Physics/Astronomy 226, Problem set 5, Due 2/17

Reading: Ch. 3, start Ch. 4

1. Prove that $R_{\lambda\nu\alpha\beta} + R_{\lambda\alpha\beta\nu} + R_{\lambda\beta\nu\alpha} = 0$.

2. Picture a donut in 3d Euclidean space. It may be either chocolate or glazed, as long as when viewed from the top it looks like two concentric circles of radius r_1 and $r_2 > r_1$. Let $b = (r_1 + r_2)/2$ and $a = (r_2 - r_1)/2$.

   (a) Set up coordinates θ, ϕ on the donut surface (for consistency, let θ label the angle about the center of the donut as measured from above, and ϕ measure the angle around a circular cross-section of the donut.)

   (b) Write down the metric g_{ij} this surface inherits from the Euclidean space it is embedded in.

   (c) Compute all non-vanishing connection coefficients $\Gamma^u_{\alpha\beta}$.

   (d) Compute all nonzero components of $R_{\mu\nu\alpha\beta}$, $R_{\mu\nu}$, and R.

3. The donut hole left over from the above donut has a roughly spherical surface. A sphere with coordinates (θ, ϕ) has metric

$$ds^2 = d\theta^2 + \sin^2\theta d\phi^2.$$

   (a) Show that lines of constant ϕ (longitude) are geodesics, and that the only line of constant θ (latitude) that is a geodesic is the $\theta = \pi/2$ (the equator).

   (b) Take a vector with components $V^\mu = (V^\theta, V^\phi) = (1, 0)$ and parallel-transport it once around a circle of constant latitude, $\theta = \theta_0$. What are the components of the resulting vector, as a function of θ_0?