

SCIPP 99/06
January, 1999
hep-ph/9901365

How Well Can We Predict the Mass of the Higgs Boson of the Minimal Supersymmetric Model?

Howard E. Haber

*Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, CA 95064, U.S.A.*

Abstract

The upper bound on the mass of the light CP-even Higgs boson of the minimal supersymmetric model (MSSM) depends on the supersymmetric particle spectrum via radiative loop effects. At present, complete one-loop results and partial two-loop results are known. Some simple analytic approximations have been obtained which are quite accurate over a large portion of the MSSM parameter space. Based on these results, I examine how accurately one can predict the upper bound on the mass of the lightest MSSM Higgs boson.

To appear in the Proceedings of the
4th International Symposium on Radiative Corrections (RADCOR 98):
Applications of Quantum Field Theory to Phenomenology,
Universitat Autònoma de Barcelona,
Barcelona, Catalonia, Spain, 8-12 September 1998.

HOW WELL CAN WE PREDICT THE MASS OF THE HIGGS BOSON OF THE MINIMAL SUPERSYMMETRIC MODEL?

HOWARD E. HABER

*Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, CA 95064 USA*

The upper bound on the mass of the light CP-even Higgs boson of the minimal supersymmetric model (MSSM) depends on the supersymmetric particle spectrum via radiative loop effects. At present, complete one-loop results and partial two-loop results are known. Some simple analytic approximations have been obtained which are quite accurate over a large portion of the MSSM parameter space. Based on these results, I examine how accurately one can predict the upper bound on the mass of the lightest MSSM Higgs boson.

1 Introduction

Low-energy supersymmetry [1] provides the most compelling framework for electroweak physics, in which the electroweak symmetry breaking is generated via the dynamics of an elementary scalar Higgs sector. The scalar boson masses are kept light (of order the electroweak symmetry breaking scale) due to an approximate supersymmetry in nature. Supersymmetry is broken at the TeV scale or below, and this information is transmitted to the scalar sector, thereby generating electroweak symmetry breaking dynamics at the proper scale.

The simplest model of low-energy supersymmetry is the minimal supersymmetric extension of the Standard Model (MSSM). In this model, the Higgs sector consists of eight degrees of freedom made up from two complex weak scalar doublets of hypercharge ± 1 respectively [2]. Supersymmetry requires that the hypercharge -1 [$+1$] Higgs doublets couple exclusively to down-type [up-type] fermions, respectively. After minimizing the Higgs potential, the neutral components of the Higgs doublets acquire vacuum expectation values (vevs) with $\langle H_i \rangle = v_i/\sqrt{2}$. The model possesses five physical Higgs bosons: two CP-even scalars, h^0 and H^0 (with $m_{h^0} < m_{H^0}$), a CP-odd Higgs scalar A^0 and a charged Higgs pair H^\pm . As usual, I define $\tan \beta \equiv v_2/v_1$ and normalize $v^2 \equiv v_1^2 + v_2^2 \equiv 4m_W^2/g^2 = (246 \text{ GeV})^2$. Due to the form of the Higgs-fermion interaction, the third generation quark masses are given by $m_b = h_b v_1/\sqrt{2}$ and $m_t = h_t v_2/\sqrt{2}$, where h_q ($q = t, b$) are the corresponding Yukawa couplings.

The tree-level physical Higgs spectrum is easily computed [2]. Its most noteworthy feature is the upper bound on the light CP-even Higgs scalar: $m_{h^0} \leq m_Z |\cos 2\beta| \leq m_Z$. The maximum tree-level upper bound of m_Z is saturated when one of the vevs vanishes (and $m_{A^0} > m_Z$). It is convenient to

consider a limiting case of the MSSM Higgs sector where $v_1 = 0$. For finite h_b , this limit corresponds to $m_b = 0$, which is a reasonable approximation.^a In the $v_1 = 0$ model, the Higgs sector degenerates to a one-doublet model with:

$$V_{\text{Higgs}} = m^2 \Phi^\dagger \Phi + \frac{1}{2} \lambda (\Phi^\dagger \Phi)^2, \quad \lambda \equiv \frac{1}{4}(g^2 + g'^2). \quad (1)$$

The supersymmetric constraint on the value of λ is a consequence of the fact that the MSSM Higgs quartic couplings originate from the D -term contributions to the scalar potential. The squared-mass of the light CP-even Higgs boson of the $v_1 = 0$ model is given by $m_{h^0}^2 = \lambda v^2 = m_Z^2$.

2 Upper bound of m_{h^0} in the MSSM

The upper bound of $m_{h^0} \leq m_Z$ will be modified by radiative corrections [3,4]. In order to obtain the radiatively corrected upper bound of m_{h^0} , it suffices to compute radiative corrections in the $v_1 = 0$ model. Let us focus on the real part of the neutral scalar component: $\Phi^0 = (v + h)/\sqrt{2}$. The bare Higgs potential takes the following form: $V_{\text{Higgs}} = t_0 h + \frac{1}{2}(m_{h^0}^2)_0 h^2 + \mathcal{O}(h^3)$, where

$$t_0 = v_0 \left[\frac{1}{2} \lambda_0 v_0^2 + m_0^2 \right], \quad (m_{h^0}^2)_0 = m_0^2 + \frac{3}{2} \lambda_0 v_0^2, \quad (2)$$

and the subscript 0 indicates bare parameters. We also introduce

$$(m_Z^2)_0 = \frac{1}{4}(g_0^2 + g_0'^2)v_0^2. \quad (3)$$

The on-shell renormalization scheme is defined such that m_Z and m_{h^0} are physical masses corresponding to zeros of the corresponding inverse propagators. Let the sum of all one-loop (and higher) Feynman graphs contributing to the Z and h^0 two-point functions be denoted by $iA_{ZZ}(q^2)g^{\mu\nu} + iB_{ZZ}(q^2)q^\mu q^\nu$ and $-iA_{hh}(q^2)$, respectively, where q is the four-momentum of one of the external legs. The physical masses are given by:

$$m_Z^2 = (m_Z^2)_0 + \text{Re } A_{ZZ}(m_Z^2), \quad (4)$$

$$m_{h^0}^2 = (m_{h^0}^2)_0 + \text{Re } A_{hh}(m_{h^0}^2). \quad (5)$$

Since v is the vev of the scalar field at the true minimum of the potential, we require that the sum of all tadpoles must vanish. That is,

$$t_0 + A_h(0) = 0, \quad (6)$$

^aIn practice, it is sufficient to take $v_1 \ll v_2$, and then fix the value of h_b to be consistent with the observed b -quark mass.

where $-iA_h(0)$ is the sum of all one-loop (and higher) Feynman graphs contributing to the h^0 one-point function. Combining eqs. (2)–(6), we obtain

$$m_{h^0}^2 = m_Z^2 + \text{Re} [A_{hh}(m_Z^2) - A_{ZZ}(m_Z^2)] - \frac{A_h(0)}{v} + [\lambda_0 - \frac{1}{4}(g_0^2 + g_0'^2)] v_0^2. \quad (7)$$

This result is accurate at one-loop order, since we have put $m_{h^0} = m_Z$ and $v_0 = v$ on the right hand side where possible.

