Compton Telescopes:
Goals, Design Options, Data Analysis

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Prologue:
Details about Compton scattering
A Compton Scattering Primer

Compton equation: \[ \cos \varphi = 1 - \frac{E_0}{E_g} + \frac{E_0}{E_e + E_g} \]

In the case the electron direction is known:

\[ \vec{e}_i = \frac{\sqrt{E_e^2 + 2E_eE_0\vec{e}_e + E_g\vec{e}_g}}{E_e + E_g} \]

Energy of initial gamma ray
Energy of the recoil electron
Energy of the scattered gamma ray
Rest energy of the electron
Total relativistic energy of the electron
“Compton scatter angle” of the gamma ray
“Electron scatter angle” of the recoil electron
Total scatter angle
Direction of the initial gamma ray
Direction of the recoil electron
Direction of the scattered gamma ray
“Compton cone”
Compton scattering is the dominant interaction mechanism between ~100 keV and ~10 MeV (in Si).

Normalized Klein-Nishina cross-section as a function of scatter angle: The higher the energy, the stronger is the forward scattering.
Compton Scattering Preserves Polarization

\[
\frac{d\sigma}{d\Omega}_{C,\text{unbound, pol}} = \frac{r_e^2}{2} \left( \frac{E_g}{E_i} \right)^2 \left( \frac{E_g}{E_i} + \frac{E_i}{E_g} - 2\sin^2 \varphi \cos^2 \chi \right)
\]

Modulation as a function of energy and scatter angle

→ Most polarization information is preserved at low energies and medium scatter angles (60-90 degrees)
Doppler-broadening as fundamental limit

The electron is not at rest, but bound to a nucleus!

→ Electron momentum cannot be measured!

→ Using Compton equation(s) to determine origin of \(\gamma\)-ray is only an approximation!

The angular resolution of any Compton telescope is fundamentally limited by this so called “Doppler-broadening”
Chapter One:
What are Compton cameras good for?
Application areas

Physics
Astrophysics: medium-energy gamma-ray astronomy
Dark matter search

Medical imaging
SPECT - Single Photon Emission CT

“Homeland security”
nuclear thread detection & nuclear waste monitoring
Astrophysics in the Compton regime

High-energy γ-ray range (pair regime)
Pair telescopes:
EGRET, AGILE, GLAST

Medium energy γ-ray range (Compton regime)
Compton telescopes:
COMPTEL

Low-energy γ-ray range/Hard X-ray range (photo effect regime)
Spatial & temporal modulators:
SPI, Swift, RHESSI
Focusing: GRI
Collimators: HXD, SGD
Astrophysics in the Compton regime

Nucleosynthesis

- nuclear $\gamma$-lines
  $\leq 8$ MeV

Cosmic Accelerators

- non-thermal continuum:
  - Bremsstrahlung
  - synchro-cyclotron radiation
  - inverse Compton scattering
Astrophysics in the Compton regime

**Nucleosynthesis**

- SN/massive stars: nuclear reactions/isotopes
- Extended galactic radioactivity: $^{26}\text{Al}$, $^{60}\text{Fe}$
- Young SNRs: $^{44}\text{Ti}$
- 847 keV: SN Ia
- 511 keV: novae, supernovae, dark matter ??

**Cosmic Accelerators**

- Unidentified EGRET Sources
- Galactic and extragalactic diffuse background
- Spectral luminosity maxima of some EGRET/COMPTEL Blazars
- non-thermal spectra in black-hole binaries, micro-quasars
- GRB spectra polarization

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Supernovae of type Ia are presumed to be standard candles and used for cosmology. However, many competing models for their explosion mechanism exist:

- **Speed of burning front:**
  - subsonic (deflagration model)
  - supersonic (detonation model)
  - mixture

- Does the explosion only happen if the white dwarf exceeds the Chandrasekhar limit, or also while it is still a sub-Chandrasekhar white dwarf?

