proportional to

$$\frac{\cos^2(\beta-\alpha)}{t-m_{h^0}^2} + \frac{\sin^2(\beta-\alpha)}{t-m_{H^0}^2} - \frac{1}{t-m_{A^0}^2} \simeq \cos^2(\beta-\alpha)\left[\frac{1}{t-m_{h^0}^2} - \frac{1}{t-m_{A^0}^2}\right]. \quad (27)$$

The u-channel amplitude behaves similarly (*i.e.*, simply interchange t and u). Thus, in the decoupling limit, where $\cos^2(\beta-\alpha) \ll 1$, we see that double charged Higgs production is suppressed.

The above considerations are based on a tree-level analysis and are confirmed in Ref. 26. However, Ref. 26 goes on to argue that the cancelation illustrated in eq. (27) is significantly softened when radiative corrections are taken into account. In a first

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[g]This is unlikely in the MSSM except at unusual points of the parameter space where the Higgs boson decays dominantly to a pair of the lightest supersymmetric particles.

Table 1. $VV \rightarrow HH$ Processes

Fusion	Final States	Decoupling?
$W^- W^-$	$H^- H^-$	yes
$\gamma W^-, ZW^-$	$\left\{ \begin{array}{l} h^0 H^- \\ H^0 H^-, A^0 H^- \end{array} \right.$	yes no
$\gamma\gamma, ZZ$	$H^+ H^-$	no
ZZ	$\left\{ \begin{array}{l} h^0 A^0 \\ h^0 h^0, H^0 H^0, A^0 A^0 \end{array} \right.$	yes no

approximation, one can account for the radiative corrections by computing the radiatively corrected CP-even Higgs squared-mass matrix, and extracting the the corresponding CP-even Higgs mixing angle α, as discussed at the end of Section 1. The radiatively-corrected value of $\cos^2(\beta - \alpha)$ (shown in Fig. 1) would then be employed in eq. (27). This procedure would result in a slight enhancement of the $H^- H^-$ production cross section due to a slightly larger value for the radiatively-corrected $\cos^2(\beta - \alpha)$. However, the resulting cross section would still exhibit the decoupling limit suppression in apparent conflict with the results of Ref. 26. In this case, a better computation is needed, in which genuine radiative corrections to the full scattering amplitude are also calculated. It is possible that one could find new terms at one-loop which do not vanish in the decoupling limit.[h] Whether such terms lead to a significant enhancement of the $H^- H^-$ production amplitude remains to be checked.

Nevertheless, I believe that the behavior of the tree-level amplitudes in the decoupling limit does provide a rough guide to the likely size of the double Higgs production rates. It should be noted that the ZZ fusion processes can involve s-channel Higgs exchange; thus these processes are potentially sensitive to triple Higgs couplings. Finally, only tree-level $\gamma\gamma$ fusion processes are listed in Table 1. With the exception of a $\gamma\gamma$ collider, the $\gamma\gamma$ fusion processes through a loop to a pair of Higgs bosons is suppressed relative to the tree-level processes of Table 1.

For completeness, one can also consider Higgs boson production in association with vector bosons via gauge boson fusion (indicated symbolically by $VV \rightarrow VH$). Again, these processes are severely suppressed by the four-body phase space. Additional suppressions in the decoupling limit can also occur, as exhibited in Table 2.

Of course, if all the non-minimal Higgs states have mass of $\mathcal{O}(m_Z)$, indicating a substantial deviation from the decoupling limit, then the attendant factors of $\cos^2(\beta - \alpha)$ in the rates for HH and HV production will not result in a significant suppression. In this case, the "decoupling" columns of Tables 1 and 2 are no longer relevant. Moreover, the relatively light final state Higgs masses could allow for a study of HH and HV production at a very high energy $e^- e^-$ collider of sufficient

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[h]However, such terms cannot be logarithmically enhanced.