Naively, one might argue that eq. (7) can be simplified by using the supersymmetric condition $\lambda_0 = \frac{1}{4}(g_0^2 + g_0'^2)$. However, this is correct only if a regularization scheme that preserves supersymmetry is employed. Of course, the physical quantity $m_{h^0}^2$ must be independent of scheme. Consider two different regularization schemes: dimensional regularization (DREG) and dimensional reduction [5] (DRED). Renormalized couplings defined via (modified) minimal subtraction in these two schemes are called $\overline{\text{MS}}$ and $\overline{\text{DR}}$ couplings, respectively. DREG does not preserve supersymmetry because the number of gauge and gaugino degrees of freedom does not match in $n \neq 4$ dimensions. In contrast, DRED preserves supersymmetry (at least at one and two-loop order). In DRED [DREG], bare quantities will be denoted with the subscript D [G]. Then, the supersymmetric condition holds in DRED:

$$\lambda_D - \frac{1}{4}(g_D^2 + g_D'^2) = 0. \quad (8)$$

We now demonstrate that the above relation is violated in DREG. First, the gauge couplings of the two schemes are related as follows [6]

$$g_D^2 = g_G^2 + \frac{g^4}{24\pi^2}, \quad g_D'^2 = g_G'^2. \quad (9)$$

For the Higgs self-coupling λ , the relation between the two schemes is derived by considering the one-loop effective potential (in the Landau gauge), $V \equiv V^{(0)} + V^{(1)}$, where $V^{(0)}$ is the tree-level scalar potential and $V^{(1)}$ is given by:

$$V^{(1)} = -\frac{1}{64\pi^2} \text{Str} \mathcal{M}^4(\Phi) \left[\Delta + K - \ln \frac{\mathcal{M}^2(\Phi)}{\mu^2} \right]. \quad (10)$$

In eq. (10), K is a scheme-dependent constant (see below), $\mathcal{M}^2(\Phi)$ denotes the squared-mass matrix as a function of the scalar Higgs fields (*i.e.*, the corresponding tree-level squared-mass matrices are obtained when Φ is replaced by its vev), and the divergences that arise in the computation of the one-loop integrals in $4 - 2\epsilon$ dimensions appear in the factor $\Delta \equiv 1/\epsilon - \gamma_E + \ln(4\pi)$ [where γ_E is the Euler constant]. We have also employed the notation

$$\text{Str} \{ \dots \} \equiv \sum_i C_i (2J_i + 1) (-1)^{2J_i + 1} \{ \dots \}_i, \quad (11)$$

where the sum is taken over the corresponding mass matrix eigenvalues, including a factor C_i which counts internal degrees of freedom (*e.g.*, charge and color) for all particles of spin J_i that couple to the Higgs bosons.

In DRED, $K = 3/2$, independent of particle i in the sum [eq. (11)]. The fact that particles of different spin yield the same constant K is an indication that DRED preserves supersymmetry at one-loop. In DREG, $K = 3/2$ for spin 0 and spin-1/2 particles, while $K = 5/6$ for spin-1 particles. However, the effective potential (expressed in terms of bare parameters) must be independent of scheme. Comparing the DREG and DRED computations, it follows that

$$\frac{1}{8}\lambda_D v^4 - \frac{1}{64\pi^2} \left(\frac{3}{2}\right) (6m_W^4 + 3m_Z^4) = \frac{1}{8}\lambda_G v^4 - \frac{1}{64\pi^2} \left(\frac{5}{6}\right) (6m_W^4 + 3m_Z^4), \quad (12)$$

which yields

$$\lambda_D = \lambda_G + \frac{g^4(m_Z^4 + 2m_W^4)}{64\pi^2 m_W^4}. \quad (13)$$

Combining the results of eqs. (9) and (13) gives the DREG result

$$\lambda_G - \frac{1}{4}(g_G^2 + g_G'^2) = -\frac{g^4}{64\pi^2 m_W^4} (m_Z^4 + \frac{4}{3}m_W^4). \quad (14)$$

Thus, in computing the one-loop corrected Higgs mass [eq. (7)] in DREG [DRED], one must use the relation between λ_0 and $\frac{1}{4}(g_0^2 + g_0'^2)$ given by eq. (14) [eq. (8)]. One can check that this difference is precisely compensated by the difference in DREG and DRED that arises in the computation of the vector boson loop contributions to the one-point and two-point functions. Henceforth, we shall always use the DRED scheme, in which case [3]

$$m_{h^0}^2 = m_Z^2 + \text{Re} [A_{hh}(m_Z^2) - A_{ZZ}(m_Z^2)] - \frac{A_h(0)}{v}. \quad (15)$$

Although the loop functions above are individually divergent, all divergences precisely cancel in the sum and yield a well-defined one-loop result for m_{h^0} .

The method described above [resulting in eq. (15)] is sometimes called the diagrammatic method [3,7] since one explicitly evaluates the one-point and two-point functions by standard Feynman diagram techniques. A second method for computing m_{h^0} , called the effective potential technique, is often employed in the literature [4,8,9]. This is not an alternate “scheme”, but simply another way of organizing the calculation. Consider the DRED one-loop effective potential introduced above (with $K = 3/2$). In the sum $V = V^{(0)} + V^{(1)}$, the $\overline{\text{DR}}$ scheme consists of absorbing the factor of Δ into the bare parameters (m_0 , λ_0 and Φ_0), which converts them into $\overline{\text{DR}}$ parameters.

