

The Origin of the Matter-Antimatter Asymmetry

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Although the origin of matter-antimatter asymmetry remains unknown, continuing advances in theory and improved experimental limits have ruled out some scenarios for baryogenesis, for example the sphaleron baryogenesis at the electroweak phase transition in the standard model. At the same time, the success of cosmological inflation and the prospects for discovering supersymmetry at the LHC have put some other models in sharper focus. We review the current state of our understanding of baryogenesis with the emphasis on those scenarios that we consider most plausible.

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I. INTRODUCTION

When we observe the universe, the most obvious, and easily studied, objects are stars and gas, made up of protons, neutrons and electrons. Astrophysicists speak of the protons and neutrons, which constitute the bulk of the mass of this matter, as the baryon content of the universe.

But we know that there is much more to the universe than baryons. By indirect means, astronomers have established that approximately 1/3 of the energy density of the universe is in the form of some non-baryonic matter, referred

to as the dark matter, while $2/3$ is in a form with negative pressure, perhaps a cosmological constant. The baryons make up a mere 5% of the total energy density of the universe.

Another, even more striking measure of the smallness of the baryon density is provided by the ratio of baryons to photons in the Cosmic Microwave Radiation Background (CMBR). Big Bang nucleosynthesis gives a good measure of the baryon density; this measurement is well supported by recent measurements of the fluctuations of the cosmic microwave radiation background. As a result, the ratio of baryons to photons is now known to about 5% (Bennett *et al.*, 2003):

$$\frac{n_B}{n_\gamma} = (6.1^{+0.3}_{-0.2}) \times 10^{-10}. \quad (1)$$

There is good evidence that there are no large regions of antimatter at any but cosmic distance scales (Cohen, De Rújula, and Glashow, 1998).

It was A. Sakharov who first suggested that the baryon density might not represent some sort of initial condition, but might be understandable in terms of microphysical laws (Sakharov, 1967). He listed three ingredients to such an understanding:

1. Baryon number violation must occur in the fundamental laws. This is necessary for two reasons. First, if at very early times, baryon number violating interactions were in equilibrium, then the universe can be said to have “started” with zero baryon number. Starting with zero baryon number, baryon number violating interactions are obviously necessary if the universe is to end up with a non-zero asymmetry.
2. CP-violation: If CP (the product of charge conjugation and parity) is conserved, every reaction which produces a particle will be accompanied by a reaction which produces its antiparticle at precisely the same rate, so no baryon number can be generated.
3. An Arrow of Time (Departure from Thermal Equilibrium): The universe, for much of its history, was very nearly in thermal equilibrium. The spectrum of the CMBR is the most perfect blackbody spectrum measured in nature. So the universe was certainly in thermal equilibrium 10^5 years after the big bang. The success of the theory of big bang nucleosynthesis (BBN) provides strong evidence that the universe was in equilibrium two-three minutes after the big bang. But if, through its early history, the universe was in thermal equilibrium, then even B and CP violating interactions could not produce a net asymmetry. One way to understand this is to recall that the CPT theorem assures strict equality of particle and antiparticle masses, so at thermal equilibrium, the densities of particles and antiparticles are equal. More precisely, since B is odd under CPT, its thermal average vanishes in an equilibrium situation. This can be generalized by saying that the universe must have an arrow of time.

One of the great successes of the Standard Model is that it explains why baryon and lepton number are conserved, to a very good approximation. To understand what this means, consider first the modern understanding of Maxwell’s equations. A quantum field theory is specified by its field content and by a lagrangian density. In the lagrangian, one distinguishes renormalizable and non-renormalizable terms. Renormalizable terms have coefficients with mass dimension greater than zero; non-renormalizable terms have coefficients (couplings) with mass dimension less than zero. For example, in quantum electrodynamics, the electron mass has dimension one, while the charge of the electron is dimensionless (throughout we use conventions where \hbar and c are dimensionless). Requiring Lorentz invariance, gauge invariance, and renormalizability leaves only one possibility for the lagrangian of electrodynamics: the Maxwell lagrangian, whose variation yields Maxwell’s equations. One can, consistent with these symmetry principles, write down an infinite number of possible non-renormalizable terms, which would yield non-linear modifications of Maxwell’s equations. There is nothing wrong with these, but they are characterized by a mass, or inverse length scale, M . So the size of non-linear corrections at wavelength λ is of order $(\lambda M)^{-n}$ for some integer n . M represents some scale at which the laws of electricity and magnetism might be significantly modified. Such corrections actually exist, and are for most purposes quite small.

Similarly, in the Standard Model, at the level of renormalizable terms, there are simply no interactions one can write which violate either baryon number or the conservation of the separate lepton numbers (electron, muon and tau number). It is possible to add dimension five operators (suppressed by some scale $1/M$) which violate lepton number, and dimension six operators (suppressed by $1/M^2$), which violate baryon number. Again, these non-renormalizable terms must be associated with some mass scale of some new baryon and lepton violating physics. The dimension five lepton-number violating operators would give rise to a mass for the neutrinos. The recent discovery of neutrino mass probably amounts to a measurement of some of these lepton number violating operators. The scale of new physics associated with these operators can not yet be determined, but theoretical arguments suggest a range of possibilities, between about 10^{11} and 10^{16} GeV.

The question, then, is what might be the scale, M_B associated with baryon number violation. At the very least, one expects quantum effects in gravity to violate all global quantum numbers (e.g. black holes swallow up any quantum numbers not connected with long range fields like the photon and graviton), so $M_B \leq M_p$, where M_p , the Planck mass, is about 10^{19} GeV, or $(2 \times 10^{-33} \text{cm})^{-1}$. The leading operators of this kind, if they have Planck mass coefficients, would lead to a proton lifetime of order 10^{34} years or so.

If quantum gravitational effects were the only source of baryon number violation, we could imagine that the baryon asymmetry of the universe was produced when the temperature of the universe was of order the Planck energy (10^{32} °K). Some complex processes associated with very energetic configurations would violate baryon number. These need not be in thermal equilibrium (indeed, in a theory of gravity, the notion of equilibrium at such a high temperature almost certainly does not make sense). The expansion of the universe at nearly the moment of the big bang would provide an arrow of time. CP is violated already at relatively low energies in the Standard Model (through the so-called Kobayashi-Maskawa (KM) mechanism), and so there is every reason to believe that it is badly violated in very high energy processes. So we could answer Sakharov by saying that the magnitude of the baryon number is the result of some very complicated, extremely high energy process, to which we will never have experimental access. It might be, in effect, an initial condition.

There are good reasons to believe that this pessimistic picture is not the correct one. First, we are trying to understand a small, dimensionless number. But in this Planck scale baryogenesis picture, it is not clear how such a small dimensionless number might arise. Second, there is growing evidence that the universe underwent a period of inflation early in its history. During this period, the universe expanded rapidly by an enormous factor (at least e^{60}). Inflation is likely to have taken place well below the scale of quantum gravity, and thus any baryon number produced in the Planck era was diluted to a totally negligible level. Third, there are a variety of proposals for new physics – as well as some experimental evidence – which suggests that baryon and lepton number violating interactions might have been important at scales well below the Planck scale. So there is some reason for optimism that we might be able to compute the observed baryon number density from some underlying framework, for which we could provide both direct (i.e. astrophysical or cosmological) and/or indirect (discovery of new particles and interactions) evidence.

Several mechanisms have been proposed to understand the baryon asymmetry:

1. Planck scale baryogenesis: this is the idea, discussed above, that Planck scale phenomena are responsible for the asymmetry. We have already advanced arguments (essentially cosmological) that this is unlikely; we will elaborate on them in the next section.
2. GUT baryogenesis: this, the earliest well-motivated scenario for the origin of the asymmetry, will be discussed more thoroughly in the next section. The main objections to this possibility come from issues associated with inflation and with the production of gravitinos. Both suggest that the universe was not hot enough in the past for this mechanism to proceed.
3. Electroweak baryogenesis: as we will explain, the Standard Model satisfies all of the conditions for baryogenesis. This is somewhat surprising, since at low temperatures the model seems to preserve baryon number, but it turns out that baryon and lepton number are badly violated at very high temperatures. The departure from thermal equilibrium can arise at the electroweak phase transition – a transition between the familiar state in which the W and Z bosons are massive and one in which they are massless. This transition can be first order, providing an arrow of time. It turns out, however, that as we will explain below, given known experimental constraints, any baryon asymmetry produced is *far* too small to account for observations. In certain extensions of the Standard Model, it is possible to obtain an adequate asymmetry, but in most cases the allowed region of parameter space is very small.
4. Leptogenesis: The observation that the weak interactions will convert some lepton number to baryon number means that if one produces a large lepton number at some stage, this will be processed into a net baryon and lepton number. The observation of neutrino masses makes this idea highly plausible. Many but not all of the relevant parameters can be directly measured.
5. Production by coherent motion of scalar fields (the Affleck-Dine mechanism): This mechanism, which can be highly efficient, might well be operative if nature is supersymmetric. So again, one might hope to measure some (but not all) of the relevant parameters. This mechanism will be the main focus of this review.

In this review we will survey these, and then focus on what are widely viewed as the most promising possibilities. The question is: can we eventually establish that one or another is correct? In order to establish or rule out particular models for the origin of the matter-antimatter asymmetry, we would hope to bring to bear both astrophysical/cosmological and particle physics arguments. Ideally, we would some day be in the position of measuring all of the parameters relevant to the asymmetry, and calculating the asymmetry in much the same way that one presently calculates the light element abundances. One question we will ask is: how close can we come to this ideal situation?

We cannot decisively rule out any of these models for baryogenesis at the present time. But we can give some general arguments against the first two, and much of the parameter space for the third has been ruled out by searches for the Higgs particle. The last two look more promising, and will be the subject of extensive experimental study over the next decade.

In the next section, after a very brief review of the standard cosmology, we survey these mechanisms, both explaining how they work and discussing their theoretical plausibility. Both electroweak baryogenesis and leptogenesis rely on the existence of processes within the standard model which violate baryon and lepton number at high temperatures, and we include a brief explanation of this phenomenon.

We then turn to a more detailed discussion of coherent production of baryons or leptons, the Affleck-Dine (AD) mechanism. This mechanism is potentially extremely efficient; it can also operate relatively late in the history of the universe. As a result, it can potentially resolve a number of cosmological puzzles. The AD mechanism presupposes low energy supersymmetry. Supersymmetry (sometimes called SUSY for short) is a hypothetical extension of Poincare invariance, a symmetry which would relate bosons to fermions. If correct, it predicts that for every boson of the standard model, there is a fermion, and vice versa. It is believed that the masses of the new particles should be about a TeV. As supersymmetry will play an important role in much of our discussion, a brief introduction to supersymmetry will be provided in the next section. The supersymmetry hypothesis will be tested over the next decade by the Tevatron and the Large Hadron Collider at CERN. Interestingly, most other proposals for baryogenesis invoke supersymmetry in some way (including electroweak baryogenesis and most detailed models for leptogenesis).

II. A BARYOGENESIS ROADMAP

A. A Cosmology Overview

Our knowledge of the Big Bang rests on a few key observational elements. First, there is the Hubble expansion of the universe. This allows us to follow the evolution of the universe to a few billion years after the Big Bang. Second, there is the CMBR. This is a relic of the time, about 10^5 years after the Big Bang, when the temperature dropped to a fraction of an electron volt and electrons and nuclei joined to form neutral atoms. Third, there is the abundance of the light elements. This is a relic of the moment of neutrino decoupling, when the temperature was about 1 MeV. As we have noted, theory and observation are now in good agreement, yielding the baryon to photon ratio of equation (1). Finally, there are the fluctuations in the temperature of the microwave background, measured recently on angular scales below one degree by BOOMERANG (de Bernardis *et al.*, 2000; Netterfield *et al.*, 2002), MAXIMA (S. Hanany *et al.*, 2000), DASI (Netterfield *et al.*, 2002), and WMAP (Bennett *et al.*, 2003). These fluctuations are probably a relic of the era of inflation (discussed in more detail below). The baryon density can be inferred independently from the CMBR data and from the BBN determination of the baryon density based on the measurements of the primordial deuterium abundance (Burles, Nollett, Turner, 2001; Kirkman *et al.*, 2003). The agreement is spectacular: $\Omega_B h^2 = 0.0214 \pm 0.002$ based on BBN (Kirkman *et al.*, 2003), while the CMBR anisotropy measurements yield $\Omega_B h^2 = 0.0224 \pm 0.0009$ (Bennett *et al.*, 2003).

The first and perhaps most striking lesson of the measurements of the CMBR is that the universe, on large scales, is extremely homogeneous and isotropic. As a result, it can be described by a Robertson-Walker metric:

$$ds^2 = dt^2 - R^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right). \quad (2)$$

$R(t)$ is known as the scale factor; the Hubble “constant” is $H = \frac{\dot{R}}{R}$. The Hubble constant is essentially the inverse of the time; in the radiation dominated era, $H = \frac{1}{2t}$; in the matter dominated era, $H = \frac{2}{3t}$. It is puzzling that the universe should be homogeneous and isotropic to such a high degree. If one runs the clock backward, one finds that vast regions of the universe which have only recently been in causal contact have essentially the same temperature.

Inflation provides an explanation for this and other puzzles (Kolb and Turner, 1990; Linde, 1990). The basic idea (Albrecht and Steinhardt, 1982; Guth, 1981; Linde, 1982) is that for a brief period, $R(t)$ grew extremely rapidly, typically exponentially. This has several effects:

- The observed universe grew from a microscopically small region, explaining homogeneity and isotropy.
- $k = 0$, i.e. the universe is spatially flat. This is now well-verified by observations.
- Small fluctuations in the metric and the field during inflation explain the observed small (part in 10^{-5}) variation in the temperature of the CMBR; detailed features of this structure, in agreement with the inflationary theory, have now been observed. These fluctuations provide the seeds for formation of the observed structure in the universe.

- Inflation also explains the absence from the universe of objects such as magnetic monopoles expected in many particle physics theories.

While it is probably fair to say that no compelling microscopic theory of inflation yet exists, as a phenomenological theory, inflation is very successful. Most pictures of inflation invoke the dynamics of a scalar field in a crucial way. This scalar field must have very special properties. Typically, for example, the curvature of its potential must be very small. The most plausible theories which achieve this invoke supersymmetry in a significant way. Supersymmetry, a hypothetical symmetry between fermions and bosons, will be discussed at greater length later in this article. It has been widely considered as a possible solution to many puzzles in particle physics. Most importantly for inflation, supersymmetry is a theoretical framework which naturally gives rise to scalars with very flat potentials. It also, almost automatically, gives rise to stable particles with just the right properties to constitute the dark matter. There are difficulties as well. One is associated with the fermionic partner of the graviton, the gravitino. In many models, this particle is very long-lived ($\tau > 10^6$ sec), and can spoil conventional Big Bang cosmology if too many are produced.

There is not space here to review the subject of inflation. Instead, we will give a “narrative” of a possible history of the universe, which will be useful to orient our discussion.

- Before $t \approx 10^{-25}$ seconds, the universe was very inhomogeneous, with an extremely large energy density. At $t \approx 10^{-25}$, inflation began in a small patch. This was associated with a scalar field, called the inflaton, which moved slowly toward the minimum of its potential.
- The scale of the inflaton potential was of order 10^{60} GeV⁴, give or take a few orders of magnitude.
- During inflation, the scale factor increased by an enormous factor. Any conserved or approximately conserved charges, such as monopole number or baryon number, were reduced by at least a factor of 10^{60} in this process.
- Inflation ended as the inflaton approached the minimum of its potential. At this point, decays of the inflaton lead to reheating of the universe to a high temperature. Depending on the detailed microscopic picture, there are constraints on the reheating temperature. If nature is supersymmetric, there is often a danger of producing too many gravitinos and other long-lived particles. Typically, this constrains the reheating temperature to be below 10^9 GeV. Even without supersymmetry, detailed inflationary models have difficulty producing high reheating temperatures without fine tuning.
- The baryon asymmetry is generated some time after the era of inflation. Any upper limit on the reheating temperature constrains the possible mechanisms for baryogenesis.