**Goal:**

Discriminate between the models through measurements of the production rate and ejection velocities of $^{56}\text{Ni}$ decay chain (847 keV, 1238 keV, 812 keV)
What is the origin of the 511 keV line at the galactic center:  
- Unresolved SN Ia, novae, black holes?  
- Positrons transported in the galactic magnetic field to the center?  
- MeV dark matter annihilation/decay?  
- Or something else?
Polarization: GRBs, pulsars

**GRBs:**

How does the inner engine work and what are the $\gamma$-ray emission processes:

- synchrotron/inverse Compton
- hadron related emission via pion production and decay

**Pulsars:**

What are the production sites

- polar cap
- outer gap
- two-pole caustic model

**Goal:**

Solve most of the open questions by high-sensitivity, high-bandwidth measurements including polarization
Science summary

Enable high sensitivity $\gamma$-ray spectroscopy, imaging and polarization measurements from 0.2 up to 10 MeV

**Life Cycles of Matter**
- Supernovae & nucleosynthesis
- Supernova remnants & interstellar medium
- Neutron stars, pulsars, novae

**Black Holes**
- Creation & evolution
- Lepton vs. hadron jets
- Deeply buried sources

**Fundamental Physics & Cosmology**
- Gamma-ray bursts & first stars
- History of star formation
- MeV dark matter
Chapter Two:
Designing a next generation Compton telescope

- **COMPTEL**
  - (0.8-30 MeV)

- **OSSE**
  - (50 keV - 10 MeV)

- **BATSE**
  - (20-600 keV)

- **EGRET**
  - (20 MeV - 30 GeV)

Interaction sequence is determined by time-of-flight measurement between detector planes.

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NCT/ACT-style Compton telescopes

- Photons interact multiple times in active detector (Si, Ge, CdTe, CZT, Xe, etc).
- The interaction sequence can be determined from redundant information (scatter angles).
- The origin of a single not-tracked event can be restricted to the so called “event circle”.
- The photon originated at the point of all overlap.
MEGA-style Compton telescopes

- First interaction occurs in an electron tracking detector (here: Si double-sided strip).
- The interaction sequence can be determined from redundant information (total scatter angles, multiple Compton interactions).
- The origin of a single tracked event can be restricted to the so called “Compton arc” whose length is determined by Molière scattering.
- The photon originated at the point of overlap.
A Compton Telescope for Improved Background Rejection With a Collimator (SGD for NeXT)

Characteristics:

- Small field-of-view
- (Almost) no imaging capabilities?
- Additional background rejection compared to collimation alone (activation!)
- Polarization measurements

Takahashi et al. 2004
A Compton Telescope As Focal Plane Detector of the Gamma-ray Lens Imager GRI

Laue diffraction of gamma rays within the crystal’s volume under Bragg condition

- Improved background rejection
- Polarization measurements
- Small all-sky monitor

Laurel Zoglauer - Compton Telescopes: Goals, Design Options, Data Analysis
What Limits the Sensitivity of a Compton Telescopes?

Sensitivity = Minimum flux in photons / cm² / s (/ keV) from a source which your telescope can significantly detect

Background!

COMPTEL (first Compton telescope in space): Source:Background ≈ 1:100

MEGA at sensitivity limit (2 MeV continuum): S:B=1:15

Science with Compton telescopes is background limited due to:
- External background (photons)
- Internal background (activation and subsequent decay)
- Incompletely absorbed events (first interaction in passive material, or photon escapes detector)
- Chance coincidences
- For strongly collimated/focused telescope: Requirement of on-off-measurements

With exception of: gamma-ray bursts & solar flares
- short time scales (seconds to minutes) - as long as satellite is not directly within the radiation belts...
The Space Radiation Environment

Sun through solar flares: photons, charged particles

Radiation belts:
Trapped protons (SAA) & resulting activation, electrons

Cosmic rays:
- Protons
- Alpha particles
- Ions
- Electrons
- Positrons

For some applications cosmic photons are also background

Secondaries induced by cosmic ray interaction with upper atmosphere:
Albedo photons, neutrons, electron, positrons
How to Reduce Background in a Compton Telescope?

By prevention

- Choose orbit wisely
  - Stay within magnetosphere
  - Avoid radiation belts
    - Equatorial low Earth orbit
- Minimize passive material → “fly your detector naked”
  - Less activation
  - Less chance of interactions in passive material
- Choose your materials wisely
  - Use elements/isotopes with low neutron/proton interaction cross sections
  - Choose materials whose activations lines don’t interfere with your science (e.g. no Aluminum if you want to do $^{26}\text{Al}$ science)
How to Reduce Background in a Compton Telescope?

By shielding and (hardware) rejection

- Use plastic anti-coincidence shield
  - Vetoes charged particles (+)
  - Increases the low-energy detection threshold (-)

- Use a heavy BGO, CsI, etc. shield
  - Protects against low-energy photon events, e.g. from Earth’s atmosphere, or satellite activation (+)
  - Allows vetoing escaping events (+)
  - Increases activation, especially 511 keV and above (-)
  - Can narrow field-of-view (-)
  - Increases overall satellite weight (-)
How to Reduce Background in a Compton Telescope?