Renormalized quantities (such as the effective potential or the n -point Green functions) will be denoted with tildes in the following. These are computed in the Landau gauge; the divergent piece Δ is removed by $\overline{\text{DR}}$ subtraction and the bare parameters are replaced by renormalized $\overline{\text{DR}}$ parameters. Finally, the $\overline{\text{DR}}$ parameters are related to physical parameters.

We proceed as follows. First, we minimize the renormalized effective potential by setting the first derivative equal to zero. This condition yields:

$$\left[\frac{1}{2}\lambda v^2 + m^2\right]v + \tilde{A}_h(0) = 0. \quad (16)$$

In eq. (16), the first term on the left hand side arises at tree-level, while the second term is a consequence of the fact that the n th derivative of $V^{(1)}$, evaluated at the potential minimum, is equal to the scalar n -point function evaluated at zero external momentum [10]. The second derivative of the effective potential, denoted by $(m_{h^0}^2)_{\text{eff}}$, is similarly given by:

$$(m_{h^0}^2)_{\text{eff}} = m^2 + \frac{3}{2}\lambda v^2 + \tilde{A}_{hh}(0). \quad (17)$$

We may use the DRED relation [eq. (8)], which is also satisfied by the renormalized $\overline{\text{DR}}$ parameters, to eliminate λ . The $\overline{\text{DR}}$ Z -mass parameter is given by $(m_Z^2)_{\overline{\text{DR}}} = \frac{1}{4}(g^2 + g'^2)v^2$. Combining the above results yields:

$$(m_{h^0}^2)_{\text{eff}} = (m_Z^2)_{\overline{\text{DR}}} + \tilde{A}_{hh}(0) - \frac{\tilde{A}_h(0)}{v}. \quad (18)$$

In the literature, $(m_{h^0}^2)_{\text{eff}}$ is sometimes used as the approximation to the one-loop-improved Higgs squared-mass. However, this is *not* a physical parameter, since it depends on an arbitrary scale that is introduced in the $\overline{\text{DR}}$ subtraction scheme. To obtain an expression for the physical mass, which corresponds to the zero of inverse propagator, we note that $(m_{h^0}^2)_{\text{eff}}$ has been computed using the two-point function evaluated at *zero* external momentum. Thus, the physical Higgs squared-mass is given by:

$$m_{h^0}^2 = (m_{h^0}^2)_{\text{eff}} + \text{Re } \tilde{A}_{hh}(m_{h^0}^2) - \tilde{A}_{hh}(0). \quad (19)$$

Likewise, we must convert from $(m_Z^2)_{\overline{\text{DR}}}$ to the physical Z squared-mass. This is accomplished using a result analogous to that of eq. (4), which guarantees that m_Z corresponds to the zero of the inverse Z propagator:

$$m_Z^2 = (m_Z^2)_{\overline{\text{DR}}} + \text{Re } \tilde{A}_{ZZ}(m_Z^2). \quad (20)$$

Combining eqs. (18)–(20), we end up with

$$m_{h^0}^2 = m_Z^2 + \text{Re } \left[\tilde{A}_{hh}(m_Z^2) - \tilde{A}_{ZZ}(m_Z^2) \right] - \frac{\tilde{A}_h(0)}{v}. \quad (21)$$

Not surprisingly, we have reproduced the diagrammatic result [eq. (15)].

3 Leading Logarithms and Renormalization Group Improvement

When the loop functions in eq. (15) are computed, one finds that the most significant contributions grow logarithmically with the top squark masses. (Terms that are logarithmically sensitive to other supersymmetric particle masses also exist.) Over a large range of supersymmetric parameter space, the radiatively corrected Higgs mass can be well approximated by just a few terms. On the other hand, if the logarithms become too large, then the validity of the perturbation theory becomes suspect. However, in this case the leading logarithms can be resummed using renormalization group (RG) techniques [11,12].

We begin with a one-loop analysis. Consider an effective field theory approach [12], and assume for simplicity that supersymmetry breaking is characterized by one mass scale, M_{SUSY} , which is assumed to be large compared with m_Z . At scales $\mu \leq M_{\text{SUSY}}$, the Higgs potential takes the form:

$$V = \frac{1}{2}m^2(\mu)[h(\mu)]^2 + \frac{1}{8}\lambda(\mu)[h(\mu)]^4. \quad (22)$$

Letting $h \rightarrow h + v$ with $m^2(\mu) < 0$, the Higgs mass is given by

$$m_{h^0}^2(\mu) = \lambda(\mu)v^2(\mu). \quad (23)$$

Since the effective theory is supersymmetric only for $\mu \geq M_{\text{SUSY}}$, we impose the supersymmetric boundary condition [see eq. (8)]:

$$\lambda(M_{\text{SUSY}}) = \frac{1}{4} [g^2(M_{\text{SUSY}}) + g'^2(M_{\text{SUSY}})]. \quad (24)$$

Scale dependent parameters satisfy renormalization group equations (RGEs). For $\mu < M_{\text{SUSY}}$, the Standard Model RGEs are relevant:

$$\begin{aligned} \beta_\lambda &\equiv \frac{d\lambda}{d\ln\mu^2} = \frac{1}{16\pi^2} \left[6\lambda^2 + \frac{3}{8}[2g^4 + (g^2 + g'^2)^2] - 2 \sum_f N_{cf} h_f^4 \right] - 2\lambda\gamma_v, \\ \gamma_v &\equiv \frac{d\ln v^2}{d\ln\mu^2} = \frac{1}{16\pi^2} \left[\frac{9}{4}g^2 + \frac{3}{4}g'^2 - \sum_f N_{cf} h_f^2 \right], \\ \beta_{g^2+g'^2} &\equiv \frac{d(g^2 + g'^2)}{d\ln\mu^2} = \frac{1}{96\pi^2} \left[(8N_g - 43)g^4 + \left(\frac{40}{3}N_g + 1\right)g'^4 \right], \end{aligned} \quad (25)$$

where $h_f = \sqrt{2}m_f/v$, $N_g = 3$ is the number of fermion generations, and $N_{cf} = 3$ [1] when f runs over quark [lepton] indices.