B. Planck Scale Baryogenesis

It is generally believed that a quantum theory of gravity cannot preserve any global quantum numbers. For example, in the collapse of a star to form a black hole, the baryon number of the star is lost; black holes are completely characterized by their mass, charge and angular momentum. Virtual processes, then, would also be expected to violate baryon number.

In string theory, the only consistent quantum theory of gravity we know, these prejudices are born out. There are no conserved global symmetries in string theory (Banks *et al.*, 1988). While we can't reliably extract detailed predictions from quantum gravity for baryon number violation, we might expect that it will be described at low energies by operators which appear in an effective field theory. The leading operators permitted by the symmetries of the Standard model which violate baryon number carry dimension six. An example is:

$$\mathcal{L}_B = \frac{1}{M^2} \bar{e} \bar{d}^* \bar{d}^* \bar{d}^* \quad (3)$$

In this equation and those which follow, the various fields, d , \bar{d} , e , \bar{e} , ν , etc. are spinors of left-handed chirality. \bar{d} , for example, contains the creation operator for a left-handed anti-d particle. \bar{d} contains the creation operator for the right-handed d quark; d^\dagger for the left-handed anti-d quark. The other two d-quark states are created by d and d^\dagger . Because the operator is of dimension six, we have indicated that its coefficient has dimensions of inverse mass-squared. This is analogous to the effective interaction in the Fermi theory of weak interactions. If quantum gravity is responsible for this term, we might expect its coefficient to be of order $1/M_p^2$, where $M_p = \sqrt{G_N^{-1}} = 10^{19}$ GeV.

Because of this very tiny coefficient, these effects could be important only at extremely early times in the universe, when, for example, $H \sim M_p$. It is probably very difficult to analyze baryon production in this era. It is certainly unclear in such a picture where the small number 10^{-10} might come from. But even if the baryon number was produced in this era, it was completely washed out in the subsequent period of inflation. So gravitational baryogenesis seems unlikely to be the source of the observed matter-antimatter asymmetry.

C. GUT baryogenesis

The earliest well-motivated scenarios for implementing Sakharov's ideas within a detailed microscopic theory were provided by grand unified theories (GUTs)(Kolb and Turner, 1990). In the Standard Model, the strong, weak and electromagnetic interactions are described by non-abelian gauge theories based on the groups $SU(3)$, $SU(2)$ and $U(1)$. Grand unification posits that the underlying theory is a gauge theory with a simple group; this gauge symmetry is broken at some very high energy scale down to the group of the Standard Model. This hypothesis immediately provides an explanation of the quantization of electric charge. It predicts that, at very high energies, the strong, weak and electromagnetic couplings (suitably normalized) should have equal strength. And most important, from the point of view of this article, it predicts violation of baryon and lepton numbers.

If nature is not supersymmetric, the GUT hypothesis fails. One can use the renormalization group to determine the values of the three gauge couplings as a function of energy, starting with their measured values. One finds that they do not meet at a point, i.e. there is no scale where the couplings are equal. Alternatively, one can take the best measured couplings, the $SU(2)$ and $U(1)$ couplings, and use the GUT hypothesis to predict the value of the strong coupling. The resulting prediction is off by 12 standard deviations (Particle Data Group, 2002). But if one assumes that nature is supersymmetric, and that the new particles predicted by supersymmetry all have masses equal to 1 TeV, one obtains unification, within 3σ . The scale of unification turns out to be $M_{gut} \approx 2 \times 10^{16}$. Relaxing the assumption that the new particles are degenerate, or assuming that there are additional, so-called threshold corrections to the couplings at the GUT scale ($\sim 4\%$) can yield complete agreement.

This value of M_{gut} is quite interesting. It is sufficiently below the Planck scale that one might hope to analyze these theories without worrying about quantum gravity corrections. Moreover, it leads to proton decay at a rate which may be accessible to current proton decay experiments. In fact, the simplest SUSY GUT, based on the gauge group $SU(5)$ is almost completely ruled out by the recent Super-Kamiokande bounds (Murayama and Pierce, 2002). However, there are many other models. For example, non-minimal $SU(5)$ or $SO(10)$ SUSY GUTs may have a proton lifetime about a factor of 5 above the present experimental limit (Altarelli, Feruglio and Masina, 2000; Babu, Pati and Wilczek, 2000; Bajc, Perez and Senjanovic, 2002; Dermisek, Mafi and Raby, 2001). Witten has recently advocated an approach to GUT model building (Friedmann and Witten, 2002; Witten, 2002) which resolves certain problems with these models, and in which proton decay might be difficult to see even in large detectors which are being considered for the future.

GUTs provide a framework which satisfies all three of Sakharov's conditions. Baryon number violation is a hallmark of these theories: they typically contain gauge bosons and other fields which mediate B violating interactions such as proton decay. CP violation is inevitable; necessarily, any model contains at least the KM mechanism for violating CP, and typically there are many new couplings which can violate CP. Departure from equilibrium is associated with the dynamics of the massive, B-violating fields. Typically one assumes that these fields are in equilibrium at temperatures well above the grand unification scale. As the temperature becomes comparable to their mass, the production rates of these particles fall below their rates of decay. Careful calculations in these models often leads to baryon densities compatible with what we observe.

We can illustrate the basic ideas with the simplest GUT model, due to Georgi and Glashow (1974). Here the unifying gauge group is $SU(5)$. A single generation of the standard model (e.g. electron, electron neutrino, u-quark, and d-quark) can be embedded in the $\bar{5}$ and 10 representation of $SU(5)$. It is natural to identify the fields in the $\bar{5}$ representation as

$$\bar{5}_i = \begin{pmatrix} \bar{d} \\ \bar{d} \\ \bar{d} \\ e \\ \nu \end{pmatrix}. \quad (4)$$

Then the generators of $SU(3)$ of color are identified as:

$$T = \begin{pmatrix} \frac{\lambda^a}{2} & 0 \\ 0 & 0 \end{pmatrix} \quad (5)$$

where λ^a are the Gell-Mann matrices, while those of $SU(2)$ are identified with:

$$T = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\sigma^i}{2} \end{pmatrix} \quad (6)$$

The $U(1)$ generator is

$$Y' = \frac{1}{\sqrt{60}} \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}. \quad (7)$$

Here the coefficient has been chosen so that the normalization is the same as that of the $SU(3)$ and $SU(2)$ matrices ($\text{Tr}(T^a T^b) = \delta_{ab}$). In this way, the corresponding gauge boson couples with the same strength as the gluons and W and Z bosons.

In the standard model, the hypercharge is related to the ordinary electric charge, Q , and the isospin generator, T_3 , by $Q = T_3 + \frac{Y}{2}$. So one sees that electric charge is quantized, and that $Y = \sqrt{3/40} Y'$. Since Y couples with the same strength as the $SU(2)$ generators, this gives a prediction of the $U(1)$ coupling of the standard model, and correspondingly of the Weinberg angle, $\sin^2(\theta_W) = 3/8$. This prediction receives radiative corrections, which, assuming supersymmetry, bring it within experimental errors of the measured value.

In a single generation, the remaining fields lie in the 10 representation. The ten transforms as the antisymmetric product of two 5's. It has the form

$$10_{ij} = \begin{pmatrix} 0 & \bar{u}_2 & -\bar{u}_1 & Q_1^1 & Q_1^2 \\ -\bar{u}_2 & 0 & \bar{u}_3 & Q_2^1 & Q_2^2 \\ \bar{u}_1 & -\bar{u}_3 & 0 & Q_3^1 & Q_3^2 \\ -Q_1^1 & -Q_2^1 & -Q_3^1 & 0 & \bar{e} \\ -Q_1^2 & -Q_2^2 & -Q_3^2 & -\bar{e} & 0 \end{pmatrix}. \quad (8)$$

$SU(5)$ is not a manifest symmetry of nature. It can be broken by the expectation value of a scalar field in the adjoint representation with the same form as Y :

$$\Phi = v \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}. \quad (9)$$

The unbroken generators are those which commute with Φ , i.e. precisely the generators of $SU(3) \times SU(2) \times U(1)$ above.

The vector bosons which correspond to the broken generators gain mass of order gv . We will refer to the corresponding gauge bosons as X ; they are associated with generators such as:

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (10)$$

They carry color and electroweak quantum numbers and mediate processes which violate baryon number. For example, examining the structure of 4, one sees that there is a coupling of the X bosons to a \bar{d} quark and an electron. Similarly, there is a coupling of the X boson to a quark doublet and a positron. Note that there is no way to assign baryon and lepton number to the X boson so that it is conserved by these couplings.

In the GUT picture of baryogenesis, it is usually assumed that at temperatures well above the GUT scale, the universe was in thermal equilibrium. As the temperature drops below the mass of the X bosons, the reactions which produce the X bosons are not sufficiently rapid to maintain equilibrium. The decays of the X bosons violate baryon number; they also violate CP. So all three conditions are readily met: baryon number violation, CP violation, and departure from equilibrium.

To understand in a bit more detail how the asymmetry can come about, note that CPT requires that the total decay rate of X is the same as that of its antiparticle \bar{X} . But it does not require equality of the decays to particular final states (partial widths). So starting with equal numbers of X and \bar{X} particles, there can be a slight asymmetry between the processes

$$X \rightarrow dL; X \rightarrow \bar{Q}\bar{u} \quad (11)$$

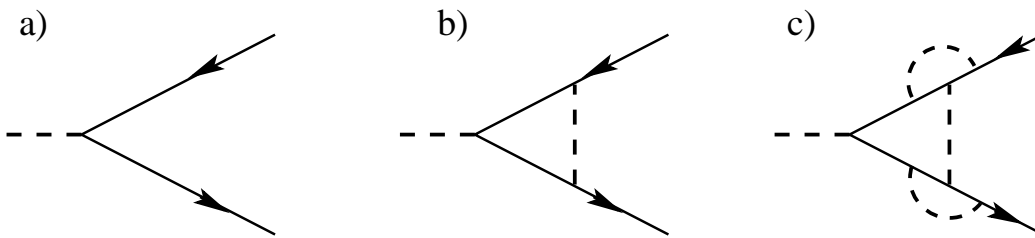


FIG. 1 Interference between the tree-level (a) and one-loop (b) diagrams with complex Yukawa couplings can provide the requisite source of CP violation for GUT baryogenesis. In viable models, to avoid the unwanted cancellations, one must often assume that the two scalars are different or go to higher loops (c) (Barr, Segrè and Weldon, 1979; Kolb and Turner, 1990).

and

$$\bar{X} \rightarrow \bar{d}\bar{L}; \bar{X} \rightarrow Qu. \quad (12)$$

The tree graphs for these processes are necessarily equal; any CP violating phase simply cancels out when we take the absolute square of the amplitude. This is not true in higher order, where additional phases associated with real intermediate states can appear. Actually computing the baryon asymmetry requires a detailed analysis, of a kind we will encounter later when we consider leptogenesis.

There are reasons to believe, however, that GUT baryogenesis is not the origin of the observed baryon asymmetry. Perhaps the most compelling of these has to do with inflation. Assuming that there was a period of inflation, any pre-existing baryon number was greatly diluted. So in order that one produce baryons through X boson decay, it is necessary that the reheating temperature after inflation be at least comparable to the X boson mass. But as we have explained, a reheating temperature greater than 10^9 GeV leads to cosmological difficulties, especially overproduction of gravitinos.

D. Electroweak Baryon Number Violation

Earlier, we stated that the renormalizable interactions of the Standard Model preserve baryon number. This statement is valid classically, but it is not quite true of the quantum theory. There are, as we will see in this section, very tiny effects which violate baryon number ('t Hooft, 1976). These effects are tiny because they are due to quantum mechanical tunneling, and are suppressed by a barrier penetration factor. At high temperatures, there is no such suppression, so baryon number violation is a rapid process, which can come to thermal equilibrium. This has at least two possible implications. First, it is conceivable that these “sphaleron” processes can themselves be responsible for generating a baryon asymmetry. This is called electroweak baryogenesis (Kuzmin, Rubakov, and Shaposhnikov, 1985). Second, as we will see, sphaleron processes can process an existing lepton number, producing a net lepton and baryon number. This is the process called leptogenesis (Fukugita and Yanagida, 1986).

In this section, we summarize the main arguments that the electroweak interactions violate baryon number at high temperature. In the next section, we explain why the electroweak interactions might produce a small baryon excess, and why this excess cannot be large enough to account for the observed asymmetry.

One of the great successes of the Standard Model is that it explains the observed conservation laws. In particular, there are no operators of dimension four or less which violate baryon number or the separate lepton numbers. The leading operators which violate baryon number are of dimension six, and thus suppressed by $\mathcal{O}(\frac{1}{M^2})$. The leading operators which violate the separate lepton numbers are of dimension five, and thus suppressed by one power of $1/M$. In each case, M should be thought of as the scale associated with some very high energy physics which violates baryon or lepton number. It cannot be determined except through measurement or by specifying a more microscopic theory.

However, it is not quite true that the standard model preserves all of these symmetries. There are tiny effects, of order

$$e^{-(2\pi/\alpha_w)} \approx 10^{-65} \quad (13)$$

which violate them. These effects are related to the fact that the separate baryon number and lepton number currents are “anomalous.” When one quantizes the theory carefully, one finds that the baryon number current, j_B^μ , is not exactly conserved but rather satisfies:

$$\partial_\mu j_B^\mu = \frac{3}{16\pi^2} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a = \frac{3}{8\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu} \quad (14)$$

Here $F_{\mu\nu}$ are the $SU(2)$ field strengths, and we have introduced matrix-valued fields in the last expression,

$$F_{\mu\nu} = \sum_a F_{\mu\nu}^a T^a, \quad (15)$$

and similarly for other fields, and the dual of F , \tilde{F} , is defined by:

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}. \quad (16)$$

In electromagnetism, $F\tilde{F} = 2\vec{E} \cdot \vec{B}$.

One might think that such a violation of current conservation would lead to dramatic violations of the symmetry. But the problem is more subtle. The right hand side of the anomaly equation is itself a total divergence:

$$\text{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu} = \partial_\mu K^\mu \quad (17)$$

where

$$K^\mu = \epsilon^{\mu\nu\rho\sigma} \text{tr} [F_{\nu\rho} A_\sigma + \frac{2}{3} A_\nu A_\rho A_\sigma] \quad (18)$$

(the reader can quickly check this for a $U(1)$ gauge theory like electromagnetism). In view of this,

$$\tilde{j} = j_B^\mu - \frac{3g^2}{8\pi^2} K^\mu \quad (19)$$

is conserved. In perturbation theory (i.e. in Feynman diagrams), the right hand side falls to zero rapidly (typically like $1/r^6$), and so its integral is zero. This fact insures that baryon number is conserved.

In abelian gauge theories, this is the end of the story. In non-abelian theories, however, there are field configurations which contribute to the right hand side. These lead to violations of baryon number and the separate lepton numbers proportional to $e^{-\frac{2\pi}{\alpha}}$. These configurations are called instantons. We will not discuss them in detail here; a pedagogical treatment is given by Coleman (1989). They correspond to calculation of a tunneling amplitude. To understand what the tunneling process is, one must consider more carefully the ground state of the field theory. Classically, the ground states are field configurations for which the energy vanishes. The trivial solution of this condition is $\vec{A} = 0$, where \vec{A} is the vector potential. More generally, one can consider \vec{A} which is a ‘‘pure gauge,’’

$$\vec{A} = \frac{1}{i} g^{-1} \vec{\nabla} g, \quad (20)$$

where g is a gauge transformation matrix. In an abelian ($U(1)$) gauge theory, fixing the gauge eliminates all but the trivial solution, $\vec{A} = 0$.¹ This is not the case for non-abelian gauge theories. There is a class of gauge transformations, labeled by a discrete index n , which do not tend to unity as $|\vec{x}| \rightarrow \infty$, which must be considered. These have the form:

$$g_n(\vec{x}) = e^{in f(\vec{x}) \hat{x} \cdot \tau / 2} \quad (21)$$

where $f(x) \rightarrow 2\pi$ as $\vec{x} \rightarrow \infty$, and $f(\vec{x}) \rightarrow 0$ as $\vec{x} \rightarrow 0$.