By increasing the data space & narrowing the point spread function

- Adding more dimensions in data space, over which background can spread
  - Using electron tracks adds another dimension
- By measuring as much information as possible
  - Ideal Compton telescopes would measure:
    - Electron tracks,
    - Time-of-flight between individual interactions
    - Multiple Compton interactions
- By improving energy resolution
- By improving angular resolution
  - Through improved energy and position resolution, as well as low-z materials for minimized Doppler-broadening
- By minimizing coincidence window
The Baseline Advanced Compton Telescope (ACT) Instrument Concept

as defined by the ACT vision mission study

D1: 27 layers of 2 mm thick Si DSD
- 10x10 cm², 64x64 strips
- 3888 detectors, 248 832 channels
- -30°C, Stirling cycle cooler

D2: 4 layers, 16 mm thick Ge DSD
- 9.2x9.2 cm², 90x90 strips
- 576 detectors, 103 680 channels
- 80 K, Turbo-Brayton cooler

BGO: 4 cm thick shield (anti coincidence)

ACD: plastic scintillator (anti coincidence)


ACT mass model on GLAST bus.

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Chapter Three:
The data analysis chain
Data analysis

All data has to pass through 4 level of analysis

- **Level:**
  - **Data:**
    - source: detector / simulation
      - hits: strips, pixels, AD units
    - calibration & low level data analysis
      - hits: position & energy
    - event reconstruction
      - events: Compton, pair, Myon, etc.
    - image reconstruction and high level data analysis
      - Images, spectra, distributions, etc.

→ Most challenging: event & image reconstruction
Event Reconstruction

Main goals are to

reconstruct the path of the original photon
determine if the event originated from a completely absorbed photon (i.e. that it is not background)

from the measured energies and positions, by utilizing:

- The kinematics of the events, i.e. the redundant Compton scatter information (electron track and/or multiple Compton interactions)
- The known response of the detector to incident gamma-ray (absorption probabilities, etc.)

The result of this analysis step are Compton, pair creation, charged particle, photo-effect events, etc. as well as an overall quality factor, describing the probability that the event was caused from an completely absorbed photon.
All event reconstruction algorithms have the following analysis steps in common to find the correct ordering of the hits:

Start with a set of hits and clusterize the hits of adjacent voxels

Find charged particle tracks of e.g. Compton or pair events

Finally find the overall Compton hit sequence and determine the quality of the event.
Electron Track Evaluation

Identify tracks for MEGA type Compton telescopes

Four steps:

A. Find all possible/reasonable paths of the electron track(s)

B. Evaluate them

C. Choose the track with the highest probability of having the correct direction of motion based on the measured data points

D. Append bremsstrahlungs hits, check if the track leaves the tracker, correct for bad strips, etc.
Electron Track Evaluation

Characteristics of a good track

- Angle change according to Molière-scattering
  - Increase of scattering at end of track (decreasing energy of electron)
- Energy deposits follow Bethe-Bloch-Equation
  - Increase of deposits at end of track
  - Large deposit at turning points
- First deposit likely to be lower than average deposit (interaction takes place somewhere within layer)

The event reconstruction has to assign to each possible track a quality factor, which describes the compliance of the track with the underlying physics!
Electron Track Evaluation

Statistical means: Pearson correlation

Goal:
Prove that for the correct track direction the energy deposits increase at the end of the track and also the angular change

Simpler:
\[ \Delta E \sim n \quad \& \quad \Delta \alpha \sim n \]

Where \( n \) is the hit number along the track
Electron Track Evaluation

Statistical means: Pearson correlation

„Pearson correlation“:
covariance divided by the variances of the involved variables

\[ P_c(E_{\text{dep}}, n) = \frac{E_{\text{dep}} \cdot n - \bar{E}_{\text{dep}} \cdot \bar{n}}{\sqrt{E_{\text{dep}}^2 - \bar{E}_{\text{dep}}^2} \sqrt{n^2 - \bar{n}^2}} \quad P_c(\Delta \alpha, n) = \frac{\Delta \alpha \cdot n - \bar{\Delta \alpha} \cdot \bar{n}}{\sqrt{\Delta \alpha^2 - \bar{\Delta \alpha}^2} \sqrt{n^2 - \bar{n}^2}} \]

