# Supersymmetry from Bottom Up and the Top Down

## Jerusalem Winter School, Jerusalem, 2011

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December, 2011

**Lecture I, Part A: The Standard Model and Beyond after $5 \text{ fb}^{-1}$ at LHC**

1. Discovery of the Higgs – almost. The Standard Model is Complete!(?)
2. Is particle physics over? What's missing from the Standard Model?

**Lecture I, part B: Supersymmetry Introduction**

1. Why supersymmetry?
2. Basics of Supersymmetry
3. R Symmetries (a theme in these lectures)
4. SUSY soft breakings
5. MSSM: counting of parameters
6. MSSM: features. Sparticle spectrum. *Higgs mass*.
7. LHC Constraints: Implications for tuning.
8. Beyond the MSSM

**Lecture II: Microscopic supersymmetry: supersymmetry breaking and its Mediation**

1. Gravity Mediation
2. Minimal Gauge Mediation (one – really three) parameter description of the MSSM.
3. General Gauge Mediation
4. Nelson-Seiberg Theorem (R symmetries)
5. O'Raifeartaigh Models
6. The Goldstino
7. Flat directions/pseudomoduli: Coleman-Weinberg potential and finding the vacuum.
8. Integrating out pseudomoduli (if time) – non-linear lagrangians.

**Lecture III: Dynamical (*Metastable*) Supersymmetry Breaking**

1. Non-renormalization theorems
2. SUSY QCD/Gaugino Condensation
3. Generalizing Gaugino condensation
4. A simple approach to Dynamical, Metastable Supersymmetry Breaking: Retrofitting.
5. Retrofitting – a second look. Why it might be right (cosmological constant!).

**Lecture IV: Naturalness Revisited**

Landscape as a Model to Confront Questions of Naturalness

1. What can we learn from String Theory?
2. Landscape Models: Definition
3. Landscape Models: Implications
4. Assessment.

For the rest of these lectures, I will accept that there is a Higgs with mass approximately 125 GeV, with couplings close to those predicted by the Standard Model. I am willing to bet that this is the case, though, as you'll see, I wish the mass had been a bit smaller, or the couplings appreciably different than those predicted by the SM.

We should pause. This is a triumph. The Standard Model is complete. The simplest version of the Higgs model is at least an approximate description of the breaking of electroweak symmetry.

We know the Standard Model cannot be a complete theory of nature; fails to account for known facts.

Top down (energy scales)

1. Doesn't incorporate General Relativity (UV behavior, quantum mechanics of black holes...)
2. Doesn't account for inflation
3. Doesn't account for neutrino mass
4. Doesn't account for the baryon asymmetry
5. Doesn't account for dark matter
6. Strong CP problem
7. Many parameters (related: not "UV complete")

And several hierarchy problems (in rough order of severity):

1. The dark energy ($10^{-120}$) (The problem of the Cosmological Constant (The *CCP*)

2. The hierarchy between the scale of weak interactions and the Planck scale ($10^{-32}$) (The *EHP*)

3. The hierarchy between the scale of neutrino masses and the Planck scale ($10^{-8}$?) (The *NMP*)

4. The hierarchy between the scale of inflation and the Planck scale ($10^{-12}$?) (The it ISP)

## To what do these questions point?

Theorist have and will continue to address all of these questions. Some of these can be considered by themselves, e.g. the existence of consistent theories of quantum gravity. Some of these lead to models, which might be tested by observation if we are very lucky (inflation).

Of the the hierarchy problems, the weak/Planck scale seems to point directly at LHC scale physics. We will see that some solutions to this solve or at least ameliorate the others. But the existence of other hierarchies, esp. the dark energy, leads to unease as to whether we should see evidence in LHC physics for a solution to the EHP.

Two ways to understand:

1. Dimensional Analysis: $m_h^2 = \Lambda^2$. $\Lambda$: some microphysical scale. $M_p$? $M_{gut}$? $M_{string}$?

2. Feynman diagrams: already at one loop, quadratically divergent.

Suppose left Higgs out of Standard Model. Then strong interactions would "break" $SU(2) \times U(1) \rightarrow U(1)$, through the condensate

$$\bar{u}u = \bar{d}d = \Lambda_{QCD}^3. \tag{1}$$

The $W$ and $Z$ would be quite light, with masses of order 10's of MeV.

Basic idea in technicolor is very simple. Postulate a new set of strong interactions, say $SU(N)$, with $\Lambda_{TC} \sim \mathrm{TeV}$, and Techniquarks $Q, \bar{U}, \bar{D}$ in $N, \bar{N}$ representations, and doublets/singlets of $SU(2)$ with suitable hypercharge

Closely related: all models of strongly coupled Higgs, composite Higgs, Randall Sundrum. I will lump them all together.

Prior to the LHC program

1. Precision electroweak physics
2. Flavor physics – obligated to explain everything; fails. Even if one pretends, leads to phenomenological catastrophe.

Most important: we now (almost) know that there is a light, SM-like Higgs!

# Scorecard for Solutions of Hierarchy Problem

|  | Models | Pr EW | Flavor | CP | DM | Unify | DSB |
|---|---|---|---|---|---|---|---|
| Technicolor | X | X | X | X | OK | X | OK |
| Other Strong | X | X | X | X | ? | X | OK |
| RS | ? | X? | X? | X? | ? | X | OK |
| SUSY | OK | OK | OK- | OK- | OK | OK | OK |

# SUPERSYMMETRY AND THE HIERARCHY PROBLEM

Virtues

1. Hierarchy Problem
2. Unification
3. Dark matter
4. Presence in string theory (often)

1. Cancelation of quadratic divergences
2. Non-renormalization theorems (holomorphy of gauge couplings and superpotential): if supersymmetry unbroken classically, unbroken to all orders of perturbation theory, but can be broken beyond: exponentially large hierarchies.