So the ground states of the gauge theory are labeled by an integer n . Now if we evaluate the integral of the current K^o , we obtain a quantity known as the Chern-Simons number:

$$n_{CS} = \frac{1}{16\pi^2} \int d^3x K^o = \frac{2/3}{16\pi^2} \int d^3x \epsilon_{ijk} \text{Tr} (g^{-1} \partial_i g g^{-1} \partial_j g g^{-1} \partial_k g). \quad (22)$$

For $g = g_n$, $n_{CS} = n$. The reader can also check that for $g' = g_n(x)h(x)$, where h is a gauge transformation which tends to unity at infinity (a so-called ‘‘small gauge transformation’’), this quantity is unchanged. n_{CS} , the ‘‘Chern-Simons number,’’ is topological in this sense (for \vec{A} 's which are not ‘‘pure gauge,’’ n_{CS} is in no sense quantized).

¹ More precisely, this is true in axial gauge. In the gauge $A_o = 0$, it is necessary to sum over all time-independent transformations to construct a state which obeys Gauss's law.

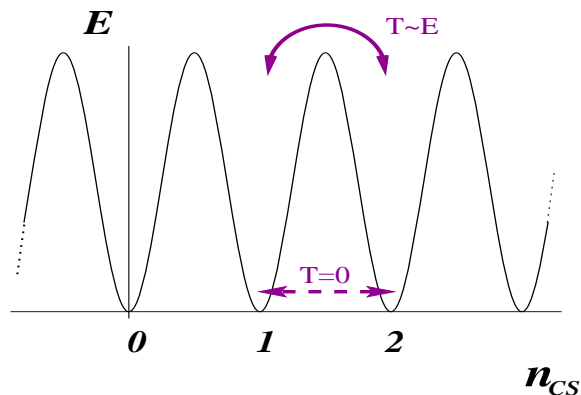


FIG. 2 Schematic Yang-Mills vacuum structure. At zero temperature, the instanton transitions between vacua with different Chern-Simons numbers are suppressed. At finite temperature, these transitions can proceed via sphalerons.

Schematically, we can thus think of the vacuum structure of a Yang-Mills theory as indicated in Fig. 2. We have, at weak coupling, an infinite set of states, labeled by integers, and separated by barriers from one another. In tunneling processes which change the Chern-Simons number, because of the anomaly, the baryon and lepton numbers will change. The exponential suppression found in the instanton calculation is typical of tunneling processes, and in fact the instanton calculation which leads to the result for the amplitude is nothing but a field-theoretic WKB calculation.

At zero temperature, the decay amplitude is suppressed, not only by $e^{-\frac{2\pi}{\alpha}}$, but by factors of Yukawa couplings. The probability that a single proton has decayed through this process in the history of the universe is infinitesimal. But this picture suggests that, at finite temperature, the rate should be larger. One can determine the height of the barrier separating configurations of different n_{CS} by looking for a solution of the static equations of motion with non-zero energy, and computing its energy. This solution is known as a “sphaleron” (Manton, 1983). When one studies the small fluctuations about this solution, one finds that there is a single negative mode, corresponding to the possibility of rolling down hill into one or the other well. The sphaleron energy is of order

$$E_{sp} = \frac{c}{g^2} M_w. \quad (23)$$

The rate for thermal fluctuations to cross the barrier should be of order (Arnold and McLerran, 1988; Dine *et.al*, 1990; Kuzmin, Rubakov, and Shaposhnikov, 1985)

$$\Gamma_{sp} = T^4 f(T/M_w) \sim e^{-E_{sp}/T}. \quad (24)$$

so the rate becomes large as the temperature approaches the W boson mass. In fact, at some temperature the weak interactions undergo a phase transition to a phase in which the W boson mass vanishes. At this point, the computation of the transition rate is a difficult problem – there is no small parameter – but general scaling arguments show that the transition rate is of the form²:

$$\Gamma_{bv} = \alpha_w^4 T^4. \quad (25)$$

Returning to our original expression for the anomaly, we see that while the separate baryon and lepton numbers are violated in these processes, the combination $B - L$ is conserved. This result leads to three observations:

1. If in the early universe, one creates baryon and lepton number, but no net $B - L$, B and L will subsequently be lost through sphaleron processes.
2. If one creates a net $B - L$ (e.g. creates a lepton number) the sphaleron process will leave both baryon and lepton numbers comparable to the original $B - L$. This realization is crucial to the idea of leptogenesis, to be discussed in more detail below.
3. The standard model satisfies, by itself, all of the conditions for baryogenesis.

² more detailed considerations alter slightly the parametric form of the rate (Arnold, Son, and Yaffe, 1997)

E. Electroweak baryogenesis

As we will see, while the Standard Model satisfies all of the conditions for baryogenesis (Kuzmin, Rubakov, and Shaposhnikov, 1985), nothing like the required baryon number can be produced. It is natural to ask whether extensions of the Standard Model, such as theories with complicated Higgs, or the Minimal Supersymmetric (extension of the) Standard Model, can generate an asymmetry, using the sphaleron process discussed in the previous section. We will refer to such a possibility more generally as “Electroweak Baryogenesis.”

1. Electroweak baryogenesis in the Standard Model

How might baryons be produced in the Standard Model? From our discussion, it is clear that the first and second of Sakharov’s conditions are satisfied. What about the need for a departure from equilibrium?

Above we alluded to the fact that in the electroweak theory, there is a phase transition to a phase with massless gauge bosons. It turns out that, for a sufficiently light Higgs, this transition is first order. To see this, we can calculate the effective potential for the Higgs particle at finite temperature. At zero temperature, in the simplest version of the Standard Model with a single Higgs field, ϕ , the Higgs potential is given by

$$V(\Phi) = -\mu^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4. \quad (26)$$

The potential has a minimum at non-zero $\Phi = \frac{1}{\sqrt{2}}v_o$, breaking the gauge symmetry and giving mass to the gauge bosons by the Higgs mechanism.

What about finite temperatures? Here, one wants to compute the free energy in the presence of a background Φ . To one loop, the potential is obtained simply by computing the free energy of the system in a constant background Φ , i.e. by allowing the masses to depend on $\phi = |\Phi|$:

$$V_T(\phi) = \pm \sum_i \int \frac{d^3p}{2\pi^3} \ln \left(1 \mp e^{-\beta\sqrt{p^2+m_i^2(\phi)}} \right) \quad (27)$$

where the sum is over all particle species (physical helicity states), and the plus sign is for bosons, the minus for fermions. One obtains

$$V(\phi, T) = D(T^2 - T_o^2)\phi^2 - ET\phi^3 + \frac{\lambda}{4}\phi^4. \quad (28)$$

Here the critical temperature T_o is given by:

$$T_o^2 = \frac{1}{2D}(\mu^2 - 4Bv_o^2) = \frac{1}{4D}(m_H^2 - 8Bv_o^2), \quad (29)$$

while the parameters B , D and E are given by:

$$B = \frac{3}{64\pi^2v_o^4}(2M_W^4 + M_Z^4 - 4m_t^4), \quad D = \frac{1}{8v_o^2}(2M_W^2 + m_Z^2 + 2m_t^2), \quad E = \frac{1}{4\pi v_o^3}(2M_W^3 + m_Z^3) \sim 10^{-2}. \quad (30)$$

Were it not for the ϕ^3 term, the transition would be second order, with critical temperature T_o . Because of the ϕ^3 term in the potential, the phase transition is potentially at least weakly first order. This is indicated in Fig. 3. A first order transition is not, in general, an adiabatic process. As we lower the temperature to the transition temperature, the transition proceeds by the formation of bubbles of the equilibrium state. These bubbles then expand and fill the universe. The moving bubble walls are regions where the Higgs fields are changing, and it has been shown that baryon and lepton number can be produced there. However, it is crucial that, once the bubble has passed, the baryon violating processes should shut off. This requires that after the transition, the Higgs expectation value – and therefore the sphaleron energy – be large. But given the current limit on the Higgs mass, this is not the case, at least in the minimal model with a single Higgs doublet.

Numerical simulations (Kajantie *et al.*, 1996, 1997, 1999; Rummukainen *et al.*, 1998) have shown that, for the Higgs mass above 80 GeV (which it must be, to satisfy the present experimental constraints), the phase transition in the Standard Model turns into a smooth crossover.

However, even for an unrealistically light Higgs, the actual production of baryon asymmetry in the minimal Standard Model would be highly suppressed. This is because Standard Model CP violation must involve all three generations (Kobayashi and Maskawa, 1973). The lowest order diagram that involves three generations of fermions with

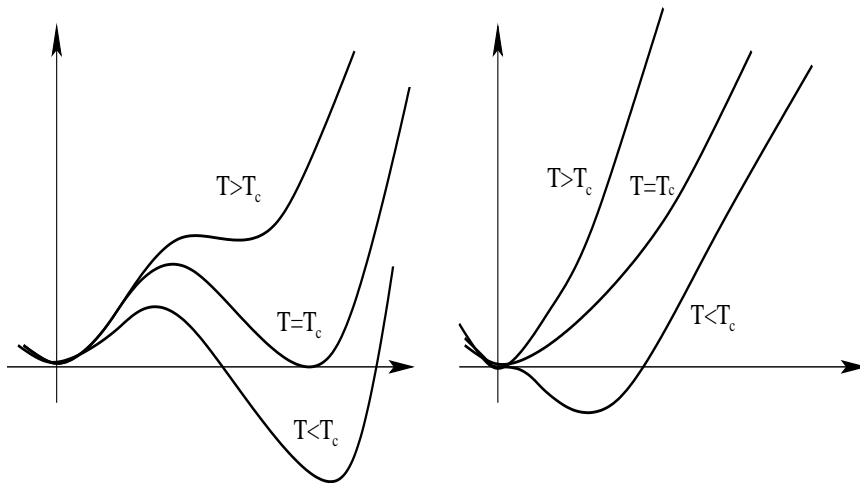


FIG. 3 First and second order phase transitions.

proper chiralities and contributes to CP violating processes relevant to baryogenesis is suppressed by 12 Yukawa couplings (Shaposhnikov, 1986, 1987). Hence, the CKM CP violation contributes a factor of 10^{-20} to the amount of baryon asymmetry that could arise in the Standard Model.

Clearly, one must look beyond the Standard Model for the origin of baryon asymmetry of the universe. One of the best motivated candidates for new physics is supersymmetry.

2. Supersymmetry, a short introduction

In this section we provide a brief introduction to supersymmetry. Much more detail can be found, e.g., in (Dine, 1996) and in many texts. Here, we will simply provide an easy to use cookbook.

Supersymmetry is a symmetry which relates bosons and fermions. The symmetry generators, Q_α , are fermionic operators. Acting on bosons they produce fermions degenerate in energy; similarly, acting on fermions, they produce degenerate bosons. Their algebra involves the total energy and momentum,

$$\{Q_\alpha, Q_\beta\} = P^\mu \gamma_{\alpha\beta}^\mu. \quad (31)$$

Neglecting gravity, supersymmetry is a global symmetry. Because of the structure of the algebra, the symmetry is broken if and only if the energy of the ground state is non-zero. If the symmetry is unbroken, for every boson there is a degenerate fermion, and conversely.

If we neglect gravity, there are two types of supermultiplets which may describe light fields. These are the chiral multiplets, containing a complex scalar and a Weyl (two-component) fermion

$$\Phi_i = (\phi_i, \psi_i), \quad (32)$$

and the vector multiplets, containing a gauge boson and a Weyl fermion:

$$V^a = (A_\mu^a, \lambda^a). \quad (33)$$

In global supersymmetry, the lagrangian is specified by the gauge symmetry and an analytic (more precisely holomorphic) function of the scalar fields, $W(\phi_i)$, known as the superpotential. For renormalizable theories,

$$W(\phi_i) = \frac{1}{2} m_{ij} \phi_i \phi_j + \lambda_{ijk} \phi_i \phi_j \phi_k. \quad (34)$$

The lagrangian then includes the following:

1. The usual covariant kinetic terms for all of the fields, for example

$$\lambda^a D\lambda^{a*}, \quad \psi D\psi^*, \quad \frac{-1}{4} F_{\mu\nu}^a{}^2, \quad |D_\mu\phi|^2. \quad (35)$$

2. Yukawa couplings with gauge strength:

$$\sqrt{2}g^a\lambda^a\phi^*T^a\psi + c.c. \quad (36)$$

3. Mass terms and Yukawa couplings from W :

$$\frac{1}{2}\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_i\psi_j \quad (37)$$

$$= m_{ij}\psi_i\psi_j + 3/2\lambda_{ijk}\phi_i\psi_j\psi_k. \quad (38)$$

4. A scalar potential:

$$V = \sum_i \left| \frac{\partial W}{\partial\phi_i} \right|^2 + \sum_a \frac{1}{2}g^{a2} \left(\sum_i \phi_i^* T^a \phi_i \right)^2 \quad (39)$$

It is convenient to define two types of auxiliary fields, the ‘‘F’’ and ‘‘D’’ fields:

$$F_i = \frac{\partial W}{\partial\phi_i} \quad D^a = g^a \sum_i \phi_i^* T^a \phi_i. \quad (40)$$

In terms of these, the potential is simply

$$V = |F_i|^2 + \frac{1}{2}|D^a|^2 \quad (41)$$

and, at the classical level, supersymmetry is unbroken if and only if all of the D and F fields vanish at the minimum of the potential.

It is useful to consider some examples. Take first a model with a single chiral field, ϕ , and superpotential

$$W = \frac{1}{2}m\phi^2. \quad (42)$$

In this case, the potential is

$$V = \left| \frac{\partial W}{\partial\phi} \right|^2 = m^2|\phi|^2. \quad (43)$$

On the other hand, the fermion mass is just m , from eqn. (38), so the model describes two bosonic and two fermionic degrees of freedom, degenerate in mass.

A more interesting model is a supersymmetric version of the standard model, known as the Minimal Supersymmetric Standard Model (MSSM). The gauge group is $SU(3) \times SU(2) \times U(1)$, and there is one vector multiplet for each gauge generator. In addition, for each of the usual fermions of the standard model, one has a chiral field with the same quantum numbers:

$$Q_a = (3, 2)_{1/3} \quad \bar{u}_a = (\bar{3}, 2)_{-4/3} \quad \bar{d}_a = (\bar{3}, 2)_{2/3} \quad L_a = (1, 2)_{-1} \quad \bar{e} = (1, 1)_2. \quad (44)$$

Here a is a generation index, $a = 1, \dots, 3$. In addition, there are two Higgs fields (two in order to cancel anomalies and to be able to give mass to all quarks and leptons)

$$H_u = (1, 2)_1 \quad H_d = (1, 2)_{-1} \quad (45)$$

The superpotential of the model is a generalization of the Yukawa couplings of the standard model:

$$W = \Gamma^{ab}Q_a\bar{u}_bH_u + \gamma^a Q_a\bar{d}_aH_d + \Gamma_a L_a\bar{e}_aH_d + \mu H_u H_d, \quad (46)$$

where we have used our freedom to make field redefinitions to make the (somewhat unconventional, but later convenient) choice that the d -quark and the lepton Yukawa couplings are diagonal.

