Swiss Re

Mathematical Concepts in the Insurance Industry

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Introduction to Swiss Re

Swiss Re

- Re-Insurance Company
- Founded 19th December 1863
- 2010 Premiums earned: 19'652 Million USD
- 2010 net income: 863 Million USD
- Combined Ratio: P&C 93.9% and L&H 88.7%
- Number of employees: 10'362

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Insurance Math

- An introduction

Example: Insure your friend's car

How much would you charge to insure your friends car?

- Simple questions: How often per year does he have an accident? (=f)
- How much money does it usually cost to repair his car? (=X)

The average loss per year: S = f*X

- You also want to make some money, so add a profit percentage (=p).

Price = f*I*(1+p)

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Example: Insure your friend's car

- That was easy!
- But, how much money do need to keep aside (=reserves) to pay your friend, in case he has an accident?
- If you insure one car only, you will have to have reserves up to the maximum possible loss, in other words, the value of the car.

More cars...

- But now we want to insure many cars.
- The yearly loss now is (X is the loss, N the number of losses):

$$S = \sum_{i=1}^{N} X_i$$

- It is obvious, that S will not be the same for every year, but has a distribution. The challenge is to find distributions for X~F(x) and N~P.

Loss distribution

- A often used distribution for the loss is Pareto

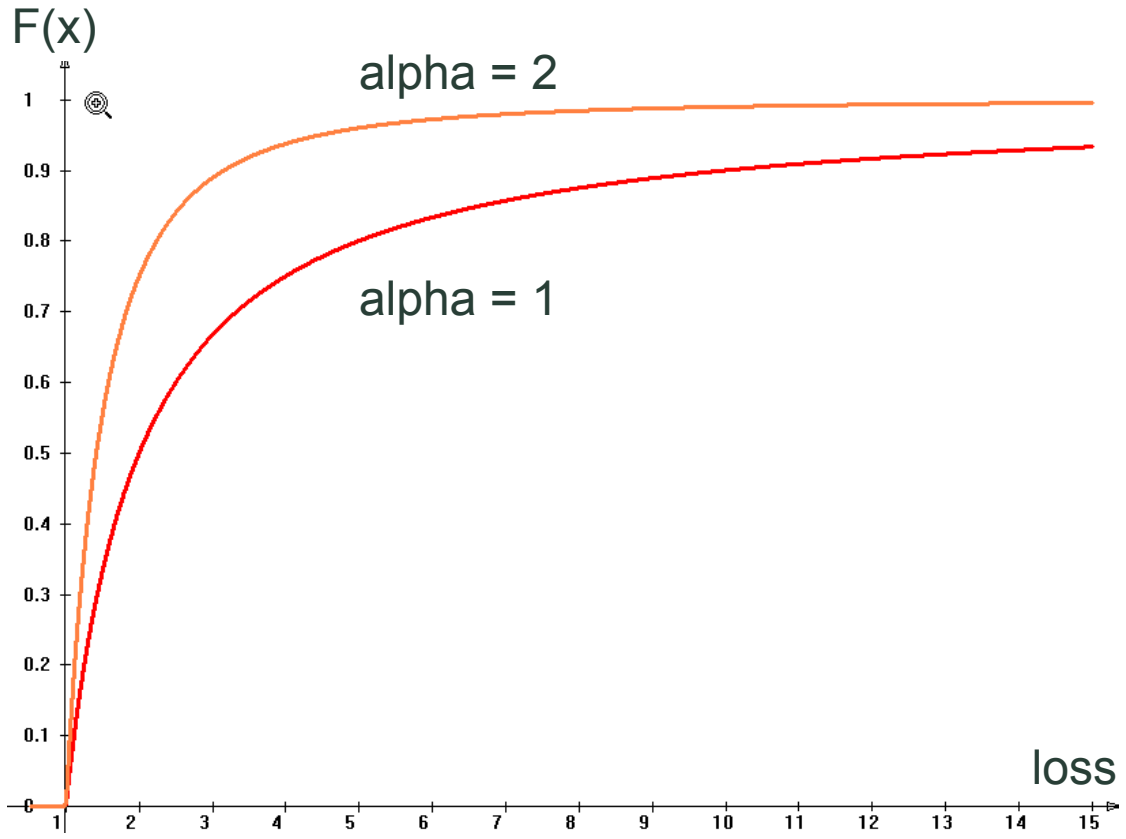
$$F_X(x) = \begin{cases} 1 - (\frac{x}{x_0})^{-\alpha} & x > x_0 \\ 0 & \text{else} \end{cases}$$

$$f_X(x) = \begin{cases} \alpha x_0^{\alpha} x^{-\alpha-1} & x > x_0 \\ 0 & \text{else} \end{cases}$$

Pareto

- The alpha depends on the type of risk.