Table 2. $VV \rightarrow VH$ Processes

Fusion	Final States	Decoupling?
$W^- W^-$	$W^- H^-$	yes
γW^-	$\begin{cases} W^- h^0 \\ W^- H^0 \end{cases}$	no yes
$Z W^-$	$\begin{cases} W^- h^0 \\ W^- H^0, Z H^- \end{cases}$	no yes
ZZ	$\begin{cases} Z H^0, Z A^0 \\ Z h^0 \end{cases}$	yes no

luminosity. The computations of Ref. 26 indicate the possibility of cross sections as large as 0.1 fb for $\sqrt{s}$ in the range of 1 to 2 TeV. Luminosities of the $e^- e^-$ collider in excess of 10^{34} cm^{-2} sec^{-1} would be required.

For probing the MSSM Higgs sector, the $e^- e^-$ collider provides little advantage over an $e^+ e^-$ or $\gamma \gamma$ collider. Rates for Higgs production tend to be very small, and would require machines of the highest possible energy and luminosity. The true motivation for an $e^- e^-$ collider must lie in more exotic processes. Ref. 27 discusses the potential of an $e^- e^-$ collider to probe a variety of exotic Higgs sectors. It remains to be seen whether the simplest Higgs sectors of the Standard Model or the MSSM will be confirmed, or whether nature chooses a more unexpected path for the dynamics underlying electroweak symmetry breaking.

References

1. H.P. Nilles, *Phys. Rep.* **110** (1984) 1; R. Arnowitt and P. Nath, in *Particles and Fields*, Proceedings of the VII Jorge Andre Swieca Summer School, Sao Paulo, Brazil, 10–23 January, 1993, edited by J.P. Eboli and V.O. Rivelles (World Scientific, Singapore, 1994) pp. 3–63; S.P. Martin, hep-ph/9709356, to appear in *Perspectives on Supersymmetry*, edited by G.L. Kane (World Scientific, Singapore, 1998).
2. H.E. Haber and G.L. Kane, *Phys. Rep.* **117** (1985) 75; R. Barbieri, *Riv. Nuovo Cimento* **11** (1988) 1;
3. H.E. Haber, *Nucl. Phys. B (Proc. Suppl.)* **62A-C** (1998) 469; H.E. Haber, in *Recent Directions in Particle Theory*, Proceedings of the 1992 Theoretical Advanced Study Institute in Elementary Particle Physics, edited by J. Harvey and J. Polchinski (World Scientific, Singapore, 1993) pp. 589–686.
4. For a pedagogical introduction to the physics of Higgs bosons, see H.E. Haber, in *Testing the Standard Model*, Proceedings of the 1990 Theoretical Advanced Study Institute in Elementary Particle Physics, edited by M. Cvetič and Paul Langacker (World Scientific, Singapore, 1991) pp. 340–475.
5. H.E. Haber and R. Hempfling, *Phys. Rev. Lett.* **66** (1991) 1815; Y. Okada, M. Yamaguchi and T. Yanagida, *Prog. Theor. Phys.* **85** (1991) 1; J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett.* **B257** (1991) 83.
6. P. Chankowski, S. Pokorski and J. Rosiek, *Phys. Lett.* **B274** (1992) 191; *Nucl. Phys.* **B423** (1994) 437; A. Dabelstein, *Z. Phys.* **C67** (1995) 495; D.M. Pierce, J.A. Bagger, K. Matchev and R.-J. Zhang, *Nucl. Phys.* **B491** (1997) 3.

7. See, *e.g.*, H.E. Haber and R. Hempfling, *Phys. Rev.* **D48** (1993) 4280.

8. R. Hempfling and A.H. Hoang, *Phys. Lett.* **B331** (1994) 99; S. Heinemeyer, W. Hollik and G. Weiglein, Karlsruhe preprint KA-TP-2-1998 [hep-ph/9803277].

9. H.E. Haber, R. Hempfling, and A.H. Hoang, *Z. Phys.* **C75** (1997) 539.

10. M. Carena, J.R. Espinosa, M. Quiros and C.E.M. Wagner, *Phys. Lett.* **B355** (1995) 209; M. Carena, M. Quiros and C.E.M. Wagner, *Nucl. Phys.* **B461** (1996) 407.