The final quality factor is:

\[ Q = 1 - \frac{P_c(E_{\text{dep}}, n) + P_c(\Delta \alpha, n)}{2} \]
Electron Track Evaluation

Statistical means: Bayesian model testing: an example

Example: Email Spam filtering

Assume we want to find out the probability that an email containing the word „Casino“ (C) is spam (ham = spam)

\[ p(\text{Spam}|C) \quad \text{Bayes rule} \quad = \quad \frac{p(C|\text{Spam}) \cdot p(\text{Spam})}{p(C|\text{Spam}) \cdot p(\text{Spam}) + p(C|\text{Ham}) \cdot p(\text{Ham})} \]

From previous emails we know:

From the last 200 Spams, 50 contained the word „Casino“

From the last 100 Hams, 1 contained the word „Casino“

\[ p(C|\text{Spam}) = \frac{50}{200}; \quad p(\text{Spam}) = \frac{200}{(100 + 200)}; \quad p(C|\text{Ham}) = \frac{1}{100}; \quad p(\text{Ham}) = \frac{100}{(100 + 200)}; \quad p(\text{Spam}|C) = \frac{50}{51} = 0.98 \]
Electron Track Evaluation

Bayesian model testing

What we want to know:

How probable is it that the track direction is correct („Ok“), given a set of measured parameters $M$ (energy deposit, angular deviation, etc.)

$$p(Ok|M) \quad \text{Bayes rule} \quad \frac{p(M|Ok) \cdot p(Ok)}{p(M|Ok) \cdot p(Ok) + p(M|Not \ Ok) \cdot p(Not \ Ok)}$$

Transition from emails to tracks:

Spam $\leftrightarrow$ Track element Ok; Ham $\leftrightarrow$ Track element Not Ok;
„Casino“ $\leftrightarrow$ Measurement Point

We need two types of matrices, spanning the complete data-space $M$:

The first one contains the numbers, how often the measured parameters $M$ occur under the condition, that the track element has the correct direction, the other that the track element has the wrong direction.
Electron Track Evaluation

Bayesian model testing: The data spaces

3 parameters describe the start of a track: $E_{\text{in}}$, $\alpha_{\text{out}}$, $E_{\text{dep}}$
5 parameters describe a central hit: $E_{\text{in}}$, $\alpha_{\text{in}}$, $E_{\text{dep}}$, $\theta_{\text{out}}$, $\varphi_{\text{out}}$
2 parameters describe a final hit: $\alpha_{\text{in}}$, $E_{\text{dep}}$

$\rightarrow$ 3x2 data-space matrices needed
Electron Track Evaluation

Bayesian model testing: Fill the data space

Evaluate each possible track combination in the same way as during tracking:

Enter each measurement point of a track ONCE into its matrix
Electron Track Evaluation

Bayesian model testing: Two data space representations

- Deposited energy per layer as a function of electron energy:
  - Shaped as a Landau-distribution

- Change of flight direction as a function of electron energy:
  - Dominated by Molière-scattering
Performance comparison for MEGA

Most of the wrongly reconstructed events can be rejected/recovered later on using also the Compton sequence
Event reconstruction: Basics

All of event reconstruction algorithms have the following analysis steps in common to find the correct ordering of the hits:

1. Start with a set of hits and clusterize the hits of adjacent voxels.
2. Find charged particle tracks of e.g. Compton or pair events.
3. Finally find the overall Compton hit sequence and determine the quality of the event.

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Compton sequence evaluation

Find the sequence of Compton interactions

Use redundant information to determine Compton sequence:

Angles $\phi_l$, $\theta_l$ can be determined via geometry and via Compton kinematics!
Compton sequence evaluation

Statistical means: „Pseudo“-χ² for completely absorbed events

Compare central Compton scatter angles, calculated via

Geometry: \[ \cos \varphi_{i,geo} = \frac{\vec{g}_{i-1} \circ \vec{g}_i}{|\vec{g}_{i-1}| \cdot |\vec{g}_i|} \]

Kinematics: \[ \cos \varphi_{i,kin} = 1 - \frac{E_0}{E_i} + \frac{E_0}{E_{i+1}} \]

\[ \chi^2_P = \sum_{i=2}^{l-1} \frac{\left( \cos \varphi_{i,geo} - \cos \varphi_{i,kin} \right)^2}{\sigma^2_{\varphi_{i,geo}} + \sigma^2_{\varphi_{i,kin}}} \]