Loss potential	α
Earthquake/storm	≈ 1
Fire	≈ 2
Fire in industry	≈ 1.5
Motor liability	≈ 2.5
General liability	≈ 1.8
Occupational injury	≈ 2



Frequency

- Very commonly used is the Poisson distribution

Poisson distribution with parameter λ:

$$\lambda > 0$$

$$P[N = k] = e^{-\lambda}\lambda^k/k! \,, \qquad k = 0, 1, \ldots$$

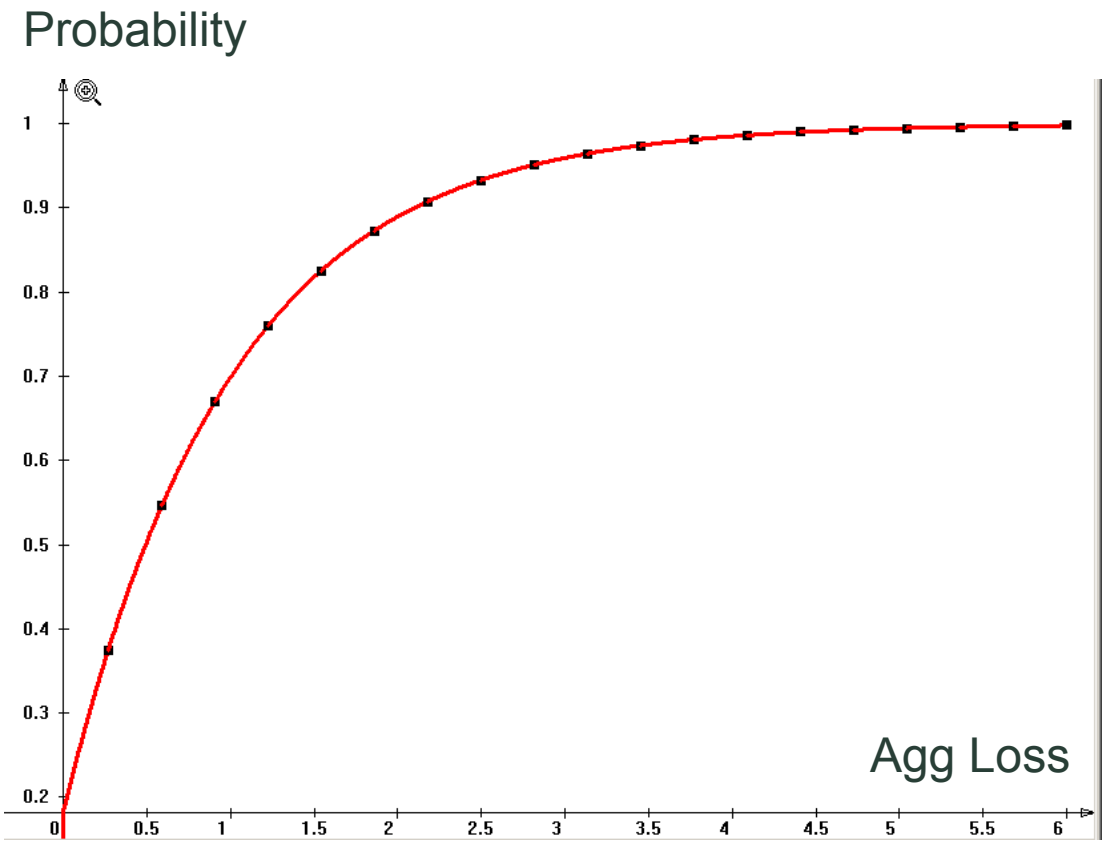
$$E[N] = Var[N] = \lambda$$

- Poisson works fine if events are rare and independent.

The Result

- We now have a distribution for the loss size and loss number to represent S.
- The aggregated cdf is usually calculated with Monte Carlo methods:
  - draw the number of losses per year
  - draw the loss amounts and add them up.
- Ordered by loss amount of the year one can calculate the aggregated CDF.
- The average of these outcomes returns the expected loss.

Aggregated CDF



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More things to consider

But there is more to think of....

- Long term/short term claims
- Capital costs
- Liquidity
- Profit margin
- Brokerage
- Recovery
- Internal costs
- Taxes

And the reserves?

- How much money do we have to reserve now?
- To hold the MPL for all contracts would be way too expensive!
- Therefore we hold reserves cover two 99% shortfall years:

  The shortfall is defined as:
  shf(S) = <S¦ S>Q(99%)>

- We calculate distribution of the losses versus the capital we hold for the whole Swiss Re group.
- There is a possibility that we go bankrupt! Otherwise we would be way too expensive.

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Research areas

- Correlations! For example Pandemic will not only trigger many life insurances, but the stock market will go down, too!
- Avoid surprises! Swiss Re is constantly looking at possible emerging risks as climate change, nano-particles, cell phone radiation etc.

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Nat Cat Modelling

- Hurricanes

Natural Catastrophes

- Swiss Re develops own models for natural catastrophes.
- This covers models for flood, hail, winter storms, earthquakes, bushfires, and tropical cyclones.