The gauge symmetries actually permit many more couplings than those written in eqn. (46). Couplings such as HL , $\bar{u}d\bar{d}$, and others would violate baryon or lepton number if they appeared. Because these are dimension four, they are unsuppressed (unless they have extremely tiny dimensionless coefficients). They can be forbidden by a symmetry, under which ordinary fields are even (quarks, leptons, and Higgs bosons) while their supersymmetric partners are odd. This symmetry is called R-parity.

By itself, this model is not realistic, since supersymmetry is unbroken and all ordinary fields (quarks, leptons, gauge bosons, higgs) are degenerate with their superpartners (squarks, sleptons, gauginos). The simplest solution to this is just to add “soft breaking terms” which explicitly break the supersymmetry. Because they are soft, they don’t spoil the good features of these theories. These soft terms include mass terms for the squarks and sleptons, Majorana mass terms for the gauginos, and cubic couplings of the scalar fields,

$$\tilde{m}_{ij}^2 |\phi^i \phi^j|^2 + m_\lambda \lambda \lambda + m_A A_{ijk} \phi^i \phi^j \phi^k \quad (47)$$

In the minimal supersymmetric standard model, there are 105 such couplings (counting real parameters). We will think of all of these mass parameters as being of order $m_{3/2} \sim m_Z$. These parameters are highly constrained, both by low energy physics (particularly by the suppression of flavor-changing processes in weak interactions) and direct searches at LEP and the Tevatron. Theoretical approaches to understanding these soft breakings can be divided broadly into two classes. Both assume that some dynamics gives rise to spontaneous breakdown of supersymmetry. In “gravity mediation,” very high energy physics is responsible for generating the soft terms; in gauge mediated models, lower energy, gauge interactions communicate supersymmetry breaking to ordinary fields.

R parity, if present, implies that the lightest of the new particles, called the “LSP” implied by supersymmetry is stable. Typically this is the partner of a neutral gauge or Higgs boson. One can calculate the abundance of these particles (neutralinos) as a function of the various supersymmetry breaking parameters. The assumption that the supersymmetry-breaking masses are hundreds of GeV leads automatically to a neutralino density of order the dark matter density of the universe, and this particle is a leading candidate for the dark matter.

3. Baryogenesis in the MSSM and the NMSSM

Supersymmetric extensions of the Standard Model contain new sources of CP violation (Dine *et al.*, 1991; Dine, Huet, and Singleton, 1992; Huet and Nelson, 1996) and an enlarged set of parameters which allow a greater possibility of a first-order transition (Bodeker *et al.*, 1997; Carena, Quiros and Wagner, 1998; Cline and Moore, 1998; Espinosa, Quiros, and Zwirner, 1993; Espinosa, 1996). So it would seem possible that electroweak baryogenesis could operate effectively in these theories.

The new sources of CP violation may come, for example, from the chargino mass matrix:

$$\bar{\psi}_R M_\chi \psi_L = (\bar{w}_1^+, \bar{h}_2^+)_R \begin{pmatrix} m_2 & gH_2(x) \\ gH_1(x) & \mu \end{pmatrix} \begin{pmatrix} \tilde{w}_1^+ \\ \tilde{h}_1^+ \end{pmatrix}_L + \text{h.c.} \quad (48)$$

Other possible sources include phases in scalar masses (by field redefinitions, some of these can be shifted from fermion to scalar mass terms). We will focus, however, on the terms in eq. (48).

As long as m_2 and μ are complex, spatially varying phases in the bubble wall provide a source of (spontaneous) CP violation (Cohen, Kaplan, and Nelson, 1991; Lee, 1974; Weinberg, 1976). However, in light of the constraints on Higgs and superpartner masses, the present window for electroweak baryogenesis in the MSSM is very narrow (Cline, Joyce and Kainulainen, 1998); several parameters must be adjusted to maximize the resulting baryon asymmetry (in particular, one must assume that the wall is very thin, take $\tan\beta < 3$, and choose the “optimal” bubble wall velocity $v_w \approx 0.02$), as shown in Fig. 4.

The origin of these difficulties lies, once again, in the strength of the electroweak phase transition. In the MSSM, the phase transition can be enhanced if the right-handed stop (the scalar partner of the top quark) is assumed to be very light, while the left-handed stop is very heavy (Carena, Quiros and Wagner, 1998). Then two-loop effects (Bodeker *et al.*, 1997; Espinosa, 1996) change the scalar potential sufficiently to allow for a first-order phase transition. However, severe constraints arise from the experimental bounds on the chargino mass, as well as the chargino contribution to the electric dipole moment of the neutron (Chang, Chang and Keung, 2002). Overall, electroweak baryogenesis in the MSSM is on the verge of being ruled out (or confirmed) by improving experimental constraints (Cline, 2002).

The strength of the phase transition can be further enhanced by adding a singlet Higgs to the model. In the next-to-minimal supersymmetric model (NMSSM), the phase transition can be more strongly first-order (Davies, Froggatt, and Moorhouse, 1996). The additional CP violation also helps increase the baryon asymmetry in this model (Huber and Schmidt, 2001).

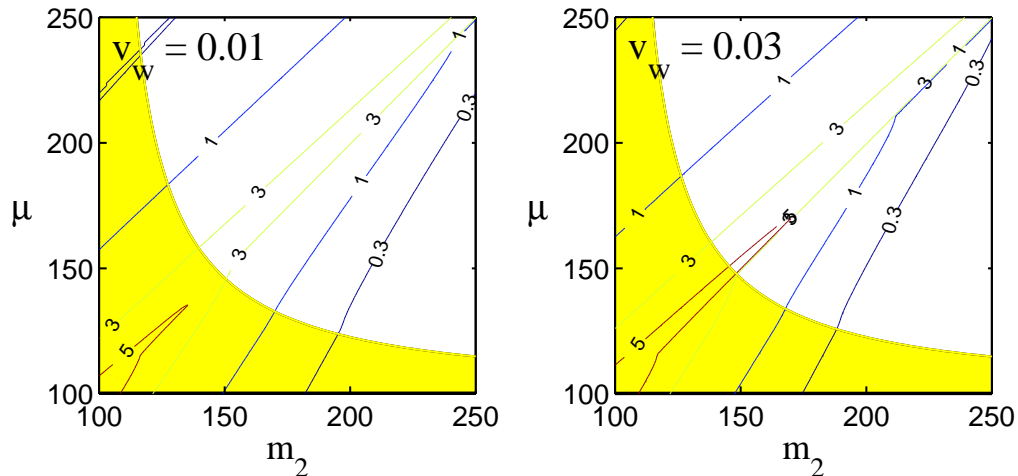


FIG. 4 Contours of constant baryon asymmetry in units 10^{-10} with $\sin \delta_\mu = 1$ for (a) $v_w = 0.01$ and (b) $v_w = 0.03$. Mass units are GeV/c^2 . Shaded regions are excluded by the LEP2 limit on the chargino mass, $m_{\chi^\pm} > 104 \text{ GeV}/c^2$. To maximize the baryon asymmetry, one assumes that $\tan \beta \lesssim 3$, that the bubble wall is very narrow, $\ell_w \simeq 6/T$, and that a number of other parameters are stretched to their limits. From a paper by Cline, Joyce and Kainulainen (1998).

4. Non-thermal electroweak baryogenesis at preheating

In light of these difficulties, various proposals have been put forth to obtain a viable picture of electroweak baryogenesis. These typically involve more drastic departures from thermal equilibrium than the weakly first order phase transitions described above. The more extreme proposals suppose that inflation occurred at the electroweak scale and kicked the universe out of equilibrium, setting the stage for baryogenesis. It is generally believed that the natural scale for inflation is much higher than 10^2 GeV . Although models with weak (German, Ross, and Sarkar, 2001; Randall and Thomas, 1995) or intermediate (Randall, Soljatic, and Guth, 1996) scale inflation have been constructed, a lower scale of inflation is generally difficult to reconcile with the observed density perturbations $(\delta\rho/\rho) \sim 10^{-5}$. As a rule, the smaller the scale of inflation, the flatter the inflaton potential must be to produce the same density fluctuations. A weak-scale inflation would require an inflaton potential to be extremely flat.

Of course, one does not have to assume that the same inflation is responsible for $(\delta\rho/\rho)$ and for baryogenesis. One could imagine that the universe has undergone more than one inflationary period. The primary inflation at a high scale could be responsible for the flatness of the universe and for the observed density perturbations. A secondary inflation at the weak scale, which could have lasted for only a couple of e-folds, could create fertile soil for baryogenesis. One can debate the plausibility of invoking a second stage of inflation just for this purpose. In favor of such a possibility, it has been argued that a low-scale inflation might ameliorate the cosmological moduli problem common to many supersymmetric theories (German, Ross, and Sarkar, 2001; Randall and Thomas, 1995).

What inhibits electroweak baryogenesis in the Standard Model is too much equilibrium and too little CP violation. Both of these problems are rectified if inflation ends with a reheat temperature just below the electroweak scale (García-Bellido *et al.*, 1999; Krauss and Trodden, 1999). Reheating, especially its variant dubbed *preheating*, involves a radical departure from thermal equilibrium (Kofman, Linde and Starobinsky, 1996).

During inflation, all matter and radiation are inflated away. When inflation is over, the energy stored in the inflaton is converted to thermal plasma. There are several possibilities for this reheating process. One possibility is that the inflaton may decay perturbatively into the light particles, which thermalize eventually. However, in a class of models, a parametric resonance may greatly enhance the production of particles in some specific energy bands (Kofman, Linde and Starobinsky, 1996). This process is known as *preheating*. A spinodal instability in the motion of the inflaton may lead to a very rapid *tachyonic* preheating (Felder *et al.*, 2001).

All of these variants of reheating force the universe into a non-equilibrium state after the end of inflation and

before thermalization takes place. This is, obviously, an opportune time for baryogenesis. The usual considerations of sphaleron transitions do not apply to a non-equilibrium system. But it turns out that baryon-number violating processes similar to sphaleron transitions do take place at preheating (Cornwall and Kusenko, 2000; García-Bellido *et al.*, 1999), as well as tachyonic preheating (Smit and Tranberg, 2002). This has been demonstrated by a combination of numerical and analytical arguments. In addition, preheating allows the coherent motions of some condensates to serve as sources of CP violation (Cornwall *et al.*, 2001). Such sources are poorly constrained by experiment and could have significant impact on baryogenesis. It is conceivable, therefore, that the electroweak-scale inflation could facilitate generation of the baryon asymmetry.

F. Leptogenesis

Of the five scenarios for baryogenesis which we have listed in the introduction, we have discussed two which are connected to very high energy physics: Gravitational and GUT baryogenesis. We have given cosmological arguments why they are not likely. These arguments depend on assumptions which we cannot now reliably establish, so it is yet possible that these mechanisms were operative. But if we tentatively accept these arguments we can significantly narrow our focus. Similarly we have seen that electroweak baryogenesis, while a beautiful idea, cannot be implemented in the Standard Model, and can, at best, be realized in only a small part of the parameter space of its minimal supersymmetric extension. So again, while we can not rule out the possibility that electroweak baryogenesis is relevant, it is tempting, for the moment, to view this possibility as unlikely. Adopting this point of view leaves leptogenesis and Affleck-Dine baryogenesis as the two most promising possibilities. What is exciting about each of these is that, if they are operative, they have consequences for experiments which will be performed at accelerators over the next few years.

While there is no experimental evidence for supersymmetry apart from the unification of couplings, in the last few years, evidence for neutrino masses has become more and more compelling (Gonzalez-Garcia and Nir, 2002). This comes from several sources: the fact that the flux of solar neutrinos does not match theoretical expectations, in the absence of masses and mixings (Super-K, 2001; SNO, 2002); the apparent observation of neutrino oscillations among atmospheric neutrinos (Super-K, 1998); and direct measurements of neutrino mixing (KamLAND, 2002).

We will not review all of these phenomena here, but just mention that the atmospheric neutrino anomaly suggests oscillations between the second and third generation of neutrinos:

$$\Delta m_\nu^2 = 10^{-2} - 10^{-4} \text{eV}^2 \quad (49)$$

with mixing of order one, while the solar neutrino deficit suggests smaller masses ($\Delta m^2 \sim 10^{-6} \text{eV}^2$). There is other evidence for neutrino oscillation from accelerator experiments. The SNO experiment has recently provided persuasive evidence in support of the hypothesis of mixing (as opposed to modifications of the standard solar model). The results from Super-Kamiokande, SNO and KamLAND are in good agreement. There is also evidence of mixing from an experiment at Los Alamos (LSND). This result should be confirmed, or not, by the MiniBoone experiment at Fermilab. The mixing suggested by atmospheric neutrinos is currently being searched for directly by accelerators. The data so far supports the mixing interpretation, but is not yet decisive.

The most economical explanation of these facts is that neutrinos have Majorana masses arising from lepton-number violating dimension five operators. (A Majorana mass is a mass for a two component fermion, which is permitted if the fermion carries no conserved charges.) We have stressed that the leading operators permitted by the symmetries of the Standard Model which violate lepton number are non-renormalizable operators of dimension five, i.e. suppressed by one power of some large mass. Explicitly, these have the form:

$$\mathcal{L}_{lv} = \frac{1}{M} LHLH \quad (50)$$

Replacing the Higgs field by its expectation value v gives a mass for the neutrino of order $\frac{v^2}{M}$. If $M = M_p$, this mass is too small to account for either set of experimental results. So one expects that some lower scale is relevant. The ‘‘see-saw’’ mechanism provides a simple picture of how this scale might arise. One supposes that in addition to the neutrinos of the Standard Model, there are some $SU(2) \times U(1)$ -singlet neutrinos, N . Nothing forbids these from obtaining a large mass. This could be of order M_{gut} , for example, or a bit smaller. These neutrinos could also couple to the left handed doublets, just like right handed charged leptons. Assuming, for the moment, that these couplings are not particularly small, one would obtain a mass matrix of the form

$$M_\nu = \begin{pmatrix} M & M_W \\ M_W & 0 \end{pmatrix} \quad (51)$$

This matrix has an eigenvalue $\frac{M_W^2}{M}$. The latter number is of the order needed to explain the neutrino anomaly for $M \sim 10^{13}$ or so, i.e. not wildly different than the GUT scale and other scales which have been proposed for new physics.

For leptogenesis (Fukugita and Yanagida, 1986), what is important in this model is that the couplings of N break lepton number. N is a heavy particle; it can decay both to $h + \nu$ and $h + \bar{\nu}$, for example. The partial widths to each of these final states need not be the same. CP violation can enter through phases in the Yukawa couplings and mass matrices of the N 's. At tree level, however, these phases will cancel out between decays to the various states and their (would be) CP conjugates, as in the case of GUTs we discussed earlier. So it is necessary to consider interference between tree and one loop diagrams with discontinuities, as in Fig. 1. In a model with three N 's, there are CP-violating phases in the Yukawa couplings of the N 's to the light Higgs. The heaviest of the right handed neutrinos, say N_1 , can decay to ℓ and a Higgs, or to $\bar{\ell}$ and a Higgs. At one loop, the decay amplitude has a discontinuity associated with the fact that the intermediate N_1 and N_2 can be on shell. So one obtains:

$$\epsilon = \frac{\Gamma(N_1 \rightarrow \ell H_2) - \Gamma(N_1 \rightarrow \bar{\ell} \bar{H}_2)}{\Gamma(N_1 \rightarrow \ell H_2) + \Gamma(N_1 \rightarrow \bar{\ell} \bar{H}_2)} \quad (52)$$

$$= \frac{1}{8\pi} \frac{1}{hh^\dagger} \sum_{i=2,3} \text{Im}[(h_\nu h_\nu^\dagger)_{1i}]^2 f\left(\frac{M_i^2}{M_1^2}\right) \quad (53)$$

where f is a function that represents radiative corrections. For example, in the Standard Model $f = \sqrt{x}[(x-2)/(x-1) + (x+1)\ln(1+1/x)]$, while in the MSSM $f = \sqrt{x}[2/(x-1) + \ln(1+1/x)]$. Here we have allowed for the possibility of multiple Higgs fields, with H_2 coupling to the leptons. The rough order of magnitude here is readily understood by simply counting loops factors. It need not be terribly small.