Degrees of freedom N := Number of „redundant informations“

\[ Q = 1 - P(\chi^2_P, N) \]
The goals:

- Use all dimensions which Compton scattering provides as one multidimensional data space – not several one-dimensional approximations!
- Get a handle on the background: Where in the data space is our background?
- Get one absolute quality factor of the event:
  
  Probability that the current sequence originated from a completely absorbed non-background photon
Compton sequence evaluation

Bayesian model estimation: The data space dimensions

1. The difference of the redundant Compton scatter angles: \( \Delta \phi = \phi_E - \phi_G \)
2. A distance factor \( d \), which describes the geometrical error of \( \Delta \phi \), resulting from pixelation of the detectors.
3. The Compton scatter angle \( \phi_E \), since larger scatter angles result in larger measurement errors and increased Doppler-broadening. In addition the Klein-Nishina equation influences \( \phi_E \).
4. The total energy \( E_{tot} \), since the distribution of \( \phi \) is given by the Klein-Nishina equation and depends on the total energy of the photon.
5. The total number of interactions, because this determines the number of combinations which have to be investigated.
6. The Compton absorption probability \( p_A \) on the photon’s path to the next interaction point. For the last interaction point the photo absorption probability is used.
7. The target material where the interaction takes place, because this determines the degree of Doppler-broadening.
Compton sequence evaluation

Bayesian model estimation: The data space dimensions

8. The difference of the redundant total scatter angle in case of electron tracking:
\[ \Delta \alpha = \alpha_E - \alpha_G \]
9. The total scatter angle \( \alpha_E \)

→ Luckily, some of those dimensions are independent
→ Several, smaller data spaces are ok
## Compton sequence evaluation

*Bayesian model estimation: The data space dimensions*

<table>
<thead>
<tr>
<th>Start</th>
<th>Central</th>
<th>Track</th>
<th>Absorption</th>
<th>Stop</th>
</tr>
</thead>
<tbody>
<tr>
<td>N hits</td>
<td>N hits</td>
<td>N hits</td>
<td>N hits</td>
<td>N hits</td>
</tr>
<tr>
<td>Material</td>
<td>Material</td>
<td>Material</td>
<td>Material</td>
<td>Material</td>
</tr>
<tr>
<td>$E_{\text{tot}}$</td>
<td>$E_{\text{tot}}$</td>
<td>$E_e$</td>
<td>$E_{\text{tot}}$</td>
<td>$E_{\text{tot}}$</td>
</tr>
<tr>
<td>$\cos \varphi$</td>
<td>$\cos \varphi$</td>
<td>$\cos \alpha$</td>
<td>$p_{\text{abs, Compton}}$</td>
<td>$p_{\text{abs, Photo}}$</td>
</tr>
<tr>
<td>$\Delta \varphi$</td>
<td>$\Delta \alpha$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>distance</td>
<td>distance</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Compton sequence evaluation

Bayesian model estimation: The data space dimensions

Event density in the $\phi_E - \Delta \phi$ data space for the correct sequences. The other dimensions of the data space are fixed to: $d = 2.6-5$ cm, $E_{\text{tot}} = 200-500$ keV, events with four hits. No other dimensions are used.

All wrong sequence points gather at larger $\Delta \phi$ values. Nevertheless there is an overlap between good and bad sequences in the data space which results in wrongly reconstructed events.
Compton sequence evaluation

Bayesian model estimation: The data space dimensions

The data space of the final measurement point contains only the photo-absorption probability $p_A$ and the deposited energy $E_{\text{tot}}$. Here, the events are roughly evenly distributed over all absorption probabilities.

A significant amount of the wrong sequence points gather at large and small probability values $p_A$, since their energy is either too high or too small for a reasonable photo absorption.
## Comparison of the Approaches

<table>
<thead>
<tr>
<th>Classical approach</th>
<th>Bayesian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uses only projections of the data space</td>
<td>Uses all dimensions of the data space</td>
</tr>
<tr>
<td></td>
<td>→ Better performance</td>
</tr>
<tr>
<td>Gives only a „figure-of-merit“</td>
<td>Results in an absolute measurement on the quality of the track/sequence</td>
</tr>
<tr>
<td></td>
<td>→ Advantages in background reduction</td>
</tr>
<tr>
<td>Simple adaptions to modified angular resolution / geometry</td>
<td>Matrices fixed to one geometry and one angular resolution</td>
</tr>
<tr>
<td>Fast</td>
<td>Large matrices</td>
</tr>
<tr>
<td></td>
<td>→ <strong>Extensive</strong> simulations</td>
</tr>
<tr>
<td>Likely to fail during strange conditions</td>
<td>Can better cope with strange conditions</td>
</tr>
<tr>
<td></td>
<td>(turning tracks, etc.)</td>
</tr>
</tbody>
</table>