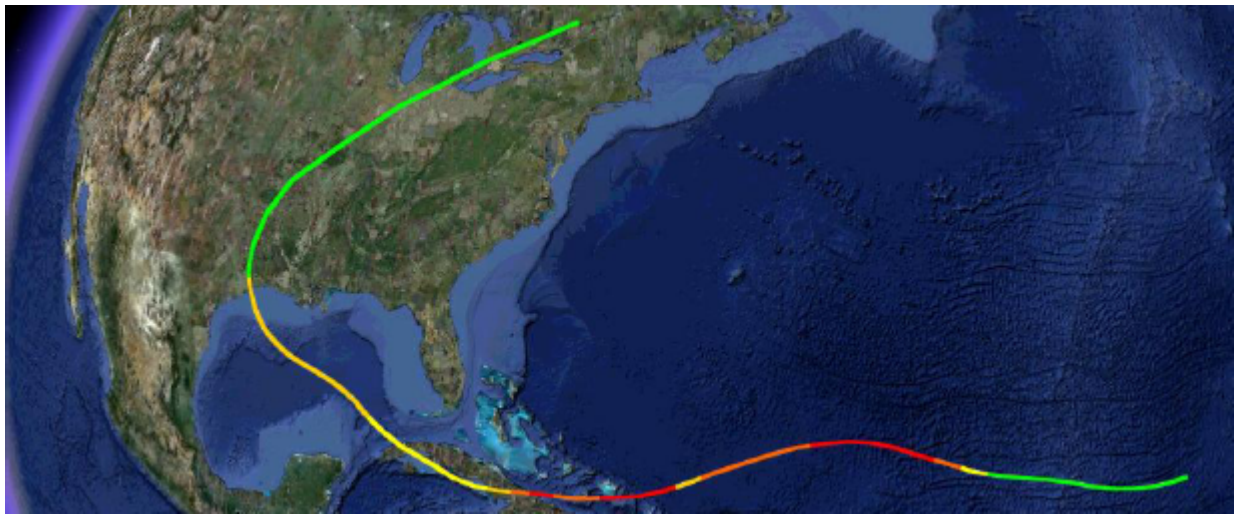
Principles of Nat Cat Modelling

Four basic sets of data are needed to be fed into a loss model:

- **Hazard:**
  Where, how often and with what intensity do events occur?
- **Vulnerability:**
  What is the extent of damage at a given event intensity?
- **Value distribution:**
  Where are the various types of insured objects located and how high is their value?
- **Insurance conditions:**
  What proportion of the loss is insured?

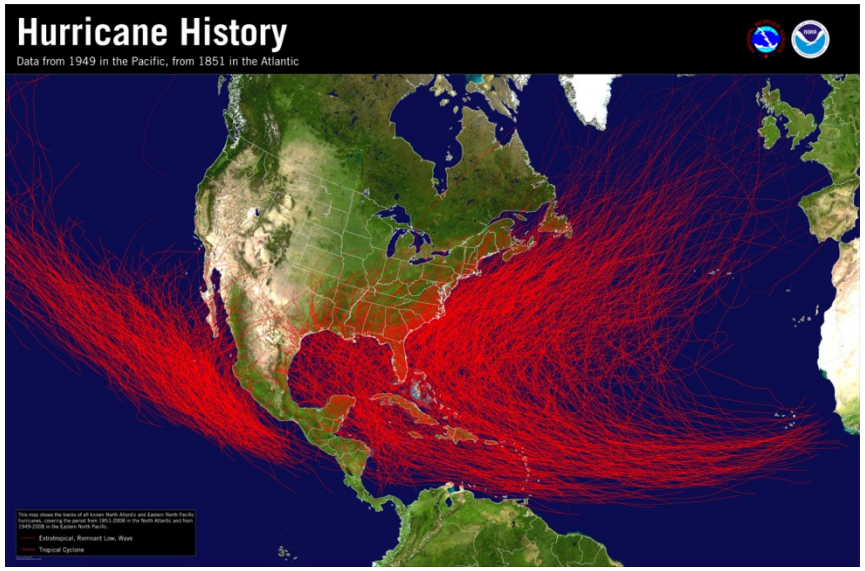
Natural Catastrophes - Hurricanes

- There is a lot of historical data about hurricanes:
- Data is recorded since 1891 of more than 1'000 hurricanes.



