VARIABILITY OF THE SOLAR NEUTRINO FLUX

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SOLAR NEUTRINO PROBLEM
STANDARD INTERPRETATION

Standard Solar Model

Neutrinos of Majorana type
Effectively zero magnetic moment

Flux Reduction due to Flavor Oscillation [MSW Effect]
Depends upon density
Independent of magnetic field

Since the solar density is spherically symmetric, the detected neutrino fluxes should be constant

Time Variation is Incompatible with this Theory
VARIABILITY TESTS

1. Look for correlation between measured flux and some solar index such as sunspot number

2. Examine variance of measurements, in comparison with simulations

3. Examine histograms of measurements, in comparison with simulations

4. Look for oscillations that can be identified with known solar oscillations

Only #4 makes full use of the timing data
<table>
<thead>
<tr>
<th>Periodicity</th>
<th>Period</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hale cycle</td>
<td>22 years</td>
<td>0.046 cpy</td>
</tr>
<tr>
<td>Sunspot cycle</td>
<td>11 years</td>
<td>0.09 cpy</td>
</tr>
<tr>
<td>Howe oscillation</td>
<td>1.3 years</td>
<td>0.77 cpy</td>
</tr>
<tr>
<td>Rieger oscillation</td>
<td>156 days</td>
<td>2.34 cpy</td>
</tr>
<tr>
<td>Rieger-type oscillations</td>
<td>78 days</td>
<td>4.68 cpy</td>
</tr>
<tr>
<td></td>
<td>52 days</td>
<td>7.02 cpy</td>
</tr>
<tr>
<td>Internal rotation - equatorial (sidereal)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radiative zone</td>
<td>13.9 +/-0.5 cpy</td>
<td></td>
</tr>
<tr>
<td>Tachocline</td>
<td>13.7 to 14.6 cpy</td>
<td></td>
</tr>
<tr>
<td>Convection zone</td>
<td>14.2 to 14.9 cpy</td>
<td></td>
</tr>
<tr>
<td>Internal rotation - equatorial (synodic)</td>
<td></td>
<td></td>
</tr>
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</tr>
<tr>
<td>Convection zone</td>
<td>13.2 to 13.9 cpy</td>
<td></td>
</tr>
<tr>
<td>Plus harmonics of rotation rate</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SOLAR NEUTRINO FLUX VARIABILITY

AVAILABLE DATA SETS

- Homestake [Radiochemical - Chlorine]
- GALLEX-GNO [Radiochemical - Gallium]
- SAGE [Radiochemical - Gallium]
- Super-Kamiokande [Cerenkov]
SOLAR NEUTRINO FLUX VARIABILITY

NOT YET AVAILABLE DATA SETS

• Kamiokande, Kamiokande II [Cerenkov]
• Sudbury Neutrino Observatory [SNO, Cerenkov]
GALLEX AND GNO DATA ANALYSIS

Histograms of flux values for (a) Gallex, and (b) GNO.

- S0505B01
Histogram of flux values for Gallex and GNO combined
Histograms of (a) $s_u$ for Gallex, (b) $s_u$ for GNO, (c) $s_l$ for Gallex, (d) $s_l$ for GNO.
GALLEX AND GNO DATA ANALYSIS

5-point running means of the experimental flux estimates for Gallex and GNO
Likelihood functions for the flux for (a) Gallex and (b) GNO

GALLEX $69.9 \pm 6.1 \text{ SNU}$

GNO $62.9 \pm 5.4 \text{ SNU}$

Flux estimates differ by 2 sigma
P = 0.02
GALLEX AND GNO DATA ANALYSIS

**TABLE 1**
GALLEX DATA

<table>
<thead>
<tr>
<th>Runs</th>
<th>Flux</th>
<th>Upper Error</th>
<th>Lower Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 13</td>
<td>78.6</td>
<td>18.3</td>
<td>18.1</td>
</tr>
<tr>
<td>14 – 26</td>
<td>79.2</td>
<td>14.0</td>
<td>13.9</td>
</tr>
<tr>
<td>27 – 39</td>
<td>63.5</td>
<td>12.3</td>
<td>11.9</td>
</tr>
<tr>
<td>40 – 52</td>
<td>44.3</td>
<td>10.6</td>
<td>10.1</td>
</tr>
<tr>
<td>53 - 65</td>
<td>108.5</td>
<td>16.4</td>
<td>15.6</td>
</tr>
</tbody>
</table>