Now, as the universe cools through temperatures of order the of masses of the N 's, they drop out of equilibrium, and their decays can lead to an excess of neutrinos over antineutrinos. Detailed predictions can be obtained by integrating a suitable set of Boltzman equations.

The resulting lepton number will be further processed by sphaleron interactions, yielding a net lepton and baryon number (recall that sphaleron interactions preserve $B - L$, but violate B and L separately). One can determine the resulting asymmetry by an elementary thermodynamics exercise. One introduces chemical potentials for each neutrino, quark and charged lepton species. One then considers the various interactions between the species at equilibrium. For any allowed chemical reaction, the sum of the chemical potentials on each side of the reaction must be equal. For neutrinos, the relations come from:

1. The sphaleron interactions themselves:

$$\sum_i (3\mu_{q_i} + \mu_{\ell_i}) = 0 \quad (54)$$

2. A similar relation for QCD sphalerons:

$$\sum_i (2\mu_{q_i} - \mu_{u_i} - \mu_{d_i}) \quad (55)$$

3. One must require that the total hypercharge of the universe vanishes

$$\sum_i (\mu_{q_i} - 2\mu_{\bar{u}_i} + \mu_{\bar{d}_i} - \mu_{\ell_i} + \mu_{\bar{e}_i}) + \frac{2}{N} \mu_H = 0. \quad (56)$$

4. From each Yukawa coupling we have a relation.

$$\mu_{q_i} - \mu_\phi - \mu_{d_j} = 0, \quad \mu_{q_i} - \mu_\phi - \mu_{u_j} = 0, \quad \mu_{\ell_i} - \mu_\phi - \mu_{e_j} = 0. \quad (57)$$

Combining these results, it is possible to solve for the chemical potentials in terms of the lepton chemical potential, and finally in terms of the initial $B - L$. With N generations,

$$B = \frac{8N + 4}{22N + 13} (B - L). \quad (58)$$

Reasonable values of the neutrino parameters give asymmetries of the order we seek to explain. Note sources of small numbers:

1. The phases in the couplings
2. The loop factor
3. The small density of the N particles when they drop out of equilibrium. Parametrically, one has, e.g., for production,

$$\Gamma \sim e^{(-M/T)} g^2 T \quad (59)$$

which is much less than $H \sim T^2/M_p$ once the density is suppressed by T/M_p , i.e. of order 10^{-6} for a 10^{13} GeV particle.

It is interesting to ask: assuming that these processes are the source of the observed asymmetry, how many parameters which enter into the computation can be measured, i.e. can we relate the observed number to microphysics. It is conceivable that over time, many of the parameters of the light neutrino mass matrices, including possible CP-violating effects, will be measured (Gonzalez-Garcia and Nir, 2002). But in order to give a precise calculation, analogous to the calculations of nucleosynthesis, of the baryon number density, one needs additional information about the masses of the fields N_i . In the absence of direct measurements, one requires a theory of these masses.

G. Baryogenesis through Coherent Scalar Fields

There are many hints that supersymmetry might play some role in nature. For example, supersymmetry seems to be an essential part of superstring theory, the only consistent theory of quantum gravity which we know. If supersymmetry is a symmetry of the laws of nature, however, it must be badly broken; otherwise we would have seen, for example, scalar electrons (“selectrons”) and fermionic photons (“photinos”). It has been widely conjectured that supersymmetry might be discovered by accelerators capable of exploring the TeV energy range. There are several reasons for this. The most compelling is the “hierarchy problem.” This is, at its most simple level, the puzzle of the wide disparity of energies between the Planck scale (or perhaps the unification scale) and the weak scale – roughly 17 orders of magnitude. While one might take this as simply a puzzling fact, within quantum theory, the question is made sharper by the fact that scalar masses (particularly the Higgs mass) receive very divergent quantum corrections. A typical expression is:

$$\delta m^2 = \frac{\alpha}{\pi} \int d^4 k \frac{1}{k^2}. \quad (60)$$

This integral diverges quadratically for large momentum (k). Presumably, the integral is cut off by some unknown physics. If the energy scale of this physics is Λ , then the corrections to the Higgs mass are *much* larger than the scale of weak interactions unless $\Lambda \sim \text{TeV}$. While various cutoffs have been proposed, one of the most compelling suggestions is that the cutoff is the scale of supersymmetry breaking. In this case, the scale must be about 1000 GeV. If this hypothesis is correct, the Large Hadron Collider under construction at CERN should discover an array of new particles and interactions (it is possible that supersymmetry could be discovered at the Tevatron beforehand).

We have seen that supersymmetry introduces new possibilities for electroweak baryogenesis. But the most striking feature of supersymmetric models, from the point of view of baryogenesis, is the appearance of scalar fields carrying baryon and lepton number. These scalars offer the possibility of coherent production of baryons. In the limit that supersymmetry is unbroken, many of these scalars have flat or nearly flat potentials. They are thus easily excited in the highly energetic environment of the early universe. Simple processes can produce substantial amounts of baryon number. This coherent production of baryons, known as Affleck-Dine baryogenesis, is the focus of the rest of this review.

III. AFFLECK-DINE BARYOGENESIS

A. Arguments for Coherent Production of the Baryon Number

In the previous section, we have reviewed several proposals for generating the baryon number. None can be firmly ruled out, however all but two seem unlikely: leptogenesis and Affleck-Dine baryogenesis. While the discovery of neutrino mass gives support to the possibility of leptogenesis, there are a number of reasons to consider coherent production:

- The Standard Model alone cannot explain the baryon asymmetry of the universe, the main obstacle being the heaviness of the Higgs. One needs new physics for baryogenesis. The requisite new physics may reside at a very high scale or at a lower scale. An increasing body of evidence implies that inflation probably took place in the early universe. Hence, baryogenesis must have happened at or after reheating. To avoid overproducing weakly interacting light particles, for example gravitinos and other new states predicted in theory, one would like the reheat temperature not to exceed 10^9 GeV. This poses a problem for GUT baryogenesis. This also limits possibilities for the leptogenesis. Affleck-Dine baryogenesis, on the other hand, is consistent with low energy and temperature scales required by inflation.
- Supersymmetry is widely regarded as a plausible, elegant, and natural candidate for physics beyond the Standard Model. Of the two simple scenarios for baryogenesis in the MSSM, the electroweak scenario is on the verge of being ruled out by accelerator constraints on supersymmetric particles, in sharp contrast with the AD scenario.
- The remaining low-reheat SUSY scenario, Affleck-Dine baryogenesis, can naturally reproduce the observed baryon asymmetry of the universe. The formation of an AD condensate can occur quite generically in cosmological models.
- The Affleck-Dine scenario can give rise simultaneously to the ordinary matter and the dark matter in the universe. This can explain why the amounts of luminous and dark matter are surprisingly close to each other, within one order of magnitude. If the two entities formed in completely unrelated processes (for example, the baryon asymmetry from leptogenesis, while the dark matter from freeze-out of neutralinos), the observed relation $\Omega_{\text{DARK}} \sim \Omega_{\text{matter}}$ is fortuitous.³
- Many particle physics models lead to significant production of entropy at relatively late times (Cohen, Kaplan and Nelson, 1993). This dilutes whatever baryon number existed previously. Coherent production can be extremely efficient, and in many models, it is precisely this late dilution which yields the small baryon density observed today.

In the rest of this section, we discuss Affleck-Dine baryogenesis in some detail.

B. Baryogenesis Through a Coherent Scalar Field:

In supersymmetric theories, the ordinary quarks and leptons are accompanied by scalar fields. These scalar fields carry baryon and lepton number. A coherent field, *i.e.*, a large classical value of such a field, can in principle carry a large amount of baryon number. As we will see, it is quite plausible that such fields were excited in the early universe.

To understand the basics of the mechanism, consider first a model with a single complex scalar field. Take the lagrangian to be

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2 \quad (61)$$

This lagrangian has a symmetry, $\phi \rightarrow e^{i\alpha} \phi$, and a corresponding conserved current, which we will refer to as baryon number:

$$j_B^\mu = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*). \quad (62)$$

It also possesses a ‘‘CP’’ symmetry:

$$\phi \leftrightarrow \phi^*. \quad (63)$$

With supersymmetry in mind, we will think of m as of order M_W .

Now let us add interactions in the following way, which will closely parallel what happens in the supersymmetric case. Include a set of quartic couplings:

$$\mathcal{L}_I = \lambda |\phi|^4 + \epsilon \phi^3 \phi^* + \delta \phi^4 + c.c. \quad (64)$$

We will take the couplings $\lambda, \epsilon, \delta \dots$ to be extremely small; in supersymmetric theories, they will be $\mathcal{O}(M_W^2/M_p^2)$ or $\mathcal{O}(M_W^2/M_{gut}^2)$. These interactions clearly violate ‘‘B’’. For general complex ϵ and δ , they also violate CP .

³ An additional *ad hoc* symmetry can also help relate the amounts of ordinary matter and dark matter (Kaplan, 1992).

In order that these tiny couplings lead to an appreciable baryon number, it is necessary that the fields, at some stage, were very large. To see how the cosmic evolution of this system can lead to a non-zero baryon number, first note that at very early times, when the Hubble constant, $H \gg m$, the mass of the field is irrelevant. It is thus reasonable to suppose that at this early time $\phi = \phi_o \gg 0$; later we will describe some specific suggestions as to how this might come about. How does the field then evolve? First ignore the quartic interactions. In a gravitational background, the equation of motion for the field is

$$D_\mu^2 \phi + \frac{\partial V}{\partial \phi} = 0, \quad (65)$$

where D_μ is the covariant derivative. For a spatially homogeneous field, $\phi(t)$, in a Robertson-Walker background, this becomes

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0. \quad (66)$$

At very early times, $H \gg m$, and so the system is highly overdamped and essentially frozen at ϕ_o . At this point, $B = 0$. However, once the universe has aged enough that $H \ll m$, ϕ begins to oscillate. Substituting $H = \frac{1}{2t}$ or $H = \frac{2}{3t}$ for the radiation and matter dominated eras, respectively, one finds that

$$\phi = \begin{cases} \frac{\phi_o}{(mt)^{3/2}} \sin(mt) & \text{(radiation)} \\ \frac{\phi_o}{(mt)} \sin(mt) & \text{(matter)}. \end{cases} \quad (67)$$

In either case, the energy behaves, in terms of the scale factor, $R(t)$, as

$$E \approx m^2 \phi_o^2 \left(\frac{R}{R_o}\right)^3 \quad (68)$$

i.e. it decreases like the energy of pressureless dust. One can think of this oscillating field as a coherent state of ϕ particles with $\vec{p} = 0$.

Now let's consider the effects of the quartic couplings. Since the field amplitude damps with time, their significance will decrease with time. Suppose, initially, that $\phi = \phi_o$ is real. Then the imaginary part of ϕ satisfies, in the approximation that ϵ and δ are small,

$$\ddot{\phi}_i + 3H\dot{\phi}_i + m^2 \phi_i \approx \text{Im}(\epsilon + \delta) \phi_o^3. \quad (69)$$

For large times, the right hand side is negligible, so this equation goes over to the free equation, with a solution of the form

$$\phi_i = a_r \frac{\text{Im}(\epsilon + \delta) \phi_o^3}{m^2 (mt)^{3/4}} \sin(mt + \delta_r) \quad \text{(radiation)}, \quad \phi_i = a_m \frac{\text{Im}(\epsilon + \delta) \phi_o^3}{m^3 t} \sin(mt + \delta_m) \quad \text{(matter)}, \quad (70)$$

in the radiation and matter dominated cases, respectively. The constants δ_m , δ_r , a_m and a_r are of order unity:

$$a_r = 0.85 \quad a_m = 0.85 \quad \delta_r = -0.91 \quad \delta_m = 1.54. \quad (71)$$

But now we have a non-zero baryon number; substituting in the expression for the current,

$$n_B = 2a_r \text{Im}(\epsilon + \delta) \frac{\phi_o^2}{m(mt)^2} \sin(\delta_r + \pi/8) \quad \text{(radiation)} \quad n_B = 2a_m \text{Im}(\epsilon + \delta) \frac{\phi_o^2}{m(mt)^2} \sin(\delta_m) \quad \text{(matter)}. \quad (72)$$

Two features of these results should be noted. First, if ϵ and δ vanish, n_B vanishes. If they are real, and ϕ_o is real, n_B vanishes. It is remarkable that the lagrangian parameters can be real, and yet ϕ_o can be complex, still giving rise to a net baryon number. We will discuss plausible initial values for the fields later, after we have discussed supersymmetry breaking in the early universe. Finally, we should point out that, as expected, n_B is conserved at late times.

This mechanism for generating baryon number could be considered without supersymmetry. In that case, it begs several questions:

- What are the scalar fields carrying baryon number?
- Why are the ϕ^4 terms so small?
- How are the scalars in the condensate converted to more familiar particles?

In the context of supersymmetry, there is a natural answer to each of these questions. First, as we have stressed, there are scalar fields carrying baryon and lepton number. As we will see, in the limit that supersymmetry is unbroken, there are typically directions in the field space in which the quartic terms in the potential vanish. Finally, the scalar quarks and leptons will be able to decay (in a baryon and lepton number conserving fashion) to ordinary quarks.

C. Flat Directions and Baryogenesis

To discuss the problem of baryon number generation, we first want to examine the theory in a limit in which we ignore the soft SUSY-breaking terms. After all, at very early times, $H \gg M_W$, and these terms are irrelevant. We want to ask whether in a model like the MSSM, some fields can have large vev's, i.e. whether there are directions in the field space for which the potential vanishes. Before considering the full MSSM, it is again helpful to consider a simpler model, in this case a theory with gauge group $U(1)$, and two chiral fields, ϕ^+ and ϕ^- with opposite charge. We take the superpotential simply to vanish. In this case the potential is

$$V = \frac{1}{2}D^2 \quad D = g(\phi^{+*}\phi^+ - \phi^{-*}\phi^-) \quad (73)$$

But D , and the potential, vanish if $\phi^+ = \phi^- = v$. It is not difficult to work out the spectrum in a vacuum of non-zero v . One finds that there is one massless chiral field, and a massive vector field containing a massive gauge boson, a massive Dirac field, and a massive scalar.

Consider, now, a somewhat more elaborate example. Take the MSSM, and give expectation values to the Higgs fields:

$$H_u = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad L = \begin{pmatrix} v \\ 0 \end{pmatrix}. \quad (74)$$

The F term vanishes in this direction, since the potentially problematic $H_u L$ term in the superpotential is absent by R parity (the other possible contributions vanish because $Q = H_U = 0$). It is easy to see that the D -term for hypercharge vanishes,

$$D_Y = g'{}^2(|H_u|^2 - |L|^2) = 0. \quad (75)$$

To see that the D terms for $SU(2)$ vanishes, one can work directly with the Pauli matrices, or use, instead, the following device which works for a general $SU(N)$ group. Just as one defines a matrix-valued gauge field,

$$(A_\mu)^i{}_j = A_\mu^a (T^a)^i{}_j, \quad (76)$$

one defines

$$(D)^i{}_j = D^a (T^a)^i{}_j. \quad (77)$$

Then, using the $SU(N)$ identity,

$$(T^a)^i{}_j (T^a)^k{}_l = \delta^i{}_l \delta^k{}_j - \frac{1}{N} \delta^i{}_j \delta^k{}_l \quad (78)$$

the contribution to $(D)^i{}_j$ from a field, ϕ , in the fundamental representation is simply

$$(D)^i{}_j = \phi^{i*} \phi_j - \frac{1}{N} |\phi|^2 \delta^i{}_j. \quad (79)$$

In the present case, this becomes

$$(D)^i{}_j = \begin{pmatrix} |v|^2 & 0 \\ 0 & |v|^2 \end{pmatrix} - \frac{1}{2} |v|^2 \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 0. \quad (80)$$

What is particularly interesting about this direction is that in this direction, fields which carry lepton number are excited. As we have seen, producing a lepton number is for all intents and purposes like producing a baryon number.