It is instructive to solve the RGEs iteratively to one-loop, by ignoring the μ dependence on the right hand sides in eq. (25). Incorporating the boundary

condition [eq. (24)], the solution for $\lambda(m_Z)$ is given by^b

$$\begin{aligned}\lambda(m_Z) &= \frac{1}{4}(g^2 + g'^2)(M_{\text{SUSY}}) - \beta_\lambda \ln \left(\frac{M_{\text{SUSY}}^2}{m_Z^2} \right) \\ &= \frac{1}{4}(g^2 + g'^2)(m_Z) + \left(\frac{1}{4}\beta_{g^2+g'^2} - \beta_\lambda \right) \ln \left(\frac{M_{\text{SUSY}}^2}{m_Z^2} \right).\end{aligned}\quad (26)$$

Finally, using eq. (23), we identify the physical Higgs mass by evaluating $m_{h^0}^2(\mu)$ at $\mu = m_Z$ and taking $v(m_Z) = 246$ GeV. We know from the previous section that this is not strictly correct. However, at the one-loop leading logarithmic level, this procedure is accurate, and we end up with:

$$(m_{h^0}^2)_{\text{1LL}} = m_Z^2 + \left(\frac{1}{4}\beta_{g^2+g'^2} - \beta_\lambda \right) v^2 \ln \left(\frac{M_{\text{SUSY}}^2}{m_Z^2} \right), \quad (27)$$

where the subscript 1LL indicates that the result is only accurate to one-loop leading logarithmic order. To obtain the full one-loop leading logarithmic expression, simply insert the results of eq. (25) into eq. (27) [in β_λ one can consistently set $\lambda = \frac{1}{4}(g^2 + g'^2)$]. We have checked [3,12] that the above result matches precisely with the diagrammatic computation [eq. (15)] in the limit of $M_{\text{SUSY}} \gg m_Z$, where M_{SUSY} characterizes the scale of supersymmetric particle masses (taken to be roughly degenerate). The dominant term at one-loop is proportional to m_t^4 and arises from the piece of β_λ proportional to h_t^4 . Inserting $\beta_\lambda = -3h_t^4/8\pi^2$ with $h_t = \sqrt{2}m_t/v$ into eq. (27), one obtains^c

$$(m_{h^0}^2)_{\text{1LT}} = m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \ln \left(\frac{M_{\text{SUSY}}^2}{m_t^2} \right). \quad (28)$$

The subscript 1LT indicates that this is the leading m_t^4 piece of $(m_{h^0}^2)_{\text{1LL}}$. However, the additional terms in $(m_{h^0}^2)_{\text{1LL}}$ are numerically significant as we shall show at the end of this section.

Thus, we see that given the RG functions, no additional diagrammatic computations are needed to extract the full one-loop leading logarithmic contribution to the Higgs mass. Thus the RG-approach provides a useful short cut for extracting the leading one-loop contributions to the Higgs mass. Of course, if the leading logarithms are large, then they should be resummed to all

^bOne subtlety consists of the proper way to run down from m_t to m_Z , since below $\mu = m_t$, the electroweak symmetry is broken. I will ignore this subtlety here although it can be addressed; see ref. [12].

^cThe lower scale of the logarithm in this case is m_t^2 (and not m_Z^2) since this term arises from the incomplete cancelation of the top quark and top squark loops.

orders. This is accomplished by computing the RG-improvement of the exact one-loop result as follows. Let $(m_{h^0}^2)_{1\text{RG}} \equiv \lambda(m_Z)v^2(m_Z)$, where $\lambda(m_Z)$ is obtained by *numerically* solving the one-loop RGEs. Write the exact one-loop result as: $m_{h^0}^2 = (m_{h^0}^2)_{1\text{LL}} + (m_{h^0}^2)_{1\text{NL}}$, where $(m_{h^0}^2)_{1\text{NL}}$ is the result obtained by subtracting the one-loop leading logarithmic contribution from the exact one-loop result. Clearly, this piece contains no term that grows logarithmically with M_{SUSY} . Then the complete one-loop RG-improved result is given by $m_{h^0}^2 = (m_{h^0}^2)_{1\text{RG}} + (m_{h^0}^2)_{1\text{NL}}$.

The RG technique can be extended to two loops as follows [13]. For simplicity, we focus on the leading corrections, which depend on $\alpha_t \equiv h_t^2/4\pi$ and $\alpha_s \equiv g_s^2/4\pi$, *i.e.*, we work in the approximation of $h_b = g = g' = 0$ and $\lambda \ll h_t$. (All two-loop results quoted in this section are based on this approximation.) The dependence on the strong coupling constant is a new feature of the two-loop analysis. We now solve the one-loop RGEs by iterating twice to two loops. In the second iteration, we need the RGE for h_t^2 , which in the above approximation is given by

$$\beta_{h_t^2} \equiv \frac{d}{d \ln \mu^2} h_t^2 = \frac{1}{16\pi^2} \left[\frac{9}{2} h_t^2 - 8g_s^2 \right] h_t^2. \quad (29)$$

This iteration produces the two-loop leading double logarithm [9], and yields

$$\lambda(m_t) = \frac{1}{4}(g^2 + g'^2) - \frac{3h_t^4(m_t)}{8\pi^2} \ln \left(\frac{M_{\text{SUSY}}^2}{m_t^2} \right) \left[1 + \left(\gamma_v + \frac{\beta_{h_t^2}}{h_t^2} \right) \ln \left(\frac{M_{\text{SUSY}}^2}{m_t^2} \right) \right]. \quad (30)$$

Next, we must incorporate the sub-dominant two loop effects. Only three modifications of our one-loop analysis are required (in the limit of $h_b = g = g' = 0$ and $\lambda \ll h_t$). First, we need only the h_t and g_s dependent parts of the two loop contribution to β_λ . That is, β_λ is modified as follows [14]

$$\beta_\lambda \longrightarrow \beta_\lambda + \frac{1}{(16\pi^2)^2} \left[30h_t^6 - 32h_t^4 g_s^2 \right]. \quad (31)$$

Including this into the iterative solution of the RGEs adds a two-loop singly logarithmic term to the result of eq. (30). Second, we must distinguish between the Higgs pole mass (denoted by m_{h^0} with no argument) and the running Higgs mass evaluated at m_t . Using the results of Sirlin and Zucchini [15],

$$m_{h^0}^2 = \frac{4m_W^2 \lambda(m_t)}{g^2} \left[1 + \frac{1}{8} \left(\frac{\alpha_t}{\pi} \right) \right]. \quad (32)$$

Third, we make use of the relation between $v^2(m_t)$ and $v^2 \equiv 4m_W^2/g^2$,

$$v^2(m_t) = \frac{4m_W^2}{g^2} \left[1 - \frac{3}{8} \left(\frac{\alpha_t}{\pi} \right) \right]. \quad (33)$$