**Fast & good enough for terrestrial applications**

**Space applications!**
Event Reconstruction Performance

ARM distribution (2 MeV photo peak events, MEGA simulation)
Wrongly reconstructed events (large angles) are reduced by roughly a factor 2

General: Bayesian reconstruction results for MEGA in a sensitivity improvement in a space radiation environment by a factor of 1.2 for 511 keV and 1.7 for 1809 keV!
Simulation of MEGA (modest energy resolution - no lines) telescope in equatorial LEO:

- Left: all triggered events
- Bottom: Within psf after reconstruction and background rejection

What happened?

- Background identification and rejection through event reconstruction and selections
- Spreading of background in tracked Compton data space
- Improvement S/B: x 450 (1809 keV)!
Data analysis

All data has to pass through 4 level of analysis

Level: Data: data source: detector / simulation
        hits: strips pixels, AD units

        calibration & low level data analysis
        hits: position & energy

        event reconstruction
        events: Compton, pair, Myon, etc.

        image reconstruction and high level data analysis
        Images, spectra, distributions, etc.

→ Most challenging: event & image reconstruction
What does a non-tracking Compton camera measure?

Step 1:
Set of channel numbers, AD converter units and timings, which are converted into energies and positions during calibration:

\[ N \times (r_i, E_i) \]

Step 2:
Event reconstruction, i.e. find the interaction sequence and thus the parameters of the original Compton interaction:

\[ E_e, r_1 \text{ and } E_g, r_2 \]
The basic imaging algorithm: ML-EM

\[ t_{nm}: \text{Probability that an event emitted in image bin m is detected as event n} \]

\[ \lambda_m(\ell): \text{Image bin m at iteration level } \ell \]

\[ \lambda_m^{(\ell+1)} = \frac{\lambda_m^{(\ell)}}{S_m} \sum_n t_{nm} \cdot c_n \]

\[ s_m: \text{Probability that an event emitted in image bin m is detected} \]

\[ \sum_k t_{nk} \cdot \lambda_k^{(\ell)} + b_n \cdot h \]

\[ b_n: \text{Background} \]

\[ h: \text{Scaling factor} \]

\[ c_n: \text{List mode: portion of the event that intersects the image} \]

\[ \text{Binned mode: Number of events in bin n} \]

Projection:

Data space \(\rightarrow\) image space

“Maximize the expectation of the log-likelihood”

Projection:

Image space \(\rightarrow\) data space

“Expectation of log-likelihood”
List mode vs. binned mode

Imaging detector response (system matrix) $t_{nm}$:

$$(E_\gamma, \lambda, \nu) \rightarrow (E_e, E_g, r_1, r_2)$$

Imaging space $\rightarrow$ Data space

Two approaches to span data space: list and binned mode:
- List mode: list of events containing each event's parameters
- Binned mode: multi-dimensional matrix

Advantages of list mode:
- Requires less memory when number of events smaller than required bins
- Most detailed handling of data (no bins!)

Disadvantages:
- Very difficult to handle background and absolute normalization
Requirements for astrophysics - 1

1. Correct reconstruction of the spatial distribution of point and extended sources

→ Single most important requirement for most terrestrial applications (medical imaging, nuclear monitoring), besides optimizing noise vs. contrast ratio

Achieved by correctly modeling the shapes of the Compton cones and arcs.
A list-mode response model - basics

Compton cone axis, Compton scatter angle, and electron direction are known
→ Fit shapes perpendicular and parallel to cone section

- Fit shapes perpendicular and parallel to cone section
A list-mode response model - the model

From simulations determine the difference between real and measured scatter angles as a function of the measured energies: “Response”

Shapes include most measurement uncertainties, incomplete absorption, etc. but constitute a simplification to 1D
Simulation of the Cygnus region in the light of radioactive $^{26}\text{Al}$ (1.8 MeV)