Non-renormalizable, higher dimension terms, with more fields, can lift the flat direction. For example, the quartic term in the superpotential:

$$\mathcal{L}_4 = \frac{1}{M} (H_u L)^2 \quad (81)$$

respects all of the gauge symmetries and is invariant under R -parity. It gives rise to a potential

$$V_{\text{lift}} = \frac{\Phi^6}{M^2} \quad (82)$$

where Φ is the superfield whose vev parameterizes the flat direction.

There are many more flat directions, and many of these do carry baryon or lepton number.⁴ A flat direction with both baryon and lepton number excited is the following:

$$\text{First generation : } Q_1^1 = b \quad \bar{u}_2 = a \quad L_2 = b \quad \text{Second : } \bar{d}_1 = \sqrt{|b|^2 + |a|^2} \quad \text{Third : } \bar{d}_3 = a. \quad (83)$$

(On Q , the upper index is a color index, the lower index an $SU(2)$ index, and we have suppressed the generation indices).

To check that this is indeed a flat direction, consider first the D terms for the various groups. Using our earlier matrix construction, we have:

$$SU(3) : \begin{pmatrix} |b|^2 - (|a|^2 + |b|^2) & 0 & 0 \\ 0 & -|a|^2 & 0 \\ 0 & 0 & -|a|^2 \end{pmatrix} - \text{trace} = 0 \quad SU(2) : \begin{pmatrix} |b|^2 & 0 \\ 0 & |b|^2 \end{pmatrix} - \text{trace} = 0 \quad (84)$$

$$U(1) : \frac{1}{3}|b|^2 - \frac{4}{3}|a|^2 + \frac{2}{3}|b|^2 + \frac{2}{3}|a|^2 + \frac{2}{3}|a|^2 - |b|^2 = 0. \quad (85)$$

The F terms also vanish:

$$\frac{\partial W}{\partial H_u} = \Gamma^{ab} Q^a \bar{u}^b = 0 \quad \text{and} \quad \frac{\partial W}{\partial H_d} = \gamma^a Q^a \bar{d}^a = 0 \quad (86)$$

The first follows since the \bar{u} fields have their expectation values in different ‘‘color slots’’ than the Q fields. The second is automatically satisfied since the \bar{d} and Q fields have expectation values in different generations, and these Yukawa couplings don’t mix generations.

Higher dimension operators again can lift this flat direction. In this case the leading term is:

$$\mathcal{L}_7 = \frac{1}{M^3} [Q^1 \bar{d}^2 L^1] [\bar{u}^1 \bar{d}^2 \bar{d}^3]. \quad (87)$$

Here the superscripts denote flavor. We have suppressed color and $SU(2)$ indices, but the braces indicate sets of fields which are contracted in $SU(3)$ and $SU(2)$ invariant ways. In addition to being completely gauge invariant, this operator is invariant under ordinary R -parity. (There are lower dimension operators, including operators of dimension 4, which violate R -parity). It gives rise to a term in the potential:

$$V_{\text{lift}} = \frac{\Phi^{10}}{M^6}. \quad (88)$$

Here Φ refers in a generic way the fields whose vev’s parameterize the flat directions (a, b) .

D. Evolution of the Condensate

For the cosmologies we wish to consider, despite the powers of $\frac{1}{M}$, these operators are quite important. During inflation, for example, such operators can determine the initial value of the field, Φ_o (here Φ denotes in a generic way the fields which parameterize the flat directions). Suppose that I is the inflaton field, and that the inflaton potential arises because of a non-zero value of the auxiliary field for I, F_I . This quantity is roughly constant during inflation. So, during inflation, supersymmetry is broken by a large amount (Dine, Randall, and Thomas, 1995), and there are all sorts of operators which contribute to the potential for Φ . These contributions to the potential have the form:

$$V_H = H^2 \Phi^2 f(\Phi^2/M_p^2). \quad (89)$$

It is perfectly possible for the second derivative of the potential near the origin to be negative. In this case, writing our higher dimension term as:

$$W_n = \frac{1}{M^n} \Phi^{n+3}. \quad (90)$$

⁴ The flat directions in the MSSM have been cataloged by Gherghetta, Kolda and Martin (1996).

the potential takes the form

$$V = -H^2|\Phi|^2 + \frac{1}{M^{2n}}|\Phi|^{2n+4}. \quad (91)$$

The minimum of the potential then lies at:

$$\Phi_0 \approx M \left(\frac{H}{M} \right)^{\frac{1}{n+1}}. \quad (92)$$

More generally, one can see that the higher the dimension of the operator which raises the flat direction, the larger the starting value of the field – and the larger the ultimate value of the baryon number. Typically, there is plenty of time for the field to find its minimum during inflation. After inflation, H decreases, and the field Φ evolves adiabatically, oscillating slowly about the local minimum for some time.

Our examples illustrate that in models with R-parity, the value of n , and hence the size of the initial field, can vary appreciably. With further symmetries, it is possible that n is even larger, and even that all operators which might lift the flat direction are forbidden (Dine, Randall, and Thomas, 1996). For the rest of this section we will continue to assume that the flat directions are lifted by terms in the superpotential. If they are not, the required analysis is different, since the lifting of the flat direction is entirely associated with supersymmetry breaking.

The term in the potential, $|\frac{\partial W}{\partial \Phi}|^2$ does not break either baryon number or CP. In most models, it turns out that the leading sources of B and CP violation come from certain supersymmetry-breaking terms. These have the form

$$am_{3/2}W + bHW. \quad (93)$$

Here a and b are *complex*, dimensionless constants. The relative phase in these two terms, δ , violates CP . This is crucial; if the two terms carry the same phase, then the phase can be eliminated by a field redefinition, and we have to look elsewhere for possible CP-violating effects. The term proportional to W violates B and/or L . In following the evolution of the field Φ , the important era occurs when $H \sim m_{3/2}$. At this point, the phase misalignment of the two terms, along with the B violating coupling, lead to the appearance of a baryon number. Examining the equation for the time rate of change of the baryon number,

$$\frac{dn_B}{dt} = \frac{\sin(\delta)m_{3/2}}{M^n}\phi^{n+3}, \quad (94)$$

one is lead to the estimate (supported by numerical studies)

$$n_B = \frac{1}{M^n} \sin(\delta) \Phi_o^{n+3}. \quad (95)$$

Here, Φ_o is determined by $H \approx m_{3/2}$, i.e. $\Phi_o^{2n+2} = m_{3/2}^2 M^{2n}$.

E. The Fate of the Condensate

Of course, we don't live in a universe dominated by a coherent scalar field. In this section, we consider the fate of a homogeneous condensate, ignoring possible inhomogeneities. The following sections will deal with inhomogeneities, and the interesting array of phenomena to which they might give rise.

A coherent field can be thought of as a collection of zero momentum particles. These particles are long-lived, since the particles to which they couple gain large mass in the flat direction. Were there no ambient plasma or other fields, these particles would eventually simply decay. However, there are a number of effects which lead the condensate to disappear more rapidly, or to produce stable remnants. Precisely what are the most important mechanisms depend on the amplitude of the oscillations. Particles which couple to Φ will typically have masses of order Φ , and these can be quite heavy. Let us suppose, again, that there has been a period of inflation connected with an inflaton, I . The decay of the inflaton leads to a gradual reheating of the universe, as described, for example, in (Kolb and Turner, 1990). As a function of time (H):

$$T \approx (T_R^2 H(t) M_p)^{1/4}. \quad (96)$$

Provided that $T \ll \Phi$, the particles in the plasma will not couple directly to Φ , but they will scatter off the background with cross sections proportional to $1/\Phi^2$. As a result, the condensate eventually evaporates. Assuming that the inflaton

decay products make up the bulk of the energy density of the universe, one can put these elements together to calculate the final baryon number density.

To get some feeling for the processes involved, we will adopt the picture for inflation described in the previous section. We will suppose that the energy scale of inflation is $E \sim 10^{15}$ GeV. We assume that inflation is due to a field, the inflaton I , whose mass, both during and after inflation is about $m_I = 10^{12}$ GeV, and that the amplitude of inflation is of order $M \approx 10^{18}$ GeV (the reduced Planck mass). Correspondingly, the Hubble constant during inflation is of order $H_I \approx E^2/M_p \approx 10^{12}$ GeV. After inflation ends, the inflaton oscillates about the minimum of its potential until it decays. We will suppose that the inflaton decays to ordinary particles with a rate suppressed by a single power of the Planck mass. The inflaton lifetime will then be of order

$$\Gamma_I = \frac{m_I^3}{M^2} \sim 1 \text{ GeV}. \quad (97)$$

The reheating temperature is of order (Kolb and Turner, 1990)

$$T_R = T(t = \Gamma_I^{-1}) \sim (\Gamma_I M_p)^{1/2} \sim 10^9 \text{ GeV} \quad (98)$$

These assumptions are true of a broad class of inflationary models, but by no means all. We adopt them in order to give some feeling for the possible physics.

In this picture, the field Φ is driven to a large value during inflation. When inflation ends, the inflaton oscillates, with amplitude decreasing with time. The field Φ oscillates about a time-dependent minimum. This minimum drops to zero when $H \sim m_{3/2}$. During this time, a baryon number develops classically. This number is frozen once $H \sim m_{3/2}$.

Eventually the condensate will decay, through a variety of processes. As we have stressed, the condensate can be thought of as a coherent state of Φ particles. These particles – linear combinations of the squark and slepton fields – are unstable and will decay. However, at this period, the lifetimes of these particles are much longer than in the absence of the condensate. The reason is that the fields to which Φ couples have mass of order Φ , and Φ is large. In most cases, the most important process which destroys the condensate is what we might call evaporation: particles in the ambient thermal bath can scatter off of the particles in the condensate, leaving final states with only ordinary particles.

The reaction rate for a particle in the condensate is of order

$$\Gamma_p \sim \alpha^2 \pi \frac{1}{\Phi^2} (T_R^2 H M)^{3/4}. \quad (99)$$

Here we have made a crude estimate for the cross section and multiplied by the thermal density. The condensate will evaporate when this quantity is of order H . Since we know the time dependence of Φ , this allows us to solve for this time. One finds that equality occurs, in the case $n = 1$, for $H_I \sim 10^2 - 10^3$ GeV. For $n > 1$, it occurs significantly later (for $n < 4$, it occurs before the decay of the inflaton; for $n \geq 4$, a slightly different analysis is required than that which follows). In other words, for the case $n = 1$, the condensate evaporates shortly after the baryon number is created (but for more complications, see below), but for larger n , it evaporates significantly later.

The expansion of the universe is unaffected by the condensate as long as the energy density in the condensate, $\rho_\Phi \sim m_\Phi^2 \Phi^2$, is much smaller than that of the inflaton, $\rho_I \sim H^2 M^2$. Assuming that $m_\Phi \sim m_{3/2} \sim 0.1 - 1$ TeV, a typical supersymmetry breaking scale, one can estimate the ratio of the two densities at the time when $H \sim m_{3/2}$ as

$$\frac{\rho_\Phi}{\rho_I} \sim \left(\frac{m_{3/2}}{M_p} \right)^{2/(n+1)} \quad (100)$$

The baryon to photon ratio can be calculated in this model easily. Our assumption is that the inflaton, at this time, carries the bulk of the energy density of the universe, and that the inflaton decays to radiation. This leads to the estimate

$$\frac{n_B}{n_\gamma} \sim \frac{n_B}{(\rho_I/T_R)} \sim \frac{n_B}{n_\Phi} \frac{T_R}{m_\Phi} \frac{\rho_\Phi}{\rho_I} \sim 10^{-10} \left(\frac{T_R}{10^9 \text{ GeV}} \right) \left(\frac{M_p}{m_{3/2}} \right)^{\frac{(n-1)}{(n+1)}} \quad (101)$$

For $n = 1$, this is in precisely the right range to explain the observed baryon asymmetry. For larger n , it can be significantly larger. While this may seem disturbing, it is potentially quite important. Many supersymmetric models lead to creation of entropy at late times. For example, in string theory one expects the existence of other “light” ($m \approx m_{3/2}$) fields, known as “moduli.” These fields lead to cosmological difficulties (Coughlan *et al.*, 1983), unless, when they decay, they reheat the universe to temperatures of order 10 MeV, after which nucleosynthesis can occur.

These decays produce a huge amount of entropy, typically increasing the energy of the universe by a factor of 10^7 . The baryon density is decreased by a corresponding factor. So in such a picture, it is necessary that the baryon number, prior to the moduli decay, should be of order 10^{-3} . This is not the only cosmological model which requires such a large baryon number density.

There are many issues in the evolution of the condensate which we have not touched upon. One of the most serious is related to interactions with the thermal bath (Allahverdi, Campbell, and Ellis, 2000; Anisimov and Dine, 2000). In the case $n = 1$, Φ_o is not so large, and, while the particles which Φ couples to get mass of order Φ_o , they may be in thermal equilibrium. In this case, the Φ particles decay much earlier. This typically leads to significant suppression of the asymmetry, and the viability of the AD mechanism depends on the precise values of the parameters.

Overall, then, there is a broad range of parameters for which the AD mechanism can generate a value for $\frac{n_B}{n_\gamma}$ equal to or larger than that observed. This baryon number is generated long after inflation, so inflationary reheating does not provide any significant constraint. It can be large, allowing for processes which might generate entropy rather late.

F. Inhomogeneities and the Condensate

1. Stability and fragmentation

To analyze the stability of the condensate (Kusenko and Shaposhnikov, 1998), we write the complex field $\phi = \rho e^{i\Omega}$ in terms of its radial component and a phase, both real functions of space-time. We are interested in the evolution of the scalar field in the small-VEV domain, where the baryon number violating processes are suppressed, and we will assume that the scalar potential preserves the U(1) symmetry: $U(\varphi) = U(\rho)$, where $U(\rho)$ may depend on time explicitly. The classical equations of motion in the spherically symmetric metric $ds^2 = dt^2 - a(t)^2 dr^2$ are

$$\ddot{\Omega} + 3H\dot{\Omega} - \frac{1}{a^2(t)}\Delta\Omega + \frac{2\dot{\rho}}{\rho}\dot{\Omega} - \frac{2}{a^2(t)\rho}(\partial_i\Omega)(\partial^i\rho) = 0, \quad (102)$$

$$\ddot{\rho} + 3H\dot{\rho} - \frac{1}{a^2(t)}\Delta\rho - \dot{\Omega}^2\rho + \frac{1}{a^2(t)}(\partial_i\Omega)^2\rho + (\partial U/\partial\rho) = 0, \quad (103)$$

where dots denote time derivatives, and the space coordinates are labeled by the Latin indices that run from 1 to 3. The Hubble constant, again, is $H = \dot{a}/a$, where $a(t)$ is the scale factor; it is equal to $t^{-2/3}$ or $t^{-1/2}$ for a matter or radiation dominated universe, respectively.