Using the above results, we end up with

$$m_{h^0}^2 = m_Z^2 + \frac{3g^2}{8\pi^2 m_W^2} m_t^4(m_t) \ln\left(\frac{M_{\text{SUSY}}^2}{m_t^2}\right) \left[1 + \left(\gamma_v + \frac{\beta_{h_t^2}}{h_t^2} \right) \ln\left(\frac{M_{\text{SUSY}}^2}{m_t^2}\right) + \frac{4}{3} \left(\frac{\alpha_s}{\pi} \right) - \frac{3}{8} \left(\frac{\alpha_t}{\pi} \right) \right], \quad (34)$$

where $h_t \equiv h_t(m_t)$ and $m_t(m_t) \equiv h_t(m_t) v(m_t)/\sqrt{2}$. Numerically, the two-loop singly logarithmic piece of eq. (34) contributes about 3% relative to the one-loop leading logarithmic contribution.

Let us compare this result with the two-loop diagrammatic computation of ref. [16]. In order to make this comparison, we must express eq. (34) in terms of the top quark pole mass, m_t . The relation between m_t and the running top-quark mass is given by [17,18]

$$m_t = m_t(m_t) \left[1 + \frac{4}{3} \left(\frac{\alpha_s}{\pi} \right) - \frac{1}{2} \left(\frac{\alpha_t}{\pi} \right) \right], \quad (35)$$

where $m_t(m_t)$ is the $\overline{\text{MS}}$ running top-quark mass evaluated at m_t .^d Inserting the above result into eq. (34) yields:

$$m_{h^0}^2 = m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \ln\left(\frac{M_{\text{SUSY}}^2}{m_t^2}\right) \left[1 + \left(\gamma_v + \frac{\beta_{h_t^2}}{h_t^2} \right) \ln\left(\frac{M_{\text{SUSY}}^2}{m_t^2}\right) - \left(\frac{4\alpha_s}{\pi} \right) + \frac{13}{8} \left(\frac{\alpha_t}{\pi} \right) \right]. \quad (36)$$

This result matches precisely the one obtained in ref. [16] in the limit of $M_{\text{SUSY}} \gg m_Z$. Note that the numerical contribution of the two-loop singly-logarithmic contribution in eq. (36) is about 10% of the corresponding one-loop contribution. Clearly, the use of the running top quark mass [as in eq. (34)] results in a slightly better behaved perturbation expansion.

Finally, we can employ a very useful trick to make our results above even more compact. The two-loop doubly-logarithmic contribution can be absorbed into the one-loop leading-logarithmic contribution by an appropriate choice of scale for the running top-quark mass. Specifically, using the iterative one-loop leading-logarithmic solution to the RGEs for h_t and v yields

$$m_t(\mu) = \frac{1}{\sqrt{2}} h_t(\mu) v(\mu) = m_t(m_t) \left[1 - \left(\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi} \right) \ln\left(\frac{\mu^2}{m_t^2}\right) \right]. \quad (37)$$

^dWe caution the reader that ref. [18] defines $m_t(m_t) = h_t(m_t) v/\sqrt{2}$, which differs slightly from the definition of $m_t(m_t)$ used here.

If we choose the scale $\mu_t \equiv \sqrt{m_t M_{\text{SUSY}}}$ to evaluate the running top-quark mass in eq. (34), we end up with:

$$m_{h^0}^2 = m_Z^2 + \frac{3g^2}{8\pi^2 m_W^2} m_t^4(\mu_t) \ln\left(\frac{M_{\text{SUSY}}^2}{m_t^2(\mu_t)}\right) \left[1 + \frac{1}{3}\left(\frac{\alpha_s}{\pi}\right) - \frac{3}{16}\left(\frac{\alpha_t}{\pi}\right)\right]. \quad (38)$$

One can check that the sum of the terms in the brackets deviates from one by less than 1%. Thus, in practice, the two-loop singly-logarithmic contribution can now be neglected since it is numerically insignificant. That is, one can incorporate the leading two-loop contributions by simply inserting the running top-quark mass evaluated at $\mu_t \equiv \sqrt{m_t M_{\text{SUSY}}}$ into the one-loop leading-logarithmic expression for $m_{h^0}^2$.

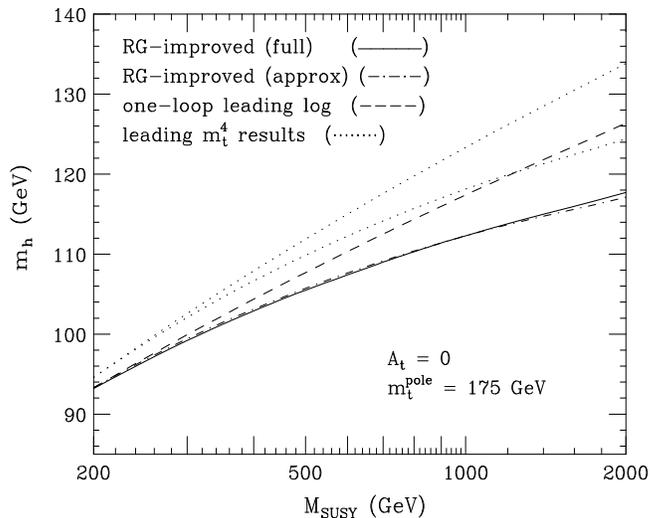


Figure 1: The upper bound to the mass of the light CP-even Higgs boson of the MSSM is plotted as a function of the common supersymmetric mass M_{SUSY} (in the absence of squark mixing). The one-loop leading logarithmic result [dashed line] is compared with the RG-improved result, which was obtained by a numerical computation [solid line] and by the simple recipe described in the text [dot-dashed line]. Also shown are the leading m_t^4 result of eq. (28) [higher dotted line], and its RG-improvement [lower dotted line]. The running top quark mass used in our numerical computations is $m_t(m_t) = 166.5 \text{ GeV}$.

Fig. 1 illustrates the results of this section. We display the results for m_{h^0} based on five different expressions for the light CP-even Higgs mass. Case (i) corresponds to the one-loop leading m_t^4 result, $(m_{h^0}^2)_{\text{1LT}}$ [eq. (28)]. In case (ii) we exhibit the full one-loop leading logarithmic expression, $(m_{h^0}^2)_{\text{1LL}}$ [eq. (27)].