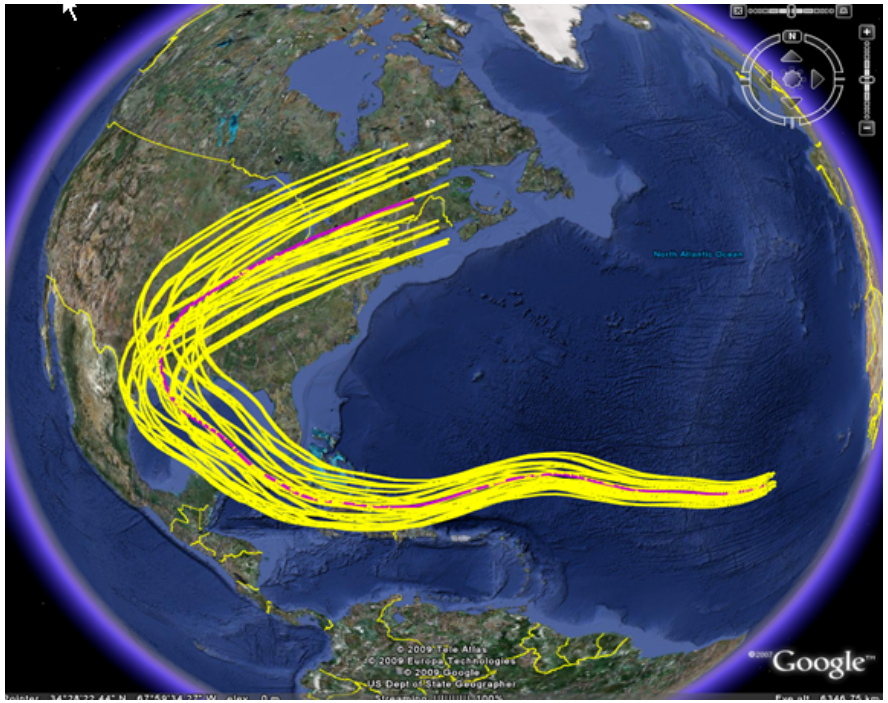
Natural Catastrophes - Hurricanes

- These historical storms can be used as a basis for hurricane modelling.
- Basic Formula to calculate the loss of hurricane
  wind speed * vulnerability * insured value * insurance conditions = loss
- These points consist of many sections, for example...
  - wind speed at a distance x from the storm
  - construction type
  - age of building
  - elevation (storm surge)



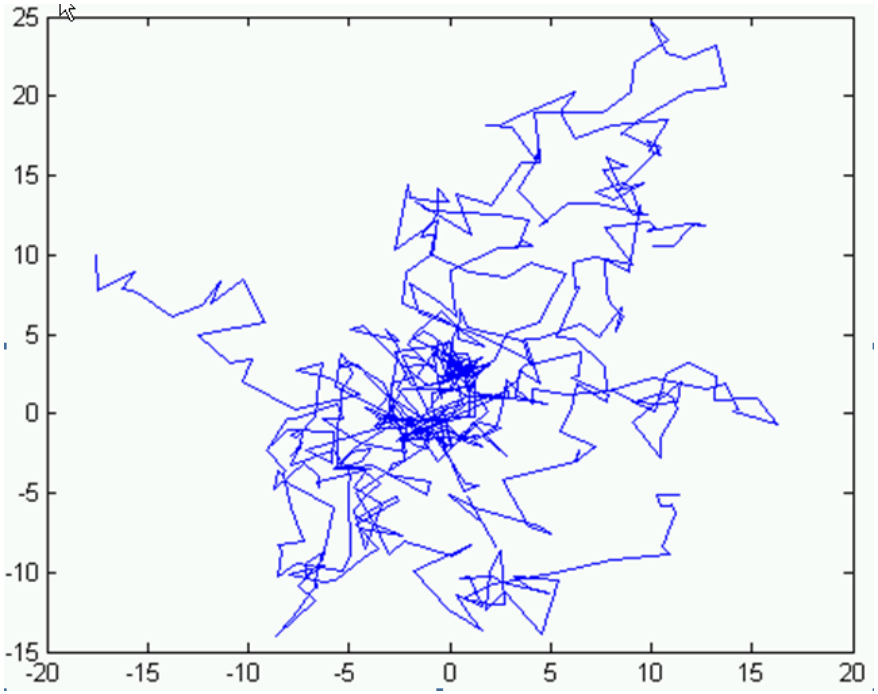
Hurricanes

- The historical storms are not granular enough....
- ... therefore storms have to be invented.