From the equations of motion (102) and (103), one can derive the equations for small perturbations $\delta\Omega$ and $\delta\rho$:

$$\delta\ddot{\Omega} + 3H(\delta\dot{\Omega}) - \frac{1}{a^2(t)}\Delta(\delta\Omega) + \frac{2\dot{\rho}}{\rho}(\delta\dot{\Omega}) + \frac{2\dot{\Omega}}{\rho}(\delta\dot{\rho}) - \frac{2\dot{\rho}\dot{\Omega}}{\rho^2}\delta\rho = 0, \quad (104)$$

$$\delta\ddot{\rho} + 3H(\delta\dot{\rho}) - \frac{1}{a^2(t)}\Delta(\delta\rho) - 2\rho\dot{\Omega}(\delta\dot{\Omega}) + U''\delta\rho - \dot{\Omega}^2\delta\rho = 0. \quad (105)$$

To examine the stability of a homogeneous solution $\varphi(x, t) = \varphi(t) \equiv \rho(t)e^{i\Omega(t)}$, let us consider a perturbation $\delta\rho, \delta\Omega \propto e^{S(t) - i\tilde{k}\tilde{x}}$ and look for growing modes, $\text{Re}\alpha > 0$, where $\alpha = dS/dt$. The value of k is the spectral index in the comoving frame and is red-shifted with respect to the physical wavenumber $\tilde{k} = k/a(t)$ in the expanding background. Of course, if an instability develops, the linear approximation is no longer valid. However, we assume that the wavelength of the fastest-growing mode sets the scale for the high and low density domains that eventually evolve into Q-balls. This assumption can be verified *post factum* by comparison with a numerical solution of the corresponding partial differential equations (102) and (103), where both large and small perturbations are taken into account.

The dispersion relation follows from the equations of motion:

$$\left[\alpha^2 + 3H\alpha + \frac{k^2}{a^2} + \frac{2\dot{\rho}}{\rho}\alpha \right] \left[\alpha^2 + 3H\alpha + \frac{k^2}{a^2} - \dot{\Omega}^2 + U''(\rho) \right] + 4\dot{\Omega}^2 \left[\alpha - \frac{\dot{\rho}}{\rho} \right] \alpha = 0. \quad (106)$$

If $(\dot{\Omega}^2 - U''(\rho)) > 0$, there is a band of growing modes that lies between the two zeros of $\alpha(k)$, $0 < k < k_{\text{max}}$, where

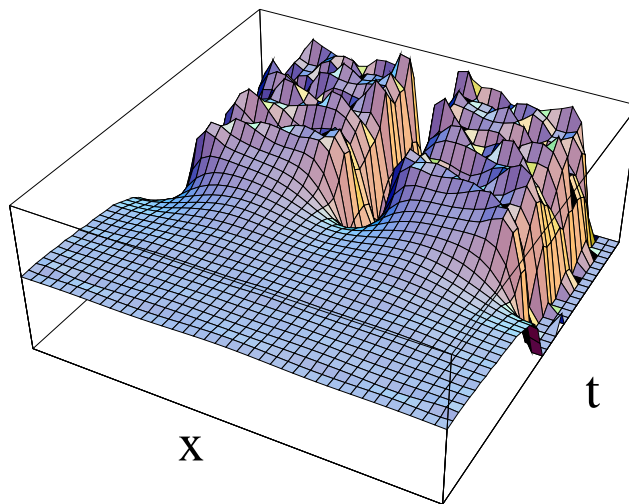


FIG. 5 The charge density per comoving volume in (1+1) dimensions for a sample potential analyzed numerically during the period when the spatially homogeneous condensate breaks up into high- and low-density domains. Two domains with high charge density are expected to form Q-balls.

$$k_{\max}(t) = a(t)\sqrt{\dot{\Omega}^2 - U''(\rho)}. \quad (107)$$

This simple linear analysis shows that when the condensate is “overloaded” with charge, that is when $\omega(t) = \dot{\Omega}$ is larger than the second derivative of the potential, an instability sets in. Depending on how $k_{\max}(t)$, defined by relation (107), varies with time, the modes in the bands of instability may or may not have time to develop fully.

Numerical analyses (Enqvist *et al.*, 2001; Kasuya and Kawasaki, 2000a,b, 2001), which can trace the evolution of unstable modes beyond the linear regime, have shown that fragmentation of the condensate is a generic phenomenon. Numerically one can also study the stability of rapidly changing solutions, hence relaxing the adiabaticity condition assumed above. This aspect is relevant to the cases where the baryon number density is small and the radial component of the condensate, $\rho(t)$, exhibits an oscillatory behavior changing significantly on small time scales. An interesting feature of this non-adiabatic regime is that both baryon and anti-baryon lumps may form as a result of fragmentation (Enqvist *et al.*, 2001).

2. Lumps of scalar condensate: Q-Balls

Field theories with scalar fields often admit non-topological solitons (Coleman, 1985; Friedberg, Lee, and Sirlin, 1976; Rosen, 1968), Q-balls, which may be stable or may decay into fermions (Cohen, Coleman, Georgi, and Manohar, 1986). Q-balls appear when a complex scalar field φ carries a conserved charge with respect to some global U(1) symmetry. In supersymmetric generalizations of the Standard Model, squarks and sleptons, which carry the conserved baryon and lepton numbers, can form Q-balls.

Let us consider a field theory with a scalar potential $U(\varphi)$ which has a global minimum $U(0) = 0$ at $\varphi = 0$. Let $U(\varphi)$ have an unbroken global U(1) symmetry at the global minimum: $\varphi \rightarrow \exp\{i\theta\}\varphi$. We will look for solutions of the classical equations by minimizing the energy

$$E = \int d^3x \left[\frac{1}{2}|\dot{\varphi}|^2 + \frac{1}{2}|\nabla\varphi|^2 + U(\varphi) \right] \quad (108)$$

subject to the constraint that the configuration has a definite charge, Q ,

$$Q = \frac{1}{2i} \int \varphi^* \overleftrightarrow{\partial}_t \varphi d^3x \quad (109)$$

To describe the essential features of Q-balls in a simple way, we will, following Coleman (1985), use a thin wall ansatz for the Q-ball

$$\phi(x, t) = e^{i\omega t} \bar{\phi}(x), \quad (110)$$

where

$$\bar{\phi}(x) = \begin{cases} 0, & \sqrt{x^2} > R \\ \phi_0 & \sqrt{x^2} \leq R \end{cases} \quad (111)$$

(for the real solution, the field varies rapidly between the two regions, changing on a scale of order the Compton wavelength of the ϕ particle).

Assuming that Q is large, let us neglect the gradient terms (relevant only for the wall energy). Then the global charge and the energy of the field configuration (110,111) are given by

$$Q = \omega \phi_0^2 V, \quad (112)$$

where $V = (4/3)\pi R^3$, and

$$E = \frac{1}{2}\omega^2 \phi_0^2 V + U(\phi_0)V = \frac{1}{2} \frac{Q^2}{V\phi_0^2} + VU(\phi_0) \quad (113)$$

We now minimize E with respect to V , obtaining

$$E = \sqrt{\frac{2U(\phi_0)}{\phi_0^2}} \quad (114)$$

It remains to minimize the energy with respect to variations of ϕ_0 . A non-trivial minimum exists as long as

$$U(\varphi)/\varphi^2 = \min, \quad \text{for } \varphi = \varphi_0 > 0; \quad (115)$$

if this condition is satisfied, a Q-ball solution exists.

So far we have assumed a particular ansatz, neglected the gradient terms, *etc.* All these flaws can be rectified by a slightly more tedious derivation (Kusenko, 1997a) using the method of Lagrange multipliers. We want to minimize

$$\mathcal{E}_\omega = E + \omega \left[Q - \frac{1}{2i} \int \varphi^* \partial_t \varphi d^3x \right], \quad (116)$$

where ω is a Lagrange multiplier. (It is no accident that we use the same letter, ω . The value of the Lagrange multiplier at the minimum will turn out to be equal to the time derivative of the phase.) Variations of $\varphi(x, t)$ and those of ω can now be treated independently, the usual advantage of the Lagrange method.

One can re-write equation (116) as

$$\mathcal{E}_\omega = \int d^3x \frac{1}{2} \left| \frac{\partial}{\partial t} \varphi - i\omega \varphi \right|^2 + \int d^3x \left[\frac{1}{2} |\nabla \varphi|^2 + \hat{U}_\omega(\varphi) \right] + \omega Q, \quad (117)$$

where

$$\hat{U}_\omega(\varphi) = U(\varphi) - \frac{1}{2} \omega^2 \varphi^2. \quad (118)$$

We are looking for a solution that extremizes \mathcal{E}_ω , while all the physical quantities, including the energy, E , are time-independent. Only the first term in equation (117) appears to depend on time explicitly, but it is positive definite and, hence, it should vanish at the minimum. To minimize this contribution to the energy, one must choose, therefore,

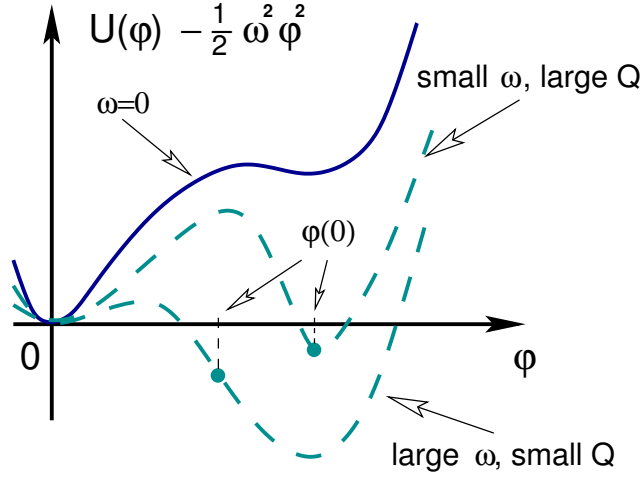


FIG. 6 Finding a Q-ball is equivalent to finding a bounce that describes tunneling in the potential $\hat{U}_\omega(\varphi) = U(\varphi) - (1/2)\omega^2\varphi^2$. The thin-wall approximation is good for large Q (upper dashed line), but breaks down when Q is small and, therefore, ω is almost as large as the mass term at the origin. In the latter case (lower dashed line), the “escape point”, $\varphi(0)$ is close to the zero of the potential and is far from the global minimum.

$$\varphi(x, t) = e^{i\omega t}\varphi(x), \quad (119)$$

where $\varphi(x)$ is real and independent of time. We have thus derived equation (110). For this solution, equation (109) yields

$$Q = \omega \int \varphi^2(x) d^3x \quad (120)$$

It remains to find an extremum of the functional

$$\mathcal{E}_\omega = \int d^3x \left[\frac{1}{2}|\nabla\varphi(x)|^2 + \hat{U}_\omega(\varphi(x)) \right] + \omega Q, \quad (121)$$

with respect to ω and the variations of $\varphi(x)$ independently. We can first minimize \mathcal{E}_ω for a fixed ω , while varying the shape of $\varphi(x)$. If this were an actual potential for a scalar field in three dimensions, one would have the possibility of tunneling between the zero energy configuration at the origin and possible lower energy configurations at non-zero ϕ (Fig. 6). Tunneling, in the semiclassical approximation, is described by the bounce, $\bar{\varphi}_\omega(x)$, the solution of the classical equations which asymptotes to the “false vacuum” at the origin (Coleman, 1977). The first term in equation (121) is then nothing but the three-dimensional Euclidean action $S_3[\bar{\varphi}_\omega(x)]$ of this bounce solution. This is a very useful correspondence. In particular, the condition for the existence of solution is simply a corollary: as long as $\hat{U}_\omega(\varphi)$ has a minimum below zero, the bounce exists, and so does the Q-ball, $\varphi(x, t) = \exp\{i\omega t\}\bar{\varphi}(x)$.

The bounce, and hence the Q-ball, exist if there exists a value of ω , for which the potential $\hat{U}_\omega(\varphi)$ has both a local minimum at $\varphi = 0$ and a global minimum at some other value of φ . This condition can be re-phrased without reference to ω : a Q-ball solution exists if

$$U(\varphi)/\varphi^2 = \min, \quad \text{for } \varphi = \varphi_0 > 0 \quad (122)$$

The corresponding effective potential $\hat{U}_{\omega_0}(\varphi)$, where $\omega_0 = \sqrt{2U(\varphi_0^2)/\varphi_0^2}$, has two degenerate minima, at $\varphi = 0$ and $\varphi = \varphi_0$. The existence of the bounce solution $\bar{\varphi}_\omega(x)$ for $\omega_0 < \omega < U''(0)$ follows (Coleman, 1977; Coleman *et al.*, 1978) from the fact that $\hat{U}_\omega(\varphi)$ has a negative global minimum in addition to the local minimum at the origin. Coleman *et al.* (1978) also showed that the solution is spherically symmetric: $\bar{\varphi}(x) = \bar{\varphi}(r)$, $r = \sqrt{x^2}$.

The soliton we want to construct is precisely this bounce for the right choice of ω , namely that which minimizes \mathcal{E}_ω . The last step is to find an extremum of

$$\mathcal{E}_\omega = S_3[\bar{\varphi}_\omega(x)] + \omega Q \quad (123)$$

with respect to ω . One can prove the existence of such an extremum (Kusenko, 1997a). Finally, the soliton is of the form (119), with ω that minimizes \mathcal{E}_ω in eq. (123).

Having obtained the solution, one can compute its energy (mass). For a finite φ_0 in eq. (122), in the limit of large Q , the Q-ball has a thin wall, and its mass is given by

$$M(Q) = \omega_0 Q \quad (124)$$

Supersymmetric generalizations of the Standard Model have scalar potentials with flat directions lifted only by supersymmetry breaking terms. Q-balls may form with a light scalar field φ that corresponds to that flat direction. If the potential $V(\varphi) = \mu^4 = \text{const}$ for large φ , then the minimum in eq. (122) is achieved for $\varphi_0 = \infty$. In this case,

$$M(Q) = \mu Q^{3/4}. \quad (125)$$

More generally, if the potential grows slower than φ^2 , *i.e.* $V(\varphi) \propto \varphi^p$, $p < 2$, condition (122) is not satisfied at any finite value of ϕ_0 , and

$$M(Q) \sim \mu Q^{(3-p/2)/(4-p)}. \quad (126)$$

It is important in what follows that the mass per unit charge is not a constant, but is a decreasing function of the total global charge Q . There is a simple reason why the soliton mass is not proportional to Q . Since $U(\varphi)/\varphi^2$ has no minimum, the scalar VEV can extend as far as the derivative terms allow it. When the next unit of charge is added, the Q-ball increases in size, which allows the scalar VEV to increase as well. Hence, the larger the charge, the greater is the VEV, and the smaller is energy per unit charge.

3. AD Q-balls

Q-balls can develop along flat directions that carry non-zero baryon number, lepton number, or both. Each flat direction can be parameterized by a gauge-invariant field, carrying these global quantum numbers. So the discussion of gauge singlet fields of the previous section also applies to baryonic and leptonic Q-balls in the MSSM. This statement may seem surprising, since all scalar baryons in the MSSM transform non-trivially under the gauge group. Although scalars with gauge interactions can also make Q-balls (Lee *et al.*, 1989), in the case of the MSSM the color structure of large Q-balls is rather simple (Kusenko, Shaposhnikov, and Tinyakov, 1998). If a Q-ball VEV points along a flat direction, its scalar constituents form a colorless combination (otherwise, that direction would not be flat because of non-vanishing D-terms).

In gauge-mediated scenarios of SUSY breaking, the flat directions are lifted by potentials which grow quadratically for small values of the fields, and then level off to a logarithmic plateau at larger ϕ . Q-balls in such a potential have masses given by eq. (125). In gravity-mediated scenarios, the potentials which arise from supersymmetry breaking grow roughly quadratically even for very large VEV. Whether Q-balls exist is thus a detailed, model-dependent question. Q-balls in these potentials have masses proportional to the first power of Q .

By construction, Q-balls are stable with respect to decay into scalars. However, they can decay by emitting fermions (Cohen, Coleman, Georgi, and Manohar, 1986). If the Q-ball has zero baryon number, it can decay by emitting light neutrinos (Cohen, Coleman, Georgi, and Manohar, 1986).