But reasons for skepticism:

1. Little hierarchy
2. Unification: why generic (grand unified models; string theory?)
3. Hierarchy: landscape (light higgs anthropic?)

Reasons for (renewed) optimism:

1. The study of metastable susy breaking (ISS) has opened rich possibilities for model building; no longer the complexity of earlier models for dynamical supersymmetry breaking.

2. Supersymmetry, even in a landscape, can account for hierarchies, as in traditional thinking about naturalness ($e^{-\frac{8\pi^2}{g^2}}$)

3. Supersymmetry, in a landscape, accounts for stability – i.e. the very existence of (metastable) states.

Basic algebra:

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu.$$

Note that taking trace:

$$Q^*_\alpha Q_\alpha + Q_\alpha Q^*_\alpha = 2P^0, \tag{2}$$

SUSY is broken iff vacuum energy vanishes.

## Superspace

It is convenient to introduce an enlargement of space-time, known as *superspace*, to describe supersymmetric systems. One does not have to attach an actual geometric interpretation to this space (though this may be possible) but can view it as a simple way to realize the supersymmetry algebra. The space has four additional, anticommuting (Grassmann) coordinates, $\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$. Fields (superfields) will be functions of $\theta, \bar{\theta}$ and $x^\mu$. Acting on this space of functions, the $Q$'s and $\bar{Q}$'s can be represented as differential operators:

$$Q_\alpha = \frac{\partial}{\partial \theta_\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu; \qquad \bar{Q}^{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} - i\theta^\alpha \sigma^\mu_{\alpha\dot{\beta}}\epsilon^{\dot{\beta}\dot{\alpha}}\partial_\mu. \qquad (3)$$

Infinitesimal supersymmetry transformations are generated by

$$\delta\Phi = \epsilon Q + \bar{\epsilon}\bar{Q}. \tag{4}$$

It is also convenient to introduce a set of *covariant derivative* operators which anticommute with the $Q_\alpha$'s, $\bar{Q}_{\dot{\alpha}}$'s:

$$D_\alpha = \frac{\partial}{\partial\theta_\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu; \qquad \bar{D}^{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\beta}}\epsilon^{\dot{\beta}\dot{\alpha}}\partial_\mu. \tag{5}$$

## Chiral and Vector Superfields

There are two irreducible representations of the algebra which are crucial to understanding field theories with $N = 1$ supersymmetry: chiral fields, $\Phi$, which satisfy $\bar{D}_{\dot\alpha}\Phi = 0$, and vector fields, defined by the reality condition $V = V^\dagger$. Both of these conditions are invariant under supersymmetry transformations, the first because $\bar{D}$ anticommutes with all of the $Q$'s. In superspace a chiral superfield may be written as

$$\Phi(x, \theta) = A(x) + \sqrt{2}\theta\psi(x) + \theta^2 F + \dots \qquad (6)$$

Here $A$ is a complex scalar, $\psi$ a (Weyl) fermion, and $F$ is an auxiliary field, and the dots denote terms containing derivatives.

More precisely, $\Phi$ can be taken to be a function of $\theta$ and

$$y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}. \tag{7}$$

Under a supersymmetry transformation with anticommuting parameter $\zeta$, the component fields transform as

$$\delta A = \sqrt{2}\zeta\psi, \tag{8}$$

$$\delta\psi = \sqrt{2}\zeta F + \sqrt{2}i\sigma^\mu\bar{\zeta}\partial_\mu A, \quad \delta F = -\sqrt{2}i\partial_\mu\psi\sigma^\mu\bar{\zeta}. \tag{9}$$

Vector fields can be written, in superspace, as

$$V = i\chi - i\chi^\dagger + \theta\sigma^\mu\bar{\lambda}A_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D. \qquad (10)$$

Here $\chi$ is a chiral field.

In order to write consistent theories of spin one fields, it is necessary to enlarge the usual notion of gauge symmetry to a transformation of $V$ and the chiral fields $\Phi$ by superfields. In the case of a $U(1)$ symmetry, one has

$$\Phi_i \rightarrow e^{q_i \Lambda} \Phi_i \quad V \rightarrow V - \Lambda - \Lambda^\dagger. \tag{11}$$

Here $\Lambda$ is a chiral field (so the transformed $\Phi_i$ is also chiral). Note that this transformation is such as to keep

$$\Phi^{i\dagger} e^{q_i V} \Phi^i \tag{12}$$

invariant. In the non-abelian case, the gauge transformation for $\Phi_i$ is as before, where $\Lambda$ is now a matrix valued field.

For the gauge fields, the physical content is most transparent in a particular gauge (really a class of gauges) know as Wess-Zumino gauge. This gauge is analogous to the Coulomb gauge in QED. In that case, the gauge choice breaks manifest Lorentz invariance (Lorentz transformations musts be accompanied by gauge transformations), but Lorentz invariance is still a property of physical amplitudes. Similarly, the choice of Wess-Zumino gauge breaks supersymmetry, but physical quantities obey the rules implied by the symmetry. In this gauge, the vector superfield may be written as

$$V = -\theta\sigma^\mu\bar{\lambda}A_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D. \tag{13}$$

The analog of the gauge invariant field strength is a chiral field:

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V \tag{14}$$

or, in terms of component fields:

$$W_\alpha = -i\lambda_\alpha + \theta_\alpha D - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha F_{\mu\nu} + \theta^2 \sigma^\mu_{\alpha\dot{\beta}}\partial_\mu\bar{\lambda}^{\dot{\beta}}. \tag{15}$$

In the non-Abelian case, the fields $V$ are matrix valued, and transform under gauge transformations as

$$V \to e^{-\Lambda^\dagger} V e^\Lambda \tag{16}$$

Correspondingly, for a chiral field transforming as

$$\Phi \to e^\Lambda \Phi \tag{17}$$

the quantity

$$\Phi^\dagger e^V \Phi \tag{18}$$

is gauge invariant.