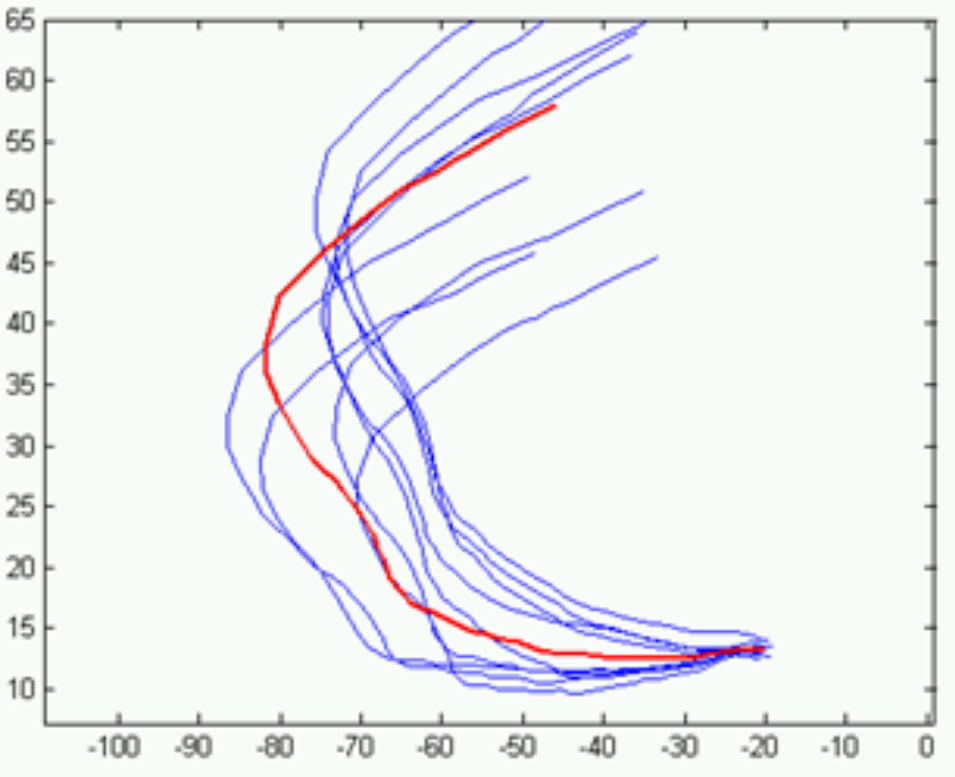
Hurricanes

- How can you create 'daughter storms'?
  → Random Walk



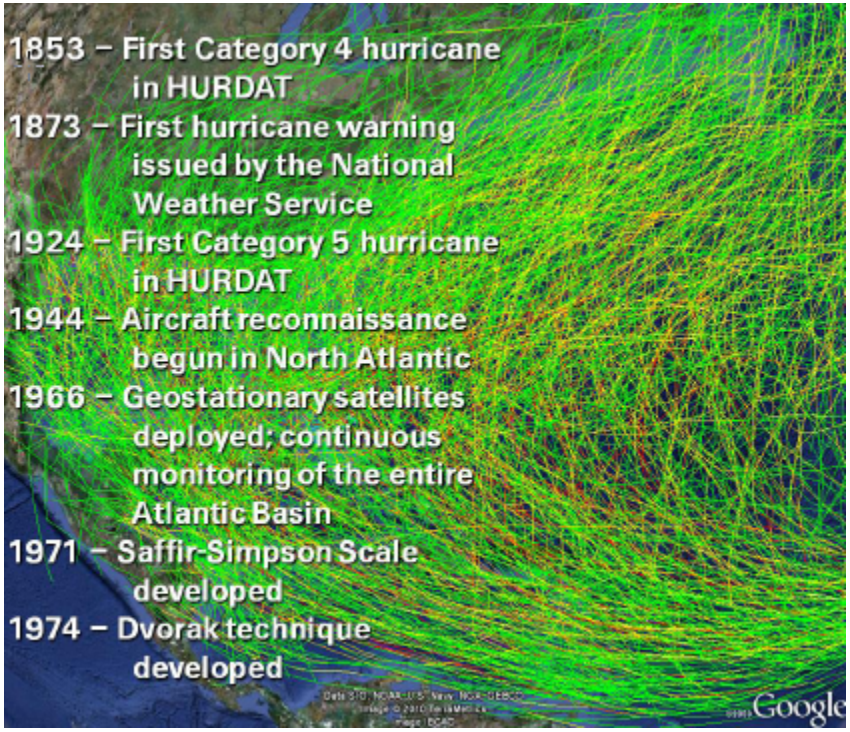
Hurricanes

- Find border conditions to limit the random walk so that the resulting tracks are realistic.



Hurricanes

- How much can a track deviate from the mother storm?
- How much can the wind speed vary?
- Landfall
- Maximum Intensity



Hurricanes - LFC

Event loss	*in millions*
No. 1	23.5
No. 2	42.5
No. 3	74.8
No. 4	8.9
No. 5	13.1
No. 6	69.6
No. 7	20.8
No. 8	33.4
No. 9	17.4
No. 10	11.2
No. 11	26.2
No. 12	58.6

Sum of all event losses	400 million
Number of model years	200 years
Expected loss per year	2 million