Chi-Square = 12.77

P = 0.012

Flux Probably Not Constant

**TABLE 2**
GNO DATA

<table>
<thead>
<tr>
<th>Runs</th>
<th>Flux</th>
<th>Upper Error</th>
<th>Lower Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 12</td>
<td>51.3</td>
<td>12.2</td>
<td>11.9</td>
</tr>
<tr>
<td>13 $ 23</td>
<td>58.2</td>
<td>12.5</td>
<td>12.1</td>
</tr>
<tr>
<td>24 $ 35</td>
<td>72.3</td>
<td>12.3</td>
<td>11.9</td>
</tr>
<tr>
<td>36 $ 46</td>
<td>55.4</td>
<td>11.8</td>
<td>11.7</td>
</tr>
<tr>
<td>47 - 58</td>
<td>38.5</td>
<td>9.9</td>
<td>9.8</td>
</tr>
</tbody>
</table>

Chi-Square = 4.90

P = 0.29

Flux Probably Constant

•S0505B08
Normalized power spectrum, calculated using std(g) as the error term, for
(a) Gallex and
(b) GNO.
JOINT SPECTRUM STATISTIC

From two (or more) power spectra, form a statistic analogous to a correlation statistic, defined so that it is distributed in the same way as a power spectrum.

For gaussian random noise,

\[ P(S) dS = e^{-S} dS \quad \text{and} \quad P_{>S}(S) = e^{-S} \]

If \( Z = S_1 S_2 \), then

\[ P(Z) dZ = \int_0^\infty dx \int_0^\infty dy e^{-x-y} \delta(Z - xy) dZ \]

We find that

\[ P_{>J}(J) = e^{-J} \]

if

\[ J = \ln \left[ 2Z^{1/2} K_1(2Z^{1/2}) \right] \]
Joint power spectrum formed from power at $nu$ and at $2nu$ for
(a) Gallex and
(b) GNO.
R-MODES

In a rotating fluid sphere, r-modes comprise oscillations for which the motion is mainly latitudinal.

In the rotating frame, the oscillation frequencies are given by

\[ \nu(l,m) = \frac{2m \nu_R}{l(l+1)} \]

where \( \nu_R \) is the rotation frequency, \( l \geq 2 \), and \( l \) and \( m \) are the usual spherical harmonic indices. (The frequencies are independent of the index \( n \).)

Consider \( \nu_R = 14 \, y^{-1} = 444 \, Hz \) and \( l = 3 \). Then

- for \( m = 1 \), \( \nu = 2.33 \, y^{-1} \) so \( P = 157 \, days \),
- for \( m = 2 \), \( \nu = 4.67 \, y^{-1} \) so \( P = 78 \, days \),
- for \( m = 3 \), \( \nu = 7.00 \, y^{-1} \) so \( P = 52 \, days \).

These frequencies are close to the periods of Rieger-type oscillations.
### GALLEX AND GNO DATA ANALYSIS

#### TABLE 3

<table>
<thead>
<tr>
<th>Mode Type</th>
<th>Frequency</th>
<th>Power</th>
<th>Frequency</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation, fundamental</td>
<td>12.50 to 13.80</td>
<td>13.07</td>
<td>6.48</td>
<td>7.22</td>
</tr>
<tr>
<td></td>
<td>13.64</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotation, harmonic</td>
<td>25.00 to 27.60</td>
<td>27.32</td>
<td>4.66</td>
<td></td>
</tr>
<tr>
<td>R-modes, 2-1 and 3-2</td>
<td>4.50 to 4.93</td>
<td>4.54</td>
<td>5.30</td>
<td></td>
</tr>
<tr>
<td>R-mode, 2-2</td>
<td>9.04 to 9.87</td>
<td></td>
<td>9.20</td>
<td>4.25</td>
</tr>
<tr>
<td>R-mode, 3-1</td>
<td>2.25 to 2.46</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-mode, 3-3</td>
<td>6.75 to 7.40</td>
<td>6.93</td>
<td>4.68</td>
<td></td>
</tr>
</tbody>
</table>

•S0505B09
HOMESTAKE AND GALLEX-GNO

- Lomb-Scargle spectrum of Homestake (left) and GALLEX-GNO (right) data in range 10 – 16 y-1
• Power spectra for SXT latitudes N60 and S60 (left) and Equator (right)
NEUTRINO AND X-RAY FLUX SPECTRA COMPARED