The generalization of $W_\alpha$ of the Abelian case is the matrix-valued field:

$$W_\alpha = -\frac{1}{4}\bar{D}^2 e^{-V} D_\alpha e^V, \qquad (19)$$

which transforms, under gauge transformations, as

$$W_\alpha \to e^{-\Lambda} W_\alpha e^\Lambda. \qquad (20)$$

## Supersymmetric Actions

To construct an action with $N = 1$ supersymmetry, it is convenient to consider integrals in superspace. The integration rules are simple:

$$\int d^2\theta\theta^2 = \int d^2\bar{\theta}\bar{\theta}^2 = 1; \quad \int d^4\theta\bar{\theta}^2\theta^2 = 1, \tag{21}$$

all others vanishing. Integrals $\int d^4x d^4\theta F(\theta, \bar{\theta})$ are invariant, for general functions $\theta$, since the action of the supersymmetry generators is either a derivative in $\theta$ or a derivative in $x$. Integrals over half of superspace of *chiral* fields are invariant as well, since, for example,

$$\bar{Q}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} + 2i\theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \tag{22}$$

so, acting on a chiral field (or any function of chiral fields, which is necessarily chiral), one obtains a derivative in superspace.

In order to build a supersymmetric lagrangian, one starts with a set of chiral superfields, $\Phi^i$, transforming in various representations of some gauge group $\mathcal{G}$. For each gauge generator, there is a vector superfield, $V^a$. The most general renormalizable lagrangian, written in superspace, is

$$\mathcal{L} = \sum_i \int d^4\theta \Phi_i^\dagger e^V \Phi_i + \sum_a \frac{1}{4g_a^2} \int d^2\theta W_\alpha^2 \qquad (23)$$

$$+ c.c. + \int d^2\theta W(\Phi_i) + c.c.$$

Here $W(\Phi)$ is a holomorphic function of chiral superfields known as the superpotential.

## Component lagrangians

In terms of the component fields, the lagrangian includes kinetic terms for the various fields (again in Wess-Zumino gauge):

$$\mathcal{L}_{kin} = \sum_i \left( |D\phi_i|^2 + i\psi_i \sigma^\mu D_\mu \psi_i^* \right) - \sum_a \frac{1}{4g_a^2} \left( F_{\mu\nu}^a F^{a\mu\nu} - i\lambda^a \sigma^\mu D_\mu \lambda^{a*} \right).$$

There are also Yukawa couplings of "matter" fermions (fermions in chiral multiplets) and scalars, as well as Yukawa couplings of matter and gauge fields:

$$\mathcal{L}_{yuk} = i\sqrt{2} \sum_{ia} (g^a \psi^i T_{ij}^a \lambda^a \phi^{*j} + c.c.) + \sum_{ij} \frac{1}{2} \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \psi^i \psi^j. \quad (25)$$

We should note here that we will often use the same label for a chiral superfield and its scalar component; this is common practice, but we will try to modify the notation when it may be confusing. The scalar potential is:

$$V = |F_i|^2 + \frac{1}{2}(D^a)^2. \tag{26}$$

The auxiliary fields $F_i$ and $D_a$ are obtained by solving their equations of motion:

$$F_i^\dagger = -\frac{\partial W}{\partial \phi_i} \qquad D^a = g^a \sum_i \phi_i^* T_{ij}^a \phi_j. \tag{27}$$

## A Simple Free Theory

To illustrate this discussion, consider first a theory of a single chiral field, with superpotential

$$W = \frac{1}{2}m\phi^2. \tag{28}$$

Then the component Lagrangian is just

$$\mathcal{L} = |\partial\phi|^2 + i\psi\sigma^\mu\partial_\mu\psi + \frac{1}{2}m\psi\psi + \text{c.c.} + m^2|\phi|^2. \tag{29}$$

So this is a theory of a free massive complex boson and a free massive Weyl fermion, each with mass $m^2$. (I have treated here $m^2$ as real; in general, one can replace $m^2$ by $|m|^2$).

## An Interacting Theory – Supersymmetry Cancelations

Now take

$$W = \frac{1}{3}\lambda\phi^3. \tag{30}$$

The interaction terms in $\mathcal{L}$ are:

$$\mathcal{L}_I = \lambda\phi\psi\psi + \lambda^2|\phi|^4. \tag{31}$$

The model has an $R$ symmetry under which

$$\phi \to e^{2i\alpha/3}\phi; \quad \psi \to e^{-2i\alpha/3}\psi; \quad W \to e^{2i\alpha}W. \tag{32}$$

## Aside 1: R Symmetries

Such symmetries will be important in our subsequent discussions. They correspond to the transformation of chiral fields:

$$\Phi_i \rightarrow e^{ir_i\alpha}\Phi_i; \ \ \theta \rightarrow e^{i\alpha\theta} \tag{33}$$