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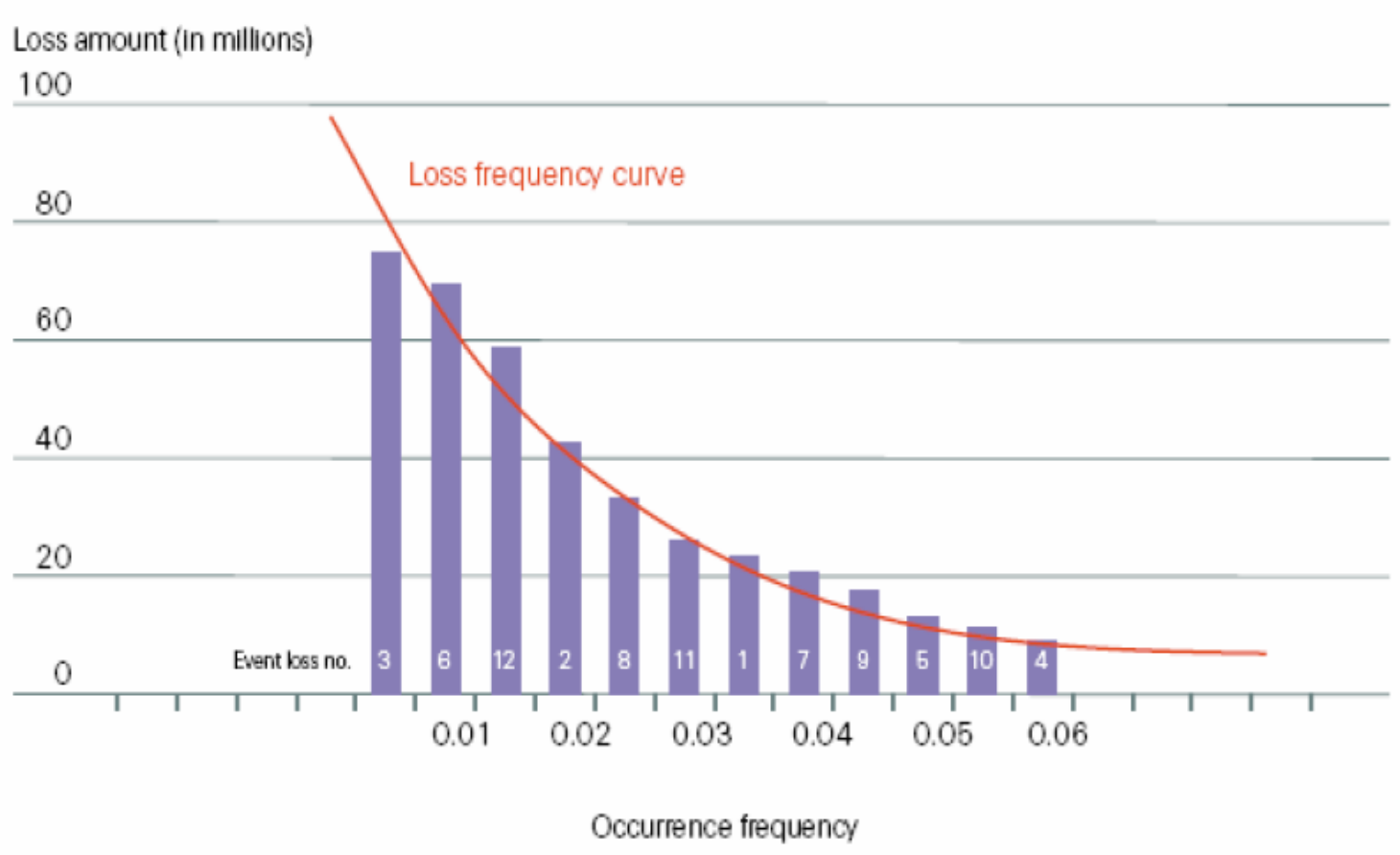
Hurricanes - LFC



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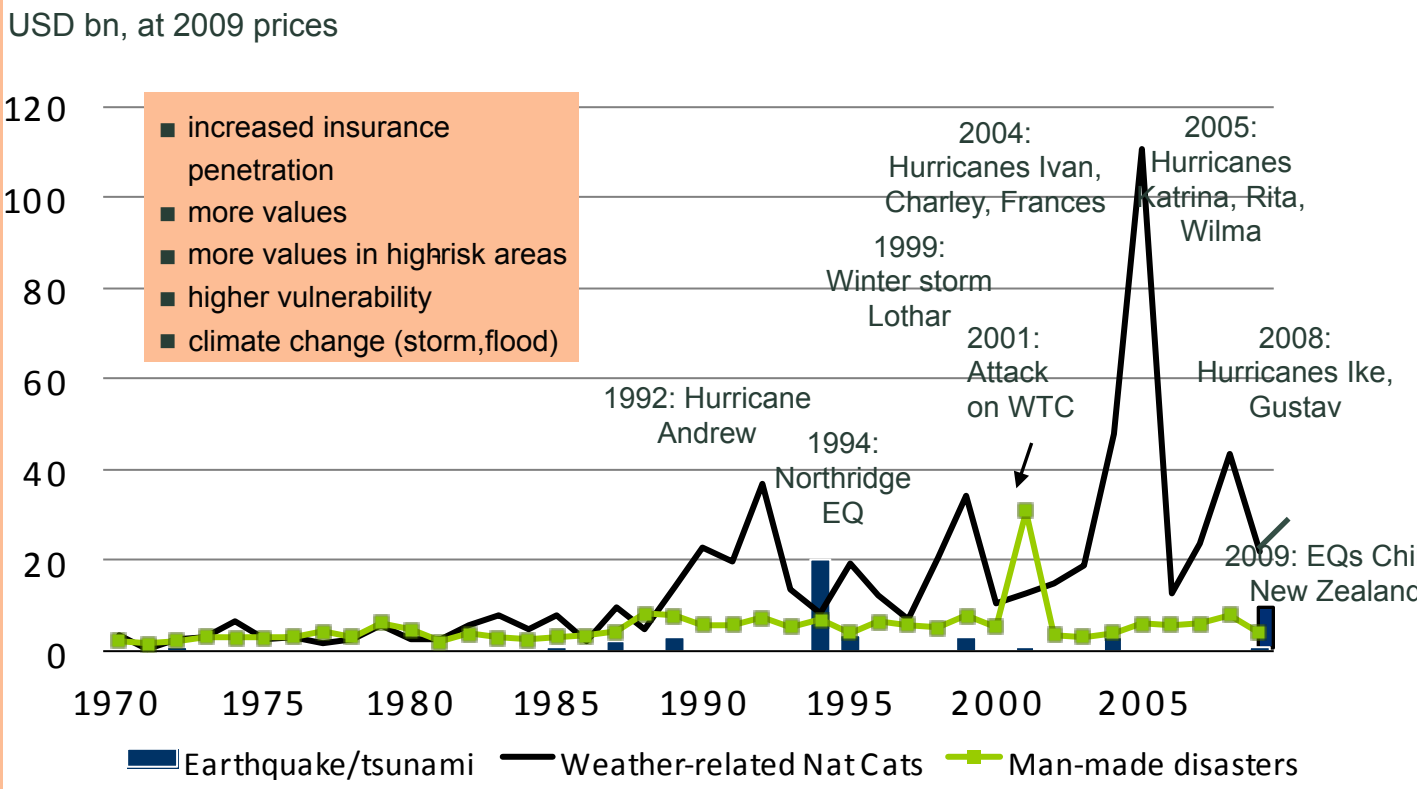
Natural Catastrophes on the rise?