In case (iii), we consider $(m_{h^0}^2)_{\text{IRG}}$ obtained by solving the one-loop RGEs numerically. Finally, case (iv) corresponds to the simple recipe proposed above, in which we evaluate $(m_{h^0}^2)_{\text{ILL}}$ by setting m_t to the running top quark mass at the scale μ_t . For completeness, we also include case (v), where we apply the same recipe to $(m_{h^0}^2)_{\text{ILT}}$.

The following general features are noteworthy. First, we observe that over the region of M_{SUSY} shown, $(m_{h^0})_{\text{IRG}} \simeq (m_{h^0})_{\text{ILL}}(m_t(\mu_t))$. Second, the difference between $(m_{h^0})_{\text{ILL}}$ and $(m_{h^0})_{\text{IRG}}$ is non-negligible for even moderate values of M_{SUSY} ; neglecting RG-improvement can lead to an overestimate of m_{h^0} which can be as large as 10 GeV (for $M_{\text{SUSY}} > 2$ TeV, the deviation grows even larger). Finally, note that although the simplest approximation, $(m_{h^0})_{\text{ILT}}$, reflects the dominant source of radiative corrections, it yields the largest overestimate of the light Higgs boson mass.

4 Additional Complications: Supersymmetric Thresholds

In the analysis of the previous section, we assumed that all supersymmetric particle masses were roughly equal and substantially larger than m_Z . To account for a non-degenerate supersymmetric spectrum, we must recompute the RGEs in steps starting from $\mu = M_{\text{SUSY}}$ and ending at m_Z . Every time the threshold of a supersymmetric particle is passed, we integrate it out of the theory, and determine a new set of RGEs for the new effective theory. Eventually, when we pass below the lightest supersymmetric threshold, we regain the RGEs of the Standard Model given in eq. (25). We can solve iteratively for $\lambda(m_Z)$ as we did in the previous section, but now using the more complicated set of RGEs. Explicit formulae can be found in refs. [12] and [13].

However, the above procedure fails to incorporate the effects of squark mixing. Since the most important contribution to the Higgs mass radiative corrections arises from the incomplete cancelation of the top quark and top squark loops, it is important to examine this sector more closely. First, we define our notation. The physical top squark squared-masses (in the $v_1 = 0$ model) are eigenvalues of the following two 2×2 matrix

$$\begin{pmatrix} M_Q^2 + m_t^2 - m_Z^2(\frac{1}{2} - e_t \sin^2 \theta_W) & m_t A_t \\ m_t A_t & M_U^2 + m_t^2 - m_Z^2 e_t \sin^2 \theta_W \end{pmatrix} \quad (39)$$

where $e_t = 2/3$ and M_Q, M_U, A_t are soft-supersymmetry-breaking parameters.

We shall treat the squark mixing perturbatively, assuming that the off-diagonal squark squared-masses are small compared to the diagonal terms.^e

^eFormally, we assume that $(M_1^2 - M_2^2)/(M_1^2 + M_2^2) \ll 1$, where M_1^2, M_2^2 are the top squark squared-masses. Thus, we demand that $m_t A_t / M_{\text{SUSY}}^2 \ll 1$.

The perturbative effect of squark mixing is to modify the supersymmetric relation between the Higgs quartic coupling and the gauge couplings [eq. (24)]. Such modifications arise from one loop squark corrections to the Higgs quartic self-coupling via: (i) corrections to the scalar two-point function on the external legs; (ii) triangle graphs involving two trilinear Higgs-squark-squark interactions and one quartic Higgs-Higgs-squark-squark interaction; and (iii) box graphs involving four trilinear Higgs-squark-squark interactions [19]. Then, eq. (24) is modified to:

$$\lambda(M_{\text{SUSY}}) = \frac{1}{4}(g^2 + g'^2) + \delta\lambda_2 + \delta\lambda_3 + \delta\lambda_4, \quad (40)$$

where the $\delta\lambda_i$ arise from the three sources quoted above. Explicitly,

$$\begin{aligned} \delta\lambda_2 &= \frac{-3(g^2 + g'^2)}{32\pi^2} A_t^2 h_t^2 B(M_Q^2, M_U^2), \\ \delta\lambda_3 &= \frac{3}{32\pi^2} \left[4h_t^4 A_t^2 h(M_Q^2, M_U^2) + g^2 h_t^2 A_t^2 p_t(M_Q^2, M_U^2) \right], \\ \delta\lambda_4 &= \frac{3}{16\pi^2} h_t^4 A_t^4 g(M_Q^2, M_U^2), \end{aligned} \quad (41)$$

where

$$\begin{aligned} B(a, b) &\equiv \frac{1}{(a-b)^2} \left[\frac{1}{2}(a+b) - \frac{ab}{a-b} \ln\left(\frac{a}{b}\right) \right], \\ h(a, b) &\equiv \frac{1}{a-b} \ln\left(\frac{a}{b}\right), \\ f(a, b) &\equiv \frac{-1}{(a-b)} \left[1 - \frac{b}{a-b} \ln\left(\frac{a}{b}\right) \right], \\ g(a, b) &\equiv \frac{1}{(a-b)^2} \left[2 - \frac{a+b}{a-b} \ln\left(\frac{a}{b}\right) \right], \\ p_t(a, b) &\equiv f(a, b) + 2e_t \sin^2 \theta_W (a-b)g(a, b). \end{aligned} \quad (42)$$

For simplicity, consider the case of $M_Q = M_U \equiv M_{\text{SUSY}}$. Using $B(a, a) = 1/6a$, $h(a, a) = 1/a$, $f(a, a) = -1/2a$ and $g(a, a) = -1/6a^2$, eq. (40) becomes:

$$\lambda(M_{\text{SUSY}}) = \frac{1}{4}(g^2 + g'^2) + \frac{3h_t^4 A_t^2}{8\pi^2 M_{\text{SUSY}}^2} \left[1 - \frac{A_t^2}{12M_{\text{SUSY}}^2} \right]. \quad (43)$$

Note that the correction term due to squark mixing has a maximum when $A_t = \sqrt{6}M_{\text{SUSY}}$. This relation is often called the maximal mixing condition, since it corresponds to the point at which the one-loop radiative corrections to $m_{h^0}^2$ are maximal.