Then

$$Q \rightarrow e^{-i\alpha}Q; W \rightarrow e^{2i\alpha}W \tag{34}$$

and

$$\phi_i \rightarrow e^{ir_i\alpha}\phi_i; \ \ \psi_i \rightarrow e^{(r_i-1)\alpha}\psi_i \ \ F_i \rightarrow e^{i(r_i-2)\alpha}F_i. \tag{35}$$

The gauginos also transform:

$$\lambda \rightarrow e^{i\alpha}\lambda. \tag{36}$$

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The symmetry means that there can be no correction to the fermion mass, or to the superpotential. Let's check, at one loop, that there is no correction to the scalar mass. Two contributions:

1. Boson loop:

$$\delta m_\phi^2 = 4\lambda^2 \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \tag{37}$$

2. Fermion loop:

$$\delta m_\phi^2 = -2\lambda^2 \frac{d^4 k}{(2\pi)^4} \frac{\text{Tr}(\sigma^\mu k_\mu \bar{\sigma}^\nu k_\nu)}{k^4}. \tag{38}$$

In the first expression, the 4 is a combinatoric factor; in the second, the minus sign arises from the fermion loop; the 2 is a combinatoric factor.

These two terms, each separately quadratically divergent, cancel.

Now add to the lagrangian a "soft", non-supersymmetric term,

$$\delta \mathcal{L} = -m^2 |\phi|^2. \tag{39}$$

This changes the scalar propagator above, so

$$\delta m_\phi^2 = 4\lambda^2 \int \frac{d^4 k}{(2\pi)^4} \left( \frac{1}{k^2 + m^2} - \frac{1}{k^2} \right) \tag{40}$$

$$= \int \frac{d^4 k}{(2\pi)^4} \frac{-m^2}{k^2(k^2 + m^2)}$$

$$\approx \frac{\lambda^2 m^2}{16\pi^2} \log(\Lambda^2/m^2).$$

Here $\Lambda$ is an ultraviolet cutoff. Note that these corrections vanish as $m^2 \to 0$.

More generally, possible soft terms are:

1. Scalar masses
2. Gaugino masses
3. Cubic scalar couplings.

All have dimension less than four.

MSSM: A supersymmetric generalization of the SM.

1. Gauge group $SU(3) \times SU(2) \times U(1)$; corresponding (12) vector multiplets.

2. Chiral field for each fermion of the SM: $Q_f$, $\bar{u}_f$, $\bar{d}_f$, $L_f$, $\bar{e}_f$.

3. Two Higgs doublets, $H_U$, $H_D$.

4. Superpotential contains a generalization of the Standard Model Yukawa couplings:

$$W = y_U H_U Q \bar{U} + y_D H_D Q \bar{D} + y_L H_D \bar{E} + \mu H_U H_D. \qquad (41)$$

$y_U$ and $y_D$ are $3 \times 3$ matrices in the space of flavors.

We have omitted a set of dangerous terms from $W$ which violate lepton and baryon number.

$$QL\bar{D}, \, QL\bar{E}, \, H_U L, \, \bar{U}\bar{D}\bar{D} \qquad (42)$$

Note that these are *dimension four* (unlike Standard Model, where leading B,L violating operators are dimension six).

To explain, postulate "R-parity", $Z_2$ under which quark, lepton superfields even, $H_U$, $H_D$ odd.

# Soft Breaking Parameters

Need also breaking of supersymmetry, potential for quarks and leptons. Introduce *explicit soft breakings*:

1. Soft mass terms for squarks, sleptons, and Higgs fields:

$$\mathcal{L}_{scalars} = Q^* m_Q^2 Q + \bar{U}^* m_U^2 \bar{U} + \bar{D}^* m_D^2 \bar{D} \qquad (43)$$
$$+ L^* m_L^2 L + \bar{E}^* m_E \bar{E}$$
$$+ m_{H_U}^2 |H_U|^2 + m_{H_U}^2 |H_U|^2 + B_\mu H_U H_D + \text{c.c.}$$

$m_Q^2$, $m_U^2$, etc., are matrices in the space of flavors.

2. Cubic couplings of the scalars:

$$\mathcal{L}_{\mathcal{A}} = H_U Q\, A_U\, \bar{U} + H_D Q\, A_D\, \bar{D} \qquad (44)$$
$$+ H_D L\, A_E \bar{E} + \text{c.c.}$$

The matrices $A_U$, $A_D$, $A_E$ are complex matrices

3. Mass terms for the U(1) (*b*), *SU(2)* (*w*), and *SU(3)* ($\lambda$) gauginos:

$$m_1 bb + m_2 ww + m_3 \lambda\lambda \qquad (45)$$

1. $\phi\phi^*$ mass matrices are $3 \times 3$ Hermitian (45 parameters)

2. Cubic terms are described by 3 complex matrices (54 parameters

3. The soft Higgs mass terms add an additional 4 parameters.

4. The $\mu$ term adds two.

5. The gaugino masses add 6.

There appear to be 111 new parameters.

But Higgs sector of SM has two parameters.

In addition, the supersymmetric part of the MSSM lagrangian has symmetries which are broken by the general soft breaking terms (including $\mu$ among the soft breakings):

1. Two of three separate lepton numbers

2. A "Peccei-Quinn" symmetry, under which $H_U$ and $H_D$ rotate by the same phase, and the quarks and leptons transform suitably.

3. A continuous "$R$" symmetry, which we will explain in more detail below.

Redefining fields using these four transformations reduces the number of parameters to 105.