Insured catastrophe losses 1970–2010

Ocean Drive, FL, 1926

Ocean Drive, FL, 2000



Source: Swiss Re, sigma No 1/2010; 1/2011, Figure 3

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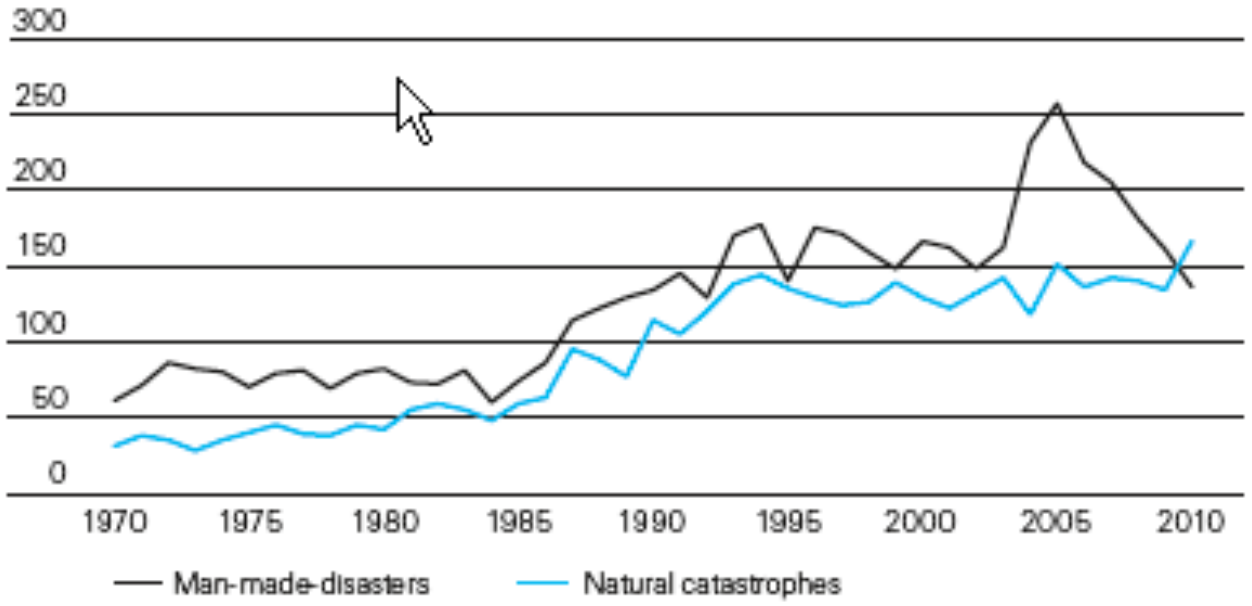
Number of events per year



An event has a loss and victim threshold.

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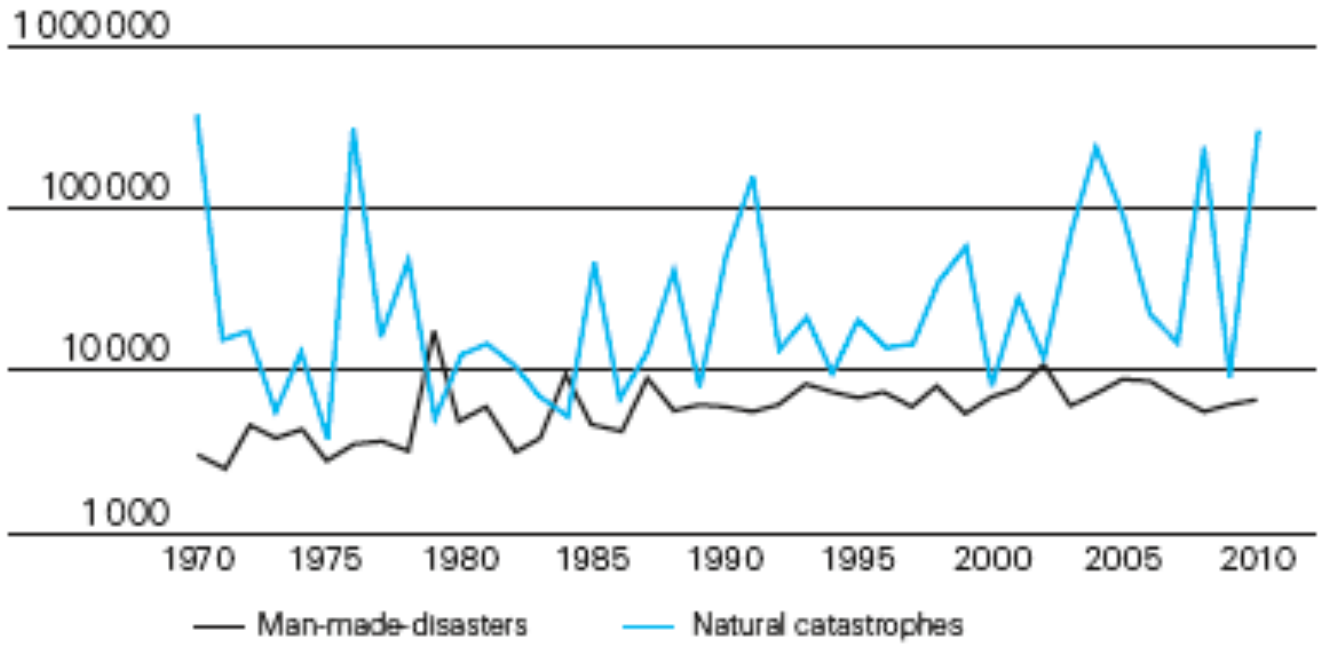
Number of victims per year