Using the new boundary condition, we may repeat the analysis of the previous section and recompute $m_{h^0}^2$. At one loop, the effect of the squark mixing is simply additive. That is, the modification of $m_{h^0}^2$ due to squark mixing at one loop is given by: $(\Delta m_h^2)_{1\text{mix}} = (\delta\lambda_2 + \delta\lambda_3 + \delta\lambda_4)v^2$. At two-loops, we solve for $\lambda(m_Z)$ by iterating the RGE for $\lambda(\mu)$ twice as in the previous section. However, the boundary condition for $\lambda(M_{\text{SUSY}})$ has been altered, and this modifies the computation. The end result is

$$(\Delta m_h^2)_{\text{mix}} = \frac{3g^2 m_t^4 A_t^2}{8\pi^2 m_W^2 M_{\text{SUSY}}^2} \left(1 - \frac{A_t^2}{12M_{\text{SUSY}}^2}\right) \left[1 + 2 \left(\gamma_v + \frac{\beta_{h_t^2}}{h_t^2}\right) \ln \left(\frac{M_{\text{SUSY}}^2}{m_t^2}\right)\right] \quad (44)$$

i.e., $(\Delta m_h^2)_{\text{mix}}$ acquires a logarithmically-enhanced piece at two loops. In this approximation, the maximum in $(\Delta m_h^2)_{\text{mix}}$ at $A_t = \sqrt{6}M_{\text{SUSY}}$ is not shifted. However, this method does *not* pick up any non-logarithmically-enhanced two-loop terms proportional to A_t . To obtain such terms, one would have to perform a two-loop computation in order to find the necessary two-loop terms that modify the boundary condition [eq. (40)].

It is again possible to absorb the two-loop singly-logarithmic term into the one-loop contribution, $(\Delta m_h^2)_{1\text{mix}}$, by an appropriate choice of scale for the top-quark mass. The end result is quite simple:

$$(\Delta m_h^2)_{\text{mix}} = \frac{3g^2 m_t^4 (M_{\text{SUSY}}) A_t^2}{8\pi^2 m_W^2 M_{\text{SUSY}}^2} \left(1 - \frac{A_t^2}{12M_{\text{SUSY}}^2}\right). \quad (45)$$

That is $(\Delta m_h^2)_{\text{mix}} = (\Delta m_h^2)_{1\text{mix}}(m_t(\mu_{\tilde{t}}))$, where the appropriate choice of scale in this case is $\mu_{\tilde{t}} \equiv M_{\text{SUSY}}$. The difference from the previous case [where $\mu_t = \sqrt{m_t M_{\text{SUSY}}}$] arises due to the extra factor of 2 multiplying the two-loop singly-logarithmic term in eq. (44) [compare this with eq. (30)]. Physically, $\mu_{\tilde{t}} = M_{\text{SUSY}}$ corresponds to the scale at which the squarks decouple and the boundary condition [eq. (43)] is modified due to squark mixing.

To illustrate the above results, we compare in Fig. 2 the value of m_{h^0} as a function of A_t based on the five cases exhibited in Fig. 1. Specifically, the effects of $(\Delta m_h^2)_{\text{mix}}$ are included at the one-loop level in cases (i) and (ii), while cases (iv) and (v) make use of the improved result given by eq. (45). In the full RG-improved result [case (iii)], the RGE for $\lambda(\mu)$ is computed numerically using the modified boundary condition [eq. (40)]. We see that $(m_{h^0}^2)_{1\text{RG}} \simeq (m_{h^0}^2)_{1\text{LL}}(m_t(\mu_t)) + (\Delta m_{h^0}^2)_{1\text{mix}}(m_t(\tilde{\mu}_t))$. Thus, once again a simple recipe provides an excellent approximation to the numerically-integrated RG-improved result over the entire region of the graph. Note that the maximal value of m_{h^0} occurs for $|A_t| \simeq 2.4M_{\text{SUSY}}$. The solid or dash-dotted line provides our best mass estimate, and we conclude that $m_{h^0} \lesssim 125$ GeV for $M_{\text{SUSY}} \leq 1$ TeV. Similar results were also obtained by Carena *et al.* [9].

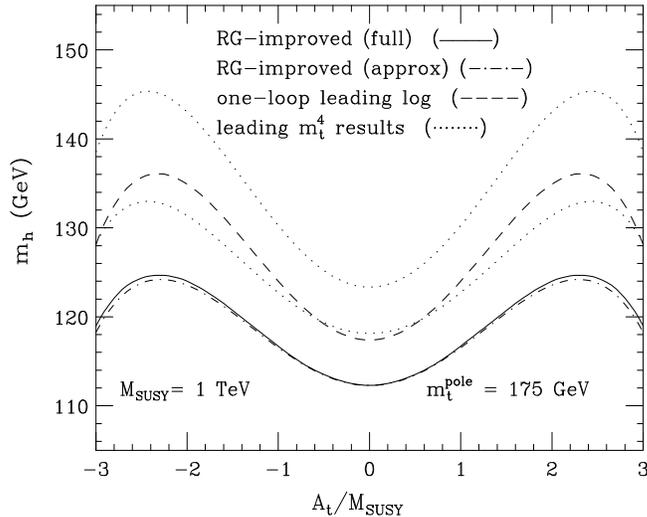


Figure 2: The upper bound to the mass of the light CP-even Higgs boson of the MSSM plotted as a function of A_t/M_{SUSY} . Squark-mixing effects are incorporated as described in the text. See the caption to Fig. 1.

During the past year, two groups have computed the A_t dependence of m_{h^0} at the two-loop level. Ref. [20] has performed a diagrammatic two-loop computation which includes all terms of $\mathcal{O}(\alpha_s)$, as a function of $\tan\beta$. Ref. [21] uses an effective potential approach to extend the computation of ref. [16] and compute directly the two-loop squark mixing contributions in the $v_1 = 0$ model. These results show that the A_t dependence of m_{h^0} is modified slightly at two loops: the maximal squark mixing point occurs at $A_t \simeq 2M_{\text{SUSY}}$, a value somewhat below the result noted above. Moreover, the value of m_{h^0} at maximal squark mixing is slightly higher than the one shown in Fig. 2; for $M_{\text{SUSY}} = 1$ TeV, the maximal value of m_{h^0} is found to be close to $m_{h^0} \simeq 130$ GeV. Presumably, these results are due to genuine two-loop non-logarithmically enhanced terms proportional to a power of A_t^2/M_{SUSY}^2 . An important check of the calculations presented in refs. [20] and [21] would be to explicitly verify the two-loop logarithmically-enhanced contribution exhibited in eq. (44).