Comparison of normalized probability distribution functions formed from power spectra:

- SXT N60&S60 (green)
- SXT Equator (red)
- Homestake (black)
- GALLEX-GNO (blue)
Flux and Error Estimates in 10-day bins
SUPER-KAMIOKANDE, 10-day bins

Flux and Error Estimates (One Year Section)
SUPER-KAMIOKANDE, 10-day bins

Power Spectrum formed from Timing Schedule
Very Strong Periodicity at 35.98 cpy
Power spectrum formed by likelihood analysis
The biggest peak is at $\nu = 26.57$ with $S = 11.11$
The second biggest peak is at $\nu = 9.41$ with $S = 7.33$
The latter is an alias of the former, due to the peak at 35.98 in the power spectrum of the sampling schedule
SUPER-KAMIOKANDE, 5-day bins

LOMB-SCARGLE ANALYSIS

Super-K, 5-day, delta function at mean live time, sx = std(x), Neu03U07.m, 030909
SUPER-KAMIOKANDE, 5-day bins

LIKELIHOOD ANALYSIS

Super-K, 5-day data, Relative Likelihood, error vector, Neu03Q24.m, 0308912
Comparison of powers of top ten peaks for five analysis procedures:
(1) basic Lomb-Scargle analysis, mean times;
(2) basic Lomb-Scargle analysis, mean live times;
(3) modified Lomb-Scargle analysis, mean live times, error data;
(4) SWW likelihood method, start times, end times, and error data;
(5) SWW likelihood method, start times, end times, mean live times, and error data.
Only the peaks at 9.43 yr\(^{-1}\) and 43.72 yr\(^{-1}\) show a monotonic increase in power.
Super-K, 5-day data, Rayleigh power of end time, Neu03Q09.m, 030805
Common peaks in 10-day, 5-day, Super-K data, Neu03ZB01.m, 031120
SUPER-KAMIOKANDE, 5-day bins

Log Signed Magnetic field, Super-K interval, Rayleigh power, Neu003N03.m, 030723
It is therefore interesting to compare the power spectrum of the neutrino measurements with the power spectrum of the magnetic field at Sun center for the period of operation of Super-Kamiokande. We obtain the estimate 13.20 +/- 0.14 for the synodic rotation frequency (or 14.20 +/- 0.14 for the sidereal rotation frequency) of the magnetic field. This leads to the band 39.60 +/- 0.42 for the third harmonic of the synodic rotation frequency of the magnetic field. We see that peak C falls within this band. When we apply the shuffle test (Bahcall & Press 1991; Sturrock, Walther, & Wheatland 1997), randomly re-assigning flux and error measurements (kept together) to time bins, we find that only 5 cases out of 1,000 yield a power 8.91 or larger in the search band.
SUPER-KAMIOKANDE, 5-day bins

Super-K 5d (red), Mag field (blue), Cumulative Rayleigh power, $\nu = 39.56$, Neu03U03.m, 030
VARIABILITY OF THE SOLAR NEUTRINO FLUX

R-modes are retrograde waves that, in a uniform and rigidly rotating sphere, have frequencies

$$\nu(l,m,\text{syn}) = m(\nu_R - 1) - \frac{2m \nu_R}{l(l+1)}$$

as seen from Earth, where $l$ and $m$ are two of the usual spherical-harmonic indices, and $\nu$ is the sidereal rotation frequency.
VARIABILITY OF THE SOLAR NEUTRINO FLUX

An observer co-rotating with the sphere would detect oscillations at the frequency

\[ \nu(l,m,\text{rot}) = \frac{2m \nu_R}{l(l+1)} \]  

(1)

Since the mode frequency does not depend upon the radial index \( n \), it seems likely that similar oscillations, with similar frequencies, could occur in thin spherical shells inside a radially stratified sphere.
VARIABILITY OF THE SOLAR NEUTRINO FLUX

R-mode oscillations may modulate the solar neutrino flux by moving magnetic regions in and out of the path of neutrinos propagating from the core to the Earth. The resulting oscillations in the neutrino flux would be formed from combinations of the frequency with which an r-mode oscillation intercepts the core-Earth line and the frequency with which a magnetic structure intercepts the core-Earth line. These combinations will have the form

$$\nu = \left| m(v_R - 1) - \frac{2m v_R}{l(l+1)} \pm m'(v_R - 1) \right|$$

(2)