11. J.M. Cornwall, D.N. Levin and G. Tiktopoulos, *Phys. Rev. Lett.* **30** (1973) 1268; *Phys. Rev.* **D10** (1974) 1145; C.H. Llewellyn Smith, *Phys. Lett.* **46B** (1973) 233; H.A. Weldon, *Phys. Rev.* **D30** (1984) 1547; J.F. Gunion, H.E. Haber and J. Wudka, *Phys. Rev.* **D43** (1991) 904.

12. J.F. Gunion, B. Grzadkowski, H.E. Haber and J. Kalinowski, *Phys. Rev. Lett.* **79** (1997) 982.

13. J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Redwood City, CA, 1990).

14. J.F. Gunion and H.E. Haber, *Nucl. Phys.* **B272** (1986) 1; **B278** (1986) 449 [E: **B402** (1993) 567].

15. M.A. Diaz, preprint IFIC/97-27 [hep-ph/9705471].

16. H.E. Haber and Y. Nir, *Nucl. Phys.* **B335** (1990) 363.

17. H.E. Haber, in *Beyond the Standard Model IV*, Proceedings of the Fourth International Conference on Physics Beyond the Standard Model, Granlibakken, Lake Tahoe, CA, 13–18 December, 1994, edited by J.F. Gunion, T. Han and J. Ohnemus (World Scientific, Singapore, 1995) pp. 151–163; and in *Perspectives for Electroweak Interactions in e^+e^- Collisions*, Proceedings of the Ringberg Workshop, Ringberg Castle, Tegernsee, Germany, 5–8 February, 1995, edited by B.A. Kniehl (World Scientific, Singapore, 1995) pp. 219–231.

18. H.E. Haber *et al.*, summary report of the Weakly-Coupled Higgs Boson and Precision Electroweak Physics Working Group, in *New Directions for High-Energy Physics*, Proceedings of the 1996 DPF/DPB Summer Study on High-Energy Physics, June 25—July 12, 1996, Snowmass, CO, edited by D.G. Cassel, L.T. Gennari, and R.H. Siemann (Stanford Linear Accelerator Center, 1997) pp. 482–498.

19. J.F. Gunion, L. Poggioli, R. Van Kooten, C. Kao and P. Rowson, in *New Directions for High-Energy Physics*, Proceedings of the 1996 DPF/DPB Summer Study on High-Energy Physics, June 25—July 12, 1996, Snowmass, CO, edited by D.G. Cassel, L.T. Gennari, and R.H. Siemann (Stanford Linear Accelerator Center, 1997) pp. 541–587.

20. J.F. Gunion, A. Stange and S. Willenbrock, in *Electroweak Symmetry Breaking and New Physics at the TeV Scale*, edited by T.L. Barklow, S. Dawson, H.E. Haber, and J.L. Siegrist (World Scientific, Singapore, 1996) pp. 23–145.

21. J.F. Gunion and H.E. Haber, *Phys. Rev.* **D48**, (1993) 5109; D.L. Borden, D.A. Bauer and D.O. Caldwell, *Phys. Rev.* **D48** (1993) 4018.

22. J.F. Gunion *et al.*, *Phys. Rev.* **D38** (1988) 3444.

23. T. Han, *Int. J. Mod. Phys.* **A11** (1996) 1541.

24. P. Minkowski, in *New Directions for High-Energy Physics*, Proceedings of the 1996 DPF/DPB Summer Study on High-Energy Physics, June 25—July 12, 1996, Snowmass, CO, edited by D.G. Cassel, L.T. Gennari, and R.H. Siemann (Stanford Linear Accelerator Center, 1997) pp. 614–618.

25. A. Djouadi, H.E. Haber and P.M. Zerwas, *Phys. Lett.* **B375** (1996) 203; F. Boudjema and E. Chopin, *Z. Phys.* **C73** (1996) 85.

26. T. Rizzo, *Int. J. Mod. Phys.* **A11** (1996) 1563.

27. J.F. Gunion, U.C. Davis preprint UCD-98-6 [hep-ph/9803222], in these Proceedings.