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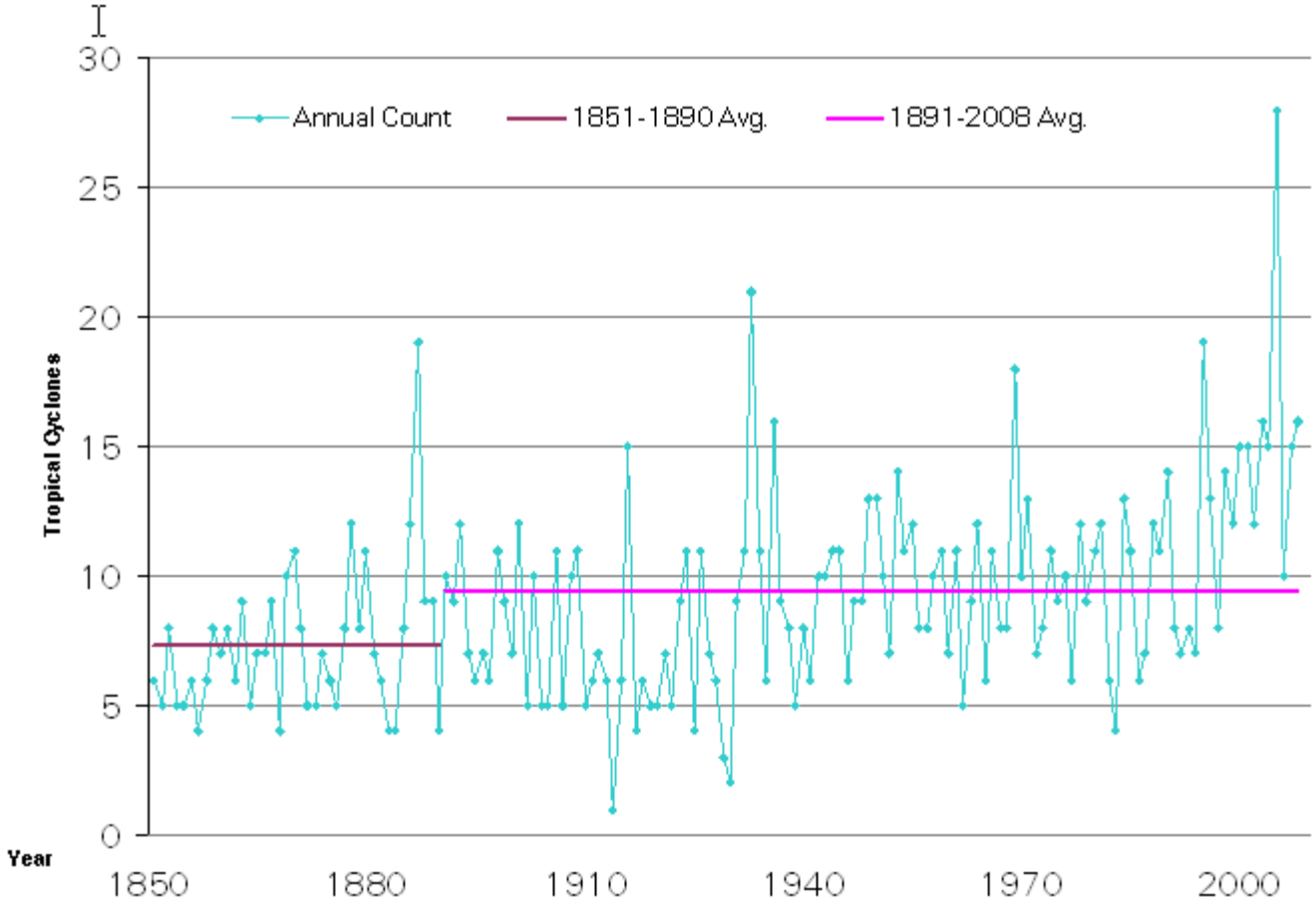
Natural Catastrophes - Hurricanes



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Thank you

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