Telescope: MEGA, 2 yrs observation

Simulation includes OB-associations, Wolf-Rayet stars and supernova remnants as well as an appropriate amount of background.
Requirements for Astrophysics - 2

1. Correct reconstruction of spatial distribution of point and extended sources

2. Correct retrieval of absolute source fluxes
   \[ \text{ph/cm}^2/\text{s/keV/sr} \]
   This requires:
   (a) Get all probabilities correct!
       → Extremely time consuming in list-mode
       → Requires extremely detailed models of the detector and its characteristics as a function of event selections
Requirements for Astrophysics - 2

1. Correct reconstruction of spatial distribution of point and extended sources

2. Correct retrieval of absolute source fluxes [ph/cm²/s/keV/sr]

(b)

Include all detector, calibration, event reconstruction, etc. imperfections in the response!

*Image space projection of response for one fixed bin in the data space.*

*i.e. pick one (E_e, E_g, r_1, r_2)*
Requirements for Astrophysics - 2

1. Correct reconstruction of spatial distribution of point and extended sources

2. Correct retrieval of absolute source fluxes [ph/cm²/s/keV/sr]

(c)

Incomplete absorption results in wrong energy and wrong spatial distribution

Spatial & spectral components are linked

→ 3D problem: Combined spatial and spectral deconvolution

Undetected escaped events, first interaction in passive material, threshold effects, etc.
1. Correct reconstruction of spatial distribution of point and extended sources

2. Correct retrieval of absolute source fluxes [ph/cm²/s/keV/sr]

3. Correct modeling of background

Simulations and experience from COMPTEL show:
Up to 99% of all photons originate from background!

Source-independent part is a complex (non-analytic) distribution in data-space and best handled during reconstruction.

\[ \hat{\chi}_m^{(l+1)} = \frac{\hat{\chi}_m^{(l)}}{S_m} \sum_n \sum_k t_{nm} \cdot \hat{\chi}_k^{(l)} + b_n \cdot h \]
1. Correct reconstruction of spatial distribution of point and extended sources

2. Correct retrieval of absolute source fluxes \([\text{ph/cm}^2/\text{s/keV/sr}]\)

3. Correct modeling of background

4. **Handle large amounts of events**

   Si-Ge based ACT (Boggs et al., 2005) will measure \(~1,500\) valid Compton events per sec

   \[
   \rightarrow \quad \sim 5 \times 10^{10} \text{ events per year}
   \]

   \[
   \rightarrow \quad \text{Estimate on a 1.6 GHz Pentium-M: 200,000 CPU days with current algorithm (50 iterations, unable to store backprojected events, all-sky, 0.5 degree resolution)}
   \]

   \[
   \rightarrow \quad \text{Number will get significantly larger, when “perfect“ response is implemented}
   \]
Requirements for Astrophysics - 5

1. Correct reconstruction of spacial distribution of point and extended sources

2. Correct retrieval of absolute source fluxes [ph/cm²/s/keV/sr]

3. Correct modeling of background

4. Handle massive amounts of events

5. Optimize “noise vs. contrast“ ratio

If points 1-4 are implemented, it is only a question of finding the perfect imaging algorithm:

- Expectation Maximization
- Maximum Entropy
- Pixon
- Etc.
Requirements for Astrophysics - 5

1. Correct reconstruction of spatial distribution of point and extended sources

2. Correct retrieval of absolute source fluxes \([\text{ph/cm}^2/\text{s/keV/sr}]\)

3. Correct modeling of background

4. Handle massive amounts of events

5. Optimize “noise vs. contrast“ ratio

6. Correct handling of all the technical details:
   - Allow for multiple observations, etc. (i.e. non-homogeneous exposure)
   - Easy handling of different event selections (earth horizon cuts, background cuts, etc.)
   - ...

Andreas Zoglauer - Compton Telescopes: Goals, Design Options, Data Analysis
Currently no algorithm exists which fulfills all requirements!

The approximations done for COMPTEL do not hold for large field-of-view telescopes

→ Required data spaces for binned mode are too large to be handled on today's hardware

→ In list-mode the absolute normalization is far too time consuming

However...
Imaging Examples

Illustration of the LM-EM algorithm recovering the image of a ring (Real measurement with MEGA prototype)

Image of the Cygnus region in the light of $^{26}$Al after 2 years exposure (MEGA detector)
Epilogue:
The future
One day there will be a mission... perhaps...

ACT ?

GRI ?

MEGA ?

2020... 2025... ??