Where m’, the azimuthal index for the magnetic structure, may be different from that of the r-mode.
VARIABILITY OF THE SOLAR NEUTRINO FLUX

For $m' = m$ and for the minus sign, this yields the frequency of equation (1), and for the plus sign it yields

$$\nu(l,m,alias) = 2m(\nu_R - 1) - \frac{2m \nu_R}{l(l+1)}$$

We may refer to this frequency for convenience as an “alias” of the frequency given by equation (1).
VARIABILITY OF THE SOLAR NEUTRINO FLUX

For $l = 2$ and $m = 2$, and for the range of values of inferred from the magnetic-field data, we find that equation (1) leads us to expect an oscillation in the band $9.47 \pm 0.09$, and equation (3) leads us to expect an oscillation in the band $43.33 \pm 0.47$.

We see that the peaks A and B fall within these two bands.

On applying the shuffle test, we find only 6 cases out of 10,000 in which a peak with power 11.51 or larger occurs in the band $9.47 \pm 0.09$, and only 5 cases out of 1,000 that yield a power 9.83 or larger in the band $43.33 \pm 0.47$. 
SUPER-KAMIOKANDE, 5-day bins

Super-K 5d, 9.42 (red), 43.72 (blue), Scaled Rayleigh powers, Neu03U06.m, 030910
Super-Kamiokande and SNO data show that Matter-Enhanced Neutrino Oscillations occur

\[ \nu_e \to \nu_{mu} \text{ or } \nu_{tau} \]

This must be the dominant process.

KamLAND data imply that similar oscillations occur involving \( \bar{\nu}_e \) rather than \( \nu_e \).

Solar and reactor data can be matched with

\[ \Delta m^2 \sim 10^{-4} \text{ eV}^2 \]

Time-variation of the solar flux must therefore be a sub-dominant process.
NEUTRINO PHYSICS  INTERPRETATION OF SOLAR FLUX MODULATION

Time-variation points to the involvement of magnetic field.

This indicates that modulation must be due to

Resonant Spin Flavor Precession (RSFP)

One possibility (Balantekin, Volpe) is

\[ \nu_e \rightarrow \bar{\nu}_{mu} \text{ or } \bar{\nu}_{tau} \]

but this effect is weak and occurs in the radiative zone.

To get the correct shape of the time-averaged neutrino energy spectrum, we need

\[ \Delta m^2 \sim 10^{-8} \text{ eV}^2 \]
The process cannot involve the three active neutrinos, since the width of the $Z^0$ resonance limits the number of light, active neutrinos to three.

Solar and reactor data require

$$\Delta m^2 \sim 10^{-4} \text{ eV}^2$$

Atmospheric $\nu_{\mu} \rightarrow \nu_{\tau}$ data require

$$\Delta m^2 \sim 10^{-3} \text{ eV}^2$$

Hence a fourth neutrino is required.

This must be sterile, to be consistent with $Z^0$ data.
Caldwell has proposed that solar-neutrino-flux time variation may be attributed to an RSFP process by which electron neutrinos are converted to sterile anti-neutrinos:

\[ \nu_e \rightarrow \bar{\nu}_s \]

via a transition magnetic moment.

The sterile neutrino does not mix with other neutrinos, so this proposal is compatible with all limitations on sterile neutrinos.

This model has been analyzed by Chauhan and Pulido. They find that:

- It gives an improved fit to time-averaged flux measurements
- It eliminates an upturn that is expected, but not found, at lower energies in Super-Kamiokande and SNO data
- It yields the correct magnitude of the modulation over the whole energy range
- It yields the correct location of the effect (in the convection zone)
SIGNIFICANCE FOR SOLAR PHYSICS OF THE VARIABILITY OF THE SOLAR NEUTRINO FLUX

• Neutrinos may be used to probe the Sun’s internal magnetic field and internal dynamics

• Variations are probably due to inhomogeneous magnetic structures with field strengths of order $10^5$ Gauss in the outer radiative zone, the tachocline, or deep convection zone

• The Rieger and related periodicities are due to internal r-mode oscillations, probably in or near the tachocline

• Neutrino observations may give advance information of the development of the solar cycle