However, if a baryonic Q-ball (“B-ball”) develops along a flat direction, it can also be stable with respect to decay into fermions. Stability requires that the baryon number be large enough. A Q-ball with baryon number Q_B and mass $M(Q_B)$ is stable if its mass is below the mass of Q_B separated baryons. For a Q-ball in a flat potential of height M_S , the mass per unit baryon number

$$\frac{M(Q_B)}{Q_B} \sim M_S Q^{-1/4} \quad (127)$$

Models of gauge-mediated supersymmetry breaking produce flat potentials with $M_S \sim 1 - 10$ TeV. If the mass per baryon number is less than the proton mass, m_p , then the Q-ball is entirely stable because it does not have enough energy to decay into a collection of nucleons with the same baryon number. This condition translates into a lower bound on Q_B :

$$\frac{M(Q_B)}{Q_B} < 1 \text{ GeV} \Rightarrow Q_B \gg \left(\frac{M_S}{1 \text{ GeV}} \right)^4 \gtrsim 10^{16}. \quad (128)$$

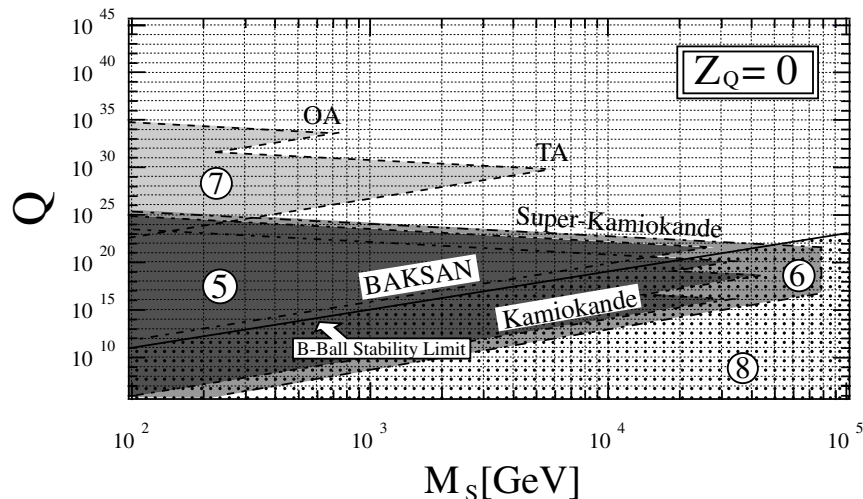


FIG. 7 Present limits on the electrically neutral dark-matter Q-balls from a paper by Arafune *et al.* (2000).

4. Dark matter in the form of stable B-balls

Stable Q-balls that form from an AD condensate are a viable candidate for dark matter. Even if they are unstable, their decay can produce neutralinos at late times, when these neutralinos are out of equilibrium. One way or another, some dark matter can arise from AD baryogenesis.

Moreover, since both the ordinary matter and the dark matter have the same origin in the AD scenario, one can try to explain why their amounts in the universe are fairly close (Banerjee and Jedamzik, 2000; Enqvist and McDonald, 1999; Fujii and Yanagida, 2002a; Laine and Shaposhnikov, 1998).

Since the MSSM with gauge-mediated supersymmetry breaking contains stable objects, baryonic Q-balls, it is natural to ask whether they can constitute the dark matter. Stable Q-balls can be copiously produced in the course of AD baryogenesis. Of the dark-matter candidates that have been considered, most were weakly interacting particles, and for good reason. If dark-matter particles have strong (*relative to their mass*) interactions with matter, these interactions might facilitate their loss of momentum and angular momentum, forcing them into the galactic disks, along with ordinary matter. But astronomical observations show that the dark matter forms spherical halos about galaxies, not disks.

Made of squarks, these Q-balls can interact strongly with ordinary matter, *via* QCD. However, if they are as heavy as the calculations show they are, the strong interactions are not strong enough to force dark-matter Q-balls to settle into the galactic disks. Analyses of Q-ball formation and partial evaporation allow one to relate the amounts of ordinary matter and dark matter. The observed ratio corresponds to Q-balls with baryon number of about $10^{26 \pm 2}$, which is in agreement with the expected Q-balls size from numerical simulations (Kasuya and Kawasaki, 2000a, 2001), as well as with the current experimental bounds summarized by Arafune *et al.* (2000), see Fig. 7. A B-ball with baryon number 10^{26} is so heavy that it could pass through ordinary stars with only a small change in its velocity ($(\delta v/v) \sim 10^{-5}$). Hence, despite the strong interactions, B-balls make a good dark-matter candidate.

The unusually large mass for a dark matter candidate means that the fluxes are very small. The experimental detection of this form of dark matter requires a large detector size (*cf.* Fig. 7).

Since B-balls have lower mass-to-baryon ratio than ordinary nuclear matter, interactions of B-balls with ordinary matter result in numerous “proton decay” events (Kusenko *et al.*, 1998a). Hence a Q-ball passing through a detector would produce a spectacular signature. The flux, however, is very low. Hence, the strongest limits come from the largest detectors, e.g. Super-Kamiokande (see Fig. 7). Some astrophysical bounds have been considered (Kusenko *et al.*, 1998c), but they do not yield very strong constraints. In addition to the existing limits discussed in (Arafune *et al.*, 2000), future experiments, such as ANTARES, Ice Cube, *etc.*, may be able to detect dark matter B-balls, or rule out the values of Q that correspond to the correct amount of dark matter.

We note that, although Q-balls are always present in the spectrum of any SUSY extension of the Standard Model, their production in the early universe requires a formation of the Affleck–Dine condensate followed by its fragmentation. Stable Q-balls are too large to form in thermal plasma by accretion (Griest and Kolb, 1989; Postma, 2002). In this sense, an observation of stable dark-matter Q-balls would be evidence of the Affleck–Dine process having taken place.

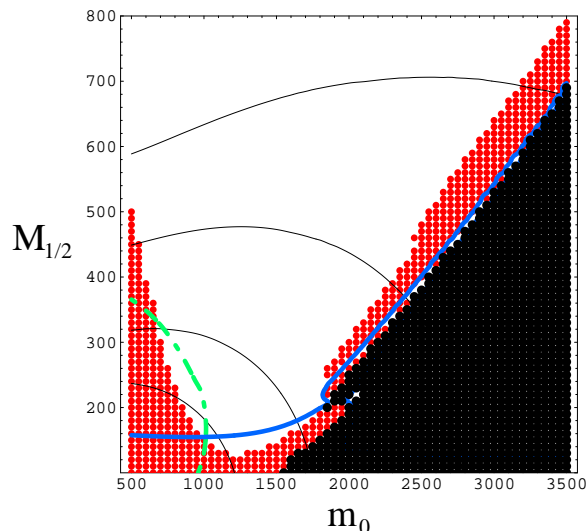


FIG. 8 The allowed range of parameters for non-thermal LSP dark matter is very different from that in the standard freeze-out case. The lightly shaded region above the solid and the dashed lines is allowed for non-thermal LSP dark matter in the minimal supergravity model with $\tan\beta = 40$ (Fujii and Hamaguchi, 2002; Fujii and Yanagida, 2002a).

5. Dark matter from unstable B-balls

If supersymmetry breaking is mediated by gravity, Q-balls are not stable as they can decay into fermions. However, Q-ball decay into fermions is a slow process because the fermions quickly fill up the Fermi sea inside the Q-ball, and further decays are limited by the rate of fermion evaporation through the surface. The rate of Q-ball decay is, therefore, suppressed by the surface-to-volume ratio (Cohen, Coleman, Georgi, and Manohar, 1986) as compared to that of free scalar particles. In a typical model, unstable baryonic Q-balls from the Affleck–Dine condensate decay when the temperature is as low as a GeV.

The lightest supersymmetric particles (LSP) are among the decay products of Q-balls. B-balls can decay and produce dark matter in the form of neutralinos at a time when they are out of equilibrium (Enqvist and McDonald, 1998, 1999; Fujii and Yanagida, 2002a). This presents another possibility for producing dark matter from the AD condensate and relating its abundance to that of ordinary matter. The requirement that neutralinos not overclose the universe constrains the parameter space of the MSSM (Fujii and Hamaguchi, 2002).

If the LSPs don't annihilate, the ratio of ordinary matter to dark matter is simply (Enqvist and McDonald, 1999)

$$\Omega_{\text{matter}}/\Omega_{\text{LSP}} \sim f^{-1} \left(\frac{m_p}{m_\chi} \right) \left(\frac{n_B}{n_\chi} \right) \quad (129)$$

where f is the fraction of the condensate trapped in Q-balls. If $f \sim 10^{-3}$, this ratio is acceptable.

However, numerical simulations (Kasuya and Kawasaki, 2000a) and some analytical calculations (McDonald, 2001) indicate that, in a wide class of AD models, practically all the baryon number may be trapped in Q-balls, that is $f \sim 1$. If that's the case, the LSP would overclose the universe, according to eq. (129). A solution, proposed by Fujii and Yanagida (2002a), is to eliminate the unwanted overdensity of neutralinos by using an LSP with a higher annihilation cross section. The LSP in the MSSM is an admixture of several neutral fermions. Depending on the parameters in the mass matrix, determined largely by the soft SUSY breaking terms, the LSP can be closely aligned with Bino (one of the SUSY partners of the $SU(2) \times U(1)$ gauge bosons), Higgsino (the fermion counterpart of the Higgs boson), or with one of the other weak eigenstates. The traditional, freeze-out scenario for LSP production favors the Bino-like LSP (Jungman, Kamionkowski, and Griest, 1996). However, according to Fujii and Yanagida (2002a), SUSY dark matter produced from the Affleck–Dine process has to be in the form of a Higgsino-like LSP. In this case, the ratio of matter densities is

$$\Omega_{\text{matter}}/\Omega_{\text{LSP}} = 10^{3-4} \left(\frac{m_\phi^2}{\langle \sigma v \rangle_\chi^{-1}} \right) \left(\frac{m_p}{m_\chi} \right) \delta_{\text{CP}}, \quad (130)$$

where $\delta_{\text{CP}} \sim 0.1$ is the effective CP violating phase of the AD condensate. For a Higgsino-like LSP, which has $\langle \sigma v \rangle \sim 10^{-(7-8)} \text{GeV}^{-2}$, this yields an acceptable result.

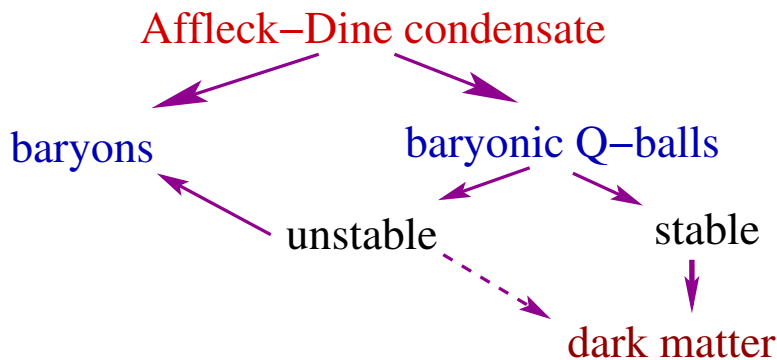


FIG. 9 The fate of the AD condensate

This has important implications for both direct dark matter searches and the collider searches for SUSY. First, the parameter space of the MSSM consistent with LSP dark matter is very different, depending on the cosmological scenario at work, that is whether the LSPs froze out of equilibrium (Arnowitz and Dutta, 2002) or were produced from the evaporation of AD B-balls (Fujii and Yanagida, 2002a). Second, higgsino and bino LSP’s interact differently with matter, so the sensitivity of direct dark-matter searches also depends on the type of the LSP.

If supersymmetry is discovered, one will be able to determine the properties of the LSP experimentally. This will, in turn, provide some information on the how the dark-matter SUSY particles could be produced. The discovery of a Higgsino-like LSP would be a evidence in favor of Affleck–Dine baryogenesis. This is yet another way in which we might be able to establish the true origin of matter-antimatter asymmetry.

IV. CONCLUSIONS

The origin of the matter-antimatter asymmetry is one of the great questions in cosmology. Yet we can obtain only limited information about the events which gave rise to the baryon asymmetry by looking at the sky. Filling out the picture requires a deeper understanding of fundamental physical law. One elegant possibility, that the Minimal Standard Model produced the baryon number near the electroweak scale, is ruled out decisively by the LEP bounds on the Higgs mass. This is a bittersweet conclusion: while one has to give up an elegant scenario, this is perhaps the strongest evidence yet for physics beyond the Standard Model – a precursor of future discoveries.

Supersymmetry is widely regarded as a prime candidate for such new physics. Theoretical arguments in favor of supersymmetry are based on the naturalness of the scale hierarchy, the success of coupling unification in supersymmetric theories, and the nearly ubiquitous role of supersymmetry in string theory. The upcoming LHC experiments will put this hypothesis to a definitive test. If low energy supersymmetry exists, there are several ways in which it might play the crucial role in baryogenesis. It could conceivably revive the electroweak baryogenesis scenario. However, the phase transition in the MSSM is only slightly stronger than that in the Standard Model; a noticeable improvement forces one into a narrow corner of the MSSM parameter space, which may soon be ruled out.

But supersymmetry opens a completely new and natural avenue for baryogenesis. If inflation took place in the early universe, for which we have an increasing body of evidence, then formation of an Affleck–Dine condensate and subsequent generation of some baryon asymmetry is natural. In a wide class of models this process produces the observed baryon asymmetry. This is true not only of conventional cosmologies, but also of cosmologies where one produces substantial entropy at late times, which might be important if nature is supersymmetric. In addition, the same process can produce dark matter, either in the form of stable SUSY Q-balls, or in the form of a thermally or non-thermally produced LSP. There are hints that the relative closeness of matter and dark matter densities may find its explanation in the same process as well. If supersymmetry is discovered, given the success of inflation theory, the Affleck–Dine scenario will appear quite plausible.

Other independent indications that the Affleck–Dine process took place in the early universe may come from detection of dark matter. If the lightest supersymmetric particle is the dark matter, its contribution to the energy density of the universe depends on its annihilation cross section and mass. A combination of accelerator limits and cosmology leaves allows some range of parameters consistent with LSP being the dark matter. In this range, the LSP, which is an admixture of several states, must be close to “Bino” if it is produced in the standard freeze-out scenario. However, if future detection will indicate that the LSP is Higgsino-like, this kind of dark matter could only arise from non-thermal production of the LSP from a fragmented Affleck–Dine condensate (Fujii and Hamaguchi, 2002; Fujii and Yanagida, 2002a). Therefore, although a standard Bino-like LSP is not inconsistent with the Affleck–Dine

scenario, a Higgsino-like LSP would provide a strong evidence in its favor. Likewise, a detection of stable baryonic Q-balls would be a definitive confirmation that an Affleck–Dine condensate formed in the early universe and fragmented into B-balls. Since stable SUSY Q-balls must be large, we know of no other cosmological scenario that could lead to their formation.

Among other possibilities, leptogenesis is also quite plausible. The discovery of neutrino mass, perhaps associated with a rather low scale of new physics, certainly gives strong support to this possibility. The question of what scales might be compatible with inflation, and what implications this might have for the underlying origin of neutrino mass is extremely important. Some pieces of the picture will be accessible to experiment, but many of the relevant parameters, including the relevant CP violation, reside at a very high scale. Perhaps, in a compelling theory of neutrino flavor, some of these questions can be pinned down.

Future experimental searches for supersymmetry, combined with the improving cosmological data on CMBR and dark matter, will undoubtedly shed further light on the origin of baryon asymmetry and will help understand both particle physics and cosmology. The study of the baryon asymmetry has already provided a compelling argument for new physics, and holds great promise of new and exciting discoveries in the future.

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