5 Conclusions

I have described in detail the theoretical basis for the computation of the upper bound of the mass of the light CP-even Higgs boson of the MSSM. It suffices

to consider the limiting case of $v_1 = 0$ which considerably simplifies the analysis. I explained how one can use renormalization group methods to provide a short-cut for obtaining the leading one-loop and two-loop contributions to m_{h^0} . These methods can also be generalized to the full MSSM Higgs sector at arbitrary $\tan\beta$. Further details and references can be found in ref. [13].

As a result of the work by many groups during this past decade, we believe that the predicted value of m_{h^0} as a function of the MSSM parameters is accurately predicted within an uncertainty of a few GeV. Simple analytic formulae provide an excellent representation of the known results over a large range of the MSSM parameter space [9,13]. The present partially known two-loop information is essential to this conclusion and provides confidence that there are no surprises lurking in some corner of the supersymmetric parameter space. Some clarification is still needed to understand more completely the dependence on the squark mixing parameters.

Acknowledgments

This paper is based on a collaboration with Ralf Hempfling and Andre Hoang. I have learned much from their wisdom. I would also like to thank Joan Sola for his kind hospitality during my visit to Barcelona and RADCOR-98. This work is partially supported by a grant from the U.S. Department of Energy.

References

1. H.P. Nilles, *Phys. Rep.* **110** (1984) 1; H.E. Haber and G.L. Kane, *Phys. Rep.* **117** (1985) 75.
2. See, *e.g.*, chapter 4 of J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley Publishing Company, Redwood City, CA, 1990).
3. H.E. Haber and R. Hempfling, *Phys. Rev. Lett.* **66** (1991) 1815.
4. Y. Okada, M. Yamaguchi and T. Yanagida, *Prog. Theor. Phys.* **85** (1991) 1; J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett.* **B257** (1991) 83; *Phys. Lett.* **B262** (1991) 477.
5. W. Siegel, *Phys. Lett.* **84B** (1979) 193; D.M. Capper, D.R.T. Jones and P. van Nieuwenhuizen, *Nucl. Phys.* **B167** (1980) 479.
6. I. Antoniadis, C. Kounnas and K. Tamvakis, *Phys. Lett.* **119B** (1982) 377; S.P. Martin and M.T. Vaughn, *Phys. Lett.* **B318** (1993) 331.
7. P.H. Chankowski, S. Pokorski, J. Rosiek, *Phys. Lett.* **B274** (1992) 191; *Nucl. Phys.* **B423** (1994) 437; A. Brignole, *Phys. Lett.* **B281** (1992) 284; A. Yamada, *Phys. Lett.* **B263** (1991) 233; *Z. Phys.* **C61** (1994)

- 247; A. Dabelstein, *Z. Phys.* **C67** (1995) 495; D.M. Pierce, J.A. Bagger, K. Matchev and R. Zhang, *Nucl. Phys.* **B491** (1997) 3.
8. S.P. Li and M. Sher, *Phys. Lett.* **B140** (1984) 339; A. Brignole, J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett.* **B271** (1991) 123 [E: **B273** (1991) 550]; M. Drees and M.M. Nojiri, *Phys. Rev.* **D45** (1992) 2482; J.A. Casas, J.R. Espinosa, M. Quiros, A. Riotto, *Nucl. Phys.* **B436** (1995) 3 [E: **B439** (1995) 466].
9. M. Carena, J.R. Espinosa, M. Quiros, and C.E.M. Wagner, *Phys. Lett.* **B355** (1995) 209; M. Carena, M. Quiros, and C.E.M. Wagner, *Nucl. Phys.* **B461** (1996) 407.
10. See, *e.g.*, S. Pokorski, *Gauge Field Theories* (Cambridge University Press, Cambridge, England, 1987) pp. 50–55.
11. R. Barbieri, M. Frigeni and F. Caravaglios, *Phys. Lett.* **B258** (1991) 167; Y. Okada, M. Yamaguchi, T. Yanagida, *Phys. Lett.* **B262** (1991) 54; J.R. Espinosa and M. Quiros, *Phys. Lett.* **B266** (1991) 389; D.M. Pierce, A. Papadopoulos and S. Johnson, *Phys. Rev. Lett.* **68** (1992) 3678. K. Sasaki, M. Carena and C.E.M. Wagner, *Nucl. Phys.* **B381** (1992) 66; J. Kodaira, Y. Yasui, K. Sasaki, *Phys. Rev.* **D50** (1994) 7035.
12. H.E. Haber and R. Hempfling, *Phys. Rev.* **D48** (1993) 4280.
13. H.E. Haber, R. Hempfling and A.H. Hoang, *Z. Phys.* **C75** (1997) 539.
14. I. Jack and H. Osborn, *J. Phys.* **A16** (1983) 1101; M.E. Machacek and M.T. Vaughn, *Nucl. Phys.* **B249** (1985) 70; C. Ford, I. Jack, and D.R.T. Jones, *Nucl. Phys.* **B387** (1992) 373. For a useful compilation of two-loop RGEs for the Standard Model and the MSSM, see B. Schrempp and M. Wimmer, *Prog. Part. Nucl. Phys.* **37** (1996) 1.
15. A. Sirlin and R. Zucchini, *Nucl. Phys.* **B266** (1986) 389.
16. R. Hempfling and A.H. Hoang, *Phys. Lett.* **B331** (1994) 99.
17. R. Tarrach, *Nucl. Phys.* **B183** (1981) 384; N. Gray, D.J. Broadhurst, W. Grafe, and K. Schilcher, *Z. Phys.* **C48** (1990) 673.
18. R. Hempfling and B.A. Kniehl, *Phys. Rev.* **D51** (1995) 1386.
19. R. Hempfling, in *Properties of SUSY Particles*, Proceedings of the 23rd Workshop of the INFN Eloisatron Project, Erice, Italy, 28 September—4 October 1992, edited by L. Cifarelli and V.A. Khoze (World Scientific, Singapore, 1993) p. 373.
20. S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rev.* **D48** (1998) 091701; *Phys. Lett.* **B440** (1998) 296; preprint KA-TP-17-1998 [hep-ph/9812472].
21. R.-J. Zhang, preprint MADPH-98-1072 [hep-ph/9808299].