*If supersymmetry is discovered, determining these parameters, and hopefully understanding them more microscopically, will be the main business of particle physics for some time. The phenomenology of these parameters has been the subject of extensive study; we will focus on a limited set of issues.*

1. Dark Matter
2. Unification

Imposing *R* parity to prevent proton decay implies that the lightest of the new supersymmetric particles (LSP) is stable. An back of the envelope estimate gives a density about right to account for the dark matter.

$$\sigma \approx \frac{1}{M_Z^2}. \tag{46}$$

Out of equilibrium when

$$n_\chi^2 \sigma < \frac{T^2}{M_p} \quad n \sim T^3 e^{-m_\chi/T}$$

Assuming exponent of order 10, gives a density at decoupling of order, for $m_\chi \sim m_Z$, a temperature of order 10 GeV. Dominates energy at temperatures of order decoupling.

Very detailed computations routine.

## Supersymmetric Unification

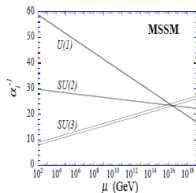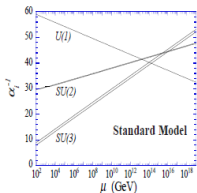In general, at one loop, gauge couplings run according to the rule:

$$\frac{8\pi^2}{g^2(\mu)} = \frac{8\pi^2}{g^2(M)} + b_0 \log(\mu/M). \tag{47}$$

The $SU(2)$ and $U(1)$ couplings are particularly well-measured at $M_Z$. If one assumes that the couplings coincide at a scale, $M_{GUT}$, one can compute $M_{GUT}$ and $\alpha_3$.
One finds $M_{GUT} \approx 2 \times 10^{16}$ GeV and good agreement for $\alpha_3$.

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$$\frac{dg_a}{d\log\mu} = \frac{g_a^3}{16\pi^2}B_a$$

SM  $B_a = \left(\frac{41}{10}, -\frac{19}{6}, -7\right)$

MSSM  $B_a = \left(\frac{33}{5}, 1, -3\right)$



(from J. Reuter)

## Constraints

Spectrum must have special features to explain

1. LHC searches severely constrain the spectrum. E.g. squark, gluino masses close to TeV over much of parameter space. Charginos (Tevatron, LEP) of order 100 GeV.

2. Absence of Flavor Changing Neutral Currents (suppression of $K \leftrightarrow \bar{K}$, $D \leftrightarrow \bar{D}$ mixing; $B \rightarrow s + \gamma$, $\mu \rightarrow e + \gamma$, ...)

3. Suppression of $CP$ violation ($d_n$; phases in $K\bar{K}$ mixing).

Latter two features might be accounted for if spectrum highly degenerate, CP violation in soft breaking suppressed.
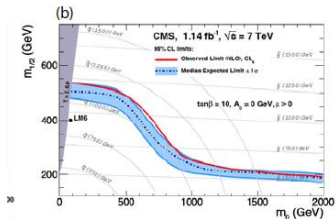
Substantially stronger limits (or discovery) will await 14 TeV runs.

Biggest radiative correction to the Higgs mass from top quark loops. Two graphs; cancel if supersymmetry is unbroken. Result of simple computation is

$$\delta m_{H_U}^2 = -6 \frac{y_t^2}{16\pi^2} \tilde{m}_t^2 \ln(\Lambda^2/\tilde{m}_t^2) \tag{48}$$

Even for modest values of the cutoff (30 TeV), given the limits on squark masses, this can be 30 times (125 GeV$^2$).

Better if stops are lighter than other squarks. Much attention focussed on this possibility for maintaining naturalness.

## The problem of the Higgs mass

At tree level in the MSSM, $m_H \leq m_Z$.

But : $m_H > 114 \,(= 125?)$ GeV.

The MSSM limit arises because the Higgs quartic coupling is controlled, in the limit of exact supersymmetry, by the gauge couplings.

Supersymmetry breaking gives rise to corrections to the quartic coupling. Loop corrections involving top quark: can substantially correct Higgs quartic, and increase mass.

$$\delta\lambda \sim 3\frac{y_t^4}{16\pi^2} \log(\tilde{m}_t^2/m_t^2). \tag{49}$$

Corresponding correction to Higgs mass:

$$m_h^2 = m_Z^2 c_{2\beta}^2 + \frac{3m_t^4}{4\pi^2 v^2} \left( \ln(m_s^2/m_t^2) + \frac{X_t^2}{m_s^2}(1 - \frac{X_t^2}{12m_s^2}) \right). \tag{50}$$

$X_t = A_t - \mu \cot\beta \quad m_S^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$
But 125 GeV typically require $\tilde{m}_t > 10$ TeV or tuning of $A$ parameter. Exacerbates tuning.

## Beyond the MSSM

A widely studied extension of the MSSM: NMSSM.

$$\lambda S H_U H_D + \kappa S^3. \tag{51}$$

Introduced to account for $\mu$ term, but $\lambda$ gives additional contribution to the Higgs quartic coupling.

We will see later that it is *in fact natural* to add to the model

$$m_S S^2 + \mu H_U H_D. \tag{52}$$

This "generalized MSSM" (discussed by Seiberg, Thomas, M.D.) allows significantly more massive Higgs (Ross et al, Hall et al). For a range of parameters, described by effective field theory:

$$\delta W = \frac{1}{M} H_U H_D H_U H_D \quad \delta K = Z^\dagger Z H_U^\dagger H_U H_U^\dagger H_U. \tag{53}$$

## Another alternative to the MSSM: R-Parity Violation

*R* parity is seductive. Simple. A beautiful consequence in the existence of a stable dark matter candidate.

But the need for such a symmetry – unlike in the SM, and the existence of other dark matter candidates (axions...), as well as the problems of $SU(5)$ models (unification) in which dimension five operators lead to conflict with current proton decay limits, suggest one should contemplate relaxing this requirement.

Most studies consider $QL\bar{d}$ and other lepton violating couplings. Following Csaki, Grosman and Heidenreich, consider

$$\lambda_{fgh}\bar{u}_f\bar{d}_g\bar{d}_h. \tag{54}$$

$$\Delta B = 1; \ \Delta L = 0.$$

Doesn't cause proton decay but constrained by
$\Delta B = 2, \Delta L = 0$ processes:

1. $N - \bar{N}$ oscillations
2. $p + p \rightarrow K^+ + K^+$

At the same time, only interesting if decay of LSP occurs quickly (i.e. in the detector).

Csaki et al: These conditions satisfied with assumption of "Minimal Flavor Violation". Basically couplings suppressed to particular quarks by their masses, mixing angles.

## Supergravity

If nature is supersymmetric, ultimately the symmetry should be local.

Some basic features:

Theory specified (at level of terms with two derivatives) by three functions:

1. Kahler potential, $K(\phi_i, \phi_i^*)$.
2. Superpotential, $W(\phi_i)$ (holomorphic).
3. Gauge coupling functions, $f_A(\phi)$ ($\frac{1}{g_A^2} = \langle f_A \rangle$).

Potential (units with $M_p = 1$):

$$V = e^K \left[ D_i W g^{\bar{i}i} D_{\bar{i}} W^* - 3|W|^2 \right] \tag{55}$$

Here $g_{i\bar{i}} = \frac{\partial^2 K}{\partial \phi_i \partial \phi_{\bar{i}}^*}$; $g^{\bar{i}i}$ is its inverse.

$D_i \phi$ is order parameter for susy breaking:

$$D_i W = \frac{\partial W}{\partial \phi_i} + \frac{\partial K}{\partial \phi_i} W. \tag{56}$$

Michael Dine    Supersymmetry from Bottom Up and the Top Down

Some general features:

1. If unbroken susy, space time is Minkowski (if $W = 0$), AdS ($W \neq 0$).

2. If flat space ($\langle V \rangle = 0$), and broken supersymmetry, then

$$m_{3/2} = \langle e^{K/2} W \rangle. \tag{57}$$

3. No continuous global symmetries.

**Mediating Supersymmetry Breaking**

Generally assumed that supersymmetry is broken by dynamics of additional fields, and some weak coupling of these fields to those of the MSSM gives rise to soft breakings. For the moment, we will model this by a field $X$, with $\langle F_X \rangle \neq 0$.

The classes of models called "gauge mediated" and "gravity mediated" are distinguished principally by the scale at which supersymmetry is broken. If terms in the supergravity lagrangian (more generally, higher dimension operators suppressed by $M_p$) are important at the weak (TeV) scale:

$$F_i = D_i W \approx (TeV)M_p \equiv M_{int}^2 \tag{58}$$

"gravity mediated". If lower, "gauge mediated"; $F_i \approx \partial_i W$.

In the low scale case, the soft breaking effects at low energies should be calculable, without requiring an ultraviolet completion; the intermediate scale case requires some theory like string theory.

# "Gravity Mediation"

Suppose a field with non-zero $F$ component, e.g.

$$W = f\, X + W_0. \tag{59}$$

Take, e.g., OR model and couple to supergravity. Add constant to $W$, $W_0$, so that $V \approx 0$ at minimum of (supergravity) potential,

$$3|W_0|^2 \approx |F_X|^2. \tag{60}$$

Suppose

$$K = X^\dagger X + \sum \phi_i^\dagger \phi_i. \tag{61}$$

Then all scalars (squarks and sleptons) gain mass from

$$|\frac{\partial W}{\partial \phi_i} + \frac{\partial K}{\partial \phi_i} W|^2 \approx |W_0|^2 |\phi_i|^2. \tag{62}$$

Universal masses for all squarks and sleptons.

$A$ terms from, e.g., with $W = W_0 + y\phi^3$

$$-3|W|^2 \approx \cdots - 3W_0 y\phi\phi\phi \tag{63}$$

i.e. $A$ term proportional to $W$.

Finally, gaugino masses from $\int d^2\theta X W_\alpha^2$.

"MSUGRA": 3 parameters, $m_o^2$, $m_{1/2}$, $A$.

$$\mathcal{L}_{soft} = m_0^2 |\phi_i|^2 + m_{1/2} \sum \lambda^A \lambda^A + A(W + W^*). \qquad (64)$$

But: if simply complicate Kahler potential:

$$K = \phi_i^* \phi_i + A_{ijk} X \phi_i^* \phi_j^* \phi_k^* + \text{c.c.} + \Gamma_{ijkl} \phi_i \phi_j \phi_k^* \phi_\ell^*. \qquad (65)$$

Generates the full set of soft breaking parameters.

So in general, problems of flavor. Possible solution might be an approximate underlying flavor symmetry; perhaps some detailed dynamics. But no completely compelling framework (generic?).

Other issues include gravitino overproduction, moduli problems (issues also in gauge mediation).

Main premiss underlying gauge mediation: in the limit that the gauge couplings vanish, the hidden and visible sectors decouple.

Simple model:

$$\langle X \rangle = x + \theta^2 F. \tag{66}$$

$X$ coupled to a vector-like set of fields, transforming as 5 and $\bar{5}$ of $SU(5)$:

$$W = X(\lambda_\ell \bar{\ell}\ell + \lambda_q \bar{q}q). \tag{67}$$

For $F < X$, $\ell, \bar{\ell}, q, \bar{q}$ are massive, with supersymmetry breaking splittings of order $F$. The fermion masses are given by:

$$m_q = \lambda_q x \quad m_\ell = \lambda_\ell x \tag{68}$$

while the scalar splittings are

$$\Delta m_q^2 = \lambda_q F \quad \Delta m_\ell^2 = \lambda_\ell F. \tag{69}$$

In such a model, masses for gauginos are generated at one loop; for scalars at two loops. The gaugino mass computation is quite simple. Even the two loop scalar masses turn out to be rather easy, as one is working at zero momentum. The latter calculation can be done quite efficiently using supergraph techniques; an elegant alternative uses background field arguments.

The result for the gaugino masses is:

$$m_{\lambda_i} = \frac{\alpha_i}{\pi}\Lambda, \tag{70}$$

For the squark and slepton masses:

$$\widetilde{m}^2 = 2\Lambda^2[C_3\left(\frac{\alpha_3}{4\pi}\right)^2 + C_2\left(\frac{\alpha_2}{4\pi}\right)^2 \tag{71}$$

$$+\frac{5}{3}\left(\frac{Y}{2}\right)^2\left(\frac{\alpha_1}{4\pi}\right)^2],$$

where $\Lambda = F_x/x$. $C_3 = 4/3$ for color triplets and zero for singlets, $C_2 = 3/4$ for weak doublets and zero for singlets.

1. One parameter describes the masses of the three gauginos and the squarks and sleptons

2. Flavor-changing neutral currents are automatically suppressed; each of the matrices $m_Q^2$, etc., is automatically proportional to the unit matrix; the *A* terms are highly suppressed (they receive no contributions before two loop order).

3. CP conservation is automatic

4. This model cannot generate a $\mu$ term; the term is protected by symmetries. Some further structure is necessary.

# General Gauge Mediation

Much work has been devoted to understanding the properties of this simple model, but it is natural to ask: just how general are these features? It turns out that they are peculiar to our assumption of a single set of messengers and just one singlet responsible for supersymmetry breaking and R symmetry breaking. Meade, Seiberg and Shih have formulated the problem of gauge mediation in a general way, and dubbed this formulation *General Gauge Mediation* (GGM). They study the problem in terms of correlation functions of (gauge) supercurrents. Analyzing the restrictions imposed by Lorentz invariance and supersymmetry on these correlation functions, they find that the general gauge-mediated spectrum is described by three complex parameters and three real parameters. Won't have time to discuss all of the features here, but the spectrum can be significantly different than that of MGM. Still, masses functions only of gauge quantum numbers of the particles, flavor problems still mitigated.

# Microscopic Models of Supersymmetry Breaking

While I won't consider string constructions per se (i.e. ultraviolet complete theories of gravity), I will focus on an important connection with gravity: the cosmological constant. I will not be attempting to provide a new explanation, but rather simply asking about the features of the low energy lagrangian in a world with approximate SUSY and small $\Lambda$.

$$\langle |W|^2 \rangle = 3 \langle |F|^2 \rangle M_p^2 + \text{ tiny.} \tag{72}$$

So not just $F$ small, but also $W$. Why?

1. Some sort of accident? E.g. KKLT assume tuning of $W$ relative to $F$ (presumably anthropically).

2. R symmetries can account for small $W$ (Banks). We we will see, $< W >$ can be correlated naturally with the scale of supersymmetry breaking. *Scale of R symmetry breaking: set by cosmological constant.*

Suggests a role for *R* symmetries. In string theory (gravity theory): discrete symmetries. Such symmetries are interesting from several points of view:

1. Cosmological constant
2. Give rise to approximate continuous R symmetries at low energies which can account for supersymmetry breaking (Nelson-Seiberg: continuous *R* symmetry necessary condition for [generic] stable supersymmetry breaking).
3. Account for small, dimensionful parameters.
4. Suppression of proton decay and other rare processes.

In general, $W$ has $R$ charge 2. Suppose fields, $X_i$, $i = 1, \ldots N$ with $R = 2$, $\phi_a$, $a = 1, \ldots M$, with $R$ charge 0. Then the superpotential has the form:

$$W = \sum_{i=1}^{N} X_i f_i(\phi_a). \tag{73}$$

Suppose, first, that $N = M$. The equations $\frac{\partial W}{\partial \Phi_i} = 0$ are solved if:

$$f_i = 0; \quad X_i = 0. \tag{74}$$

(R unbroken, $\langle W \rangle = 0$.) The first set are $N$ holomorphic equations for $N$ unknowns, and generically have solutions. Supersymmetry is unbroken; there are a discrete set of supersymmetric ground states; there are generically no massless states in these vacua.
(Again, R unbroken, $\langle W \rangle = 0$.)

Next suppose that $N < M$. Then the equations $f_i = 0$ involve more equations than unknowns; they generally have an $M - N$ dimensional space of solutions, known as a moduli space. In perturbation theory, as a consequence of non-renormalization theorems, this degeneracy is not lifted. There are massless particles associated with these moduli (it costs no energy to change the values of certain fields).

If $N > M$, the equations $F_i = 0$ in general do not have solutions; supersymmetry is broken. These are the O'Raifeartaigh models. Now the equations $\frac{\partial W}{\partial \phi_i} = 0$ do not determine the $X_i$'s, and classically, there are, again, moduli. Quantum mechanically, however, this degeneracy is lifted.

The non-renormalization theorems.

Quite generally, supersymmetric theories have the property that, if supersymmetry is not broken at tree level, then to all orders of perturbation theory, there are no corrections to the superpotential and to the gauge coupling functions. These theorems were originally proven by examining detailed properties of Feynman diagrams, but they can be understood far more simply (Seiberg).

Consider a simple Wess-Zumino model:

$$W = \left( \frac{m}{2} \phi^2 + \frac{\lambda}{3} \phi^3 \right). \tag{75}$$

Here I have chosen not to include a linear term in $\phi$; any such term can be absorbed in a redefinition of $\phi$.

Suppose that $\lambda = 0$. Then the theory has an *R* symmetry, under $\phi$ carries charge 1. We can think of $\lambda$ as itself the expectation value of a *chiral* field with charge $-1$ under this symmetry. Any correction to $\lambda$ necessarily has the form

$$\delta W = \lambda^n \phi^{3+n} \tag{76}$$

But this is exactly the structure of tree graphs. This non-renormalization theorem is a consequence of the *holomorphy* of the superpotential.

## Non-Renormalization Theorems for Gauge Couplings

Here a significant puzzle. Write the gauge field lagrangian as

$$\mathcal{L} = -\int d^2\theta \frac{1}{32\pi^2}\tau W_\alpha^2. \tag{77}$$

$\tau$ a holomorphic parameter.

$$\tau = \frac{8\pi^2}{g^2} + i\theta. \tag{78}$$

In perturbation theory, symmetry $\tau \to \tau + i\alpha$, forbids *all but one loop corrections to the coupling ($\beta$-function).*

## Exact $\beta$ function

Novikov, Shifman, Vainshtein and Zakharov: framed question in terms of "Wilsonian" vs. 1PI actions. Derived an "exact" $\beta$ function:

$$\beta(g) = -\frac{3N\frac{g^3}{16\pi^2}}{(1 + 2N\frac{g^2}{16\pi^2})}. \tag{79}$$

Agrees with two loop beta function (universal). But beyond two loops, scheme dependent. What is the scheme? Here a simple explanation (building in part on work of Arkani-Hamed and Murayama) of the result, and a clear identification of the scheme – and why it is not singled out by any compelling physical consideration.

Observation of AHM: $N = 4$ theory, suitably deformed, can serve as a (holomorphic) regulator for the $N = 1$ theory.

Three adjoint chiral fields, $\Phi_i$, $i = 1, 2, 3$, and an $SU(4)$ $R$ symmetry.

$$\mathcal{L} = \int d^4\theta \frac{1}{g^2} \Phi_i^\dagger \Phi_i - \frac{1}{32\pi^2} \int d^2\theta \left( \frac{8\pi^2}{g^2} + i\theta \right) W_\alpha^2 \qquad (80)$$

$$+ \int d^2\theta \frac{1}{g^2} \Phi_1 \Phi_2 \Phi_3 + \text{c.c.}$$

*Action is not manifestly holomorphic in $\tau$.* To exploit the power of holomorphy, necessary to rescale the adjoints so that there are no factors of $g$ in the superpotential:

$$\Phi_i \rightarrow g^{2/3} \Phi_i. \qquad (81)$$

Michael Dine    Supersymmetry from Bottom Up and the Top Down

We can add mass terms for the $\Phi_i$'s (for simplicity, we will take all masses the same, but this is not necessary, and allowing them to differ allows one to consider other questions):

$$\mathcal{L} = \int d^4\theta \frac{1}{g^{2/3}} \Phi_i^\dagger \Phi_i - \frac{1}{32\pi^2} \int d^2\theta \left( \frac{8\pi^2}{g^2} + i\theta \right) W_\alpha^2 \qquad (82)$$

$$+ \int d^2\theta (\Phi_1 \Phi_2 \Phi_3 + m_{hol} \Phi_i \Phi_i + \text{c.c.}).$$

*Holomorphic presentation* of the $N = 4$ theory.

Under a renormalization group transformation (a change from cutoff $m_{hol}^{(1)}$ to $m_{hol}^{(2)}$,

$$\frac{8\pi^2}{g^2(m_2)} = \frac{8\pi^2}{g^2(m_1)} + 3N \log(m_{hol}^{(2)}/m_{hol}^{(1)}) \qquad (83)$$

But the holomorphic masses don't correspond to the masses of physical particles; these are, at tree level:

$$m_{phys} = g^{2/3} m_{hol}. \qquad (84)$$

Substituting in the eqn. for $g$, yields the NSVZ beta function.

So it is tempting to say that the NSVZ scheme is that associated with the physical masses of the cutoff fields, i.e. some "physical" cutoff scale. However, at higher orders, the actual physical masses of the adjoints receive perturbative corrections (indeed already at one loop). So the NSVZ scheme, beyond two loop order, while easy to specify, is just one of an infinite class of schemes:

$$m_{cut} = g^{2/3}(m_{cut})(1 + \frac{g^2}{16\pi^2} f(g^2)) m_{hol}. \tag{85}$$

Intriligator, Shih and Seiberg: example of metastable supersymmetry breaking in a surprising setting: vectorlike supersymmetric QCD. At a broader level, brought the realization that metastable supersymmetry breaking is a generic phenomenon. Consider the Nelson-Seiberg theorem, which asserts that, to be generic, supersymmetry breaking requires a global, continuous $R$ symmetry. We expect that such symmetries are, at best, accidental low energy consequences of other features of some more microscopic theory.

## O'Raifeartaigh Models

This is illustrated by the simplest O'Raifeartaigh model:

$$W = \lambda X(A^2 - \mu^2) + mYA. \tag{86}$$

$R$ symmetry with

$$R_Z = R_Y = 2; R_A = 0; X(\theta) \to e^{2i\alpha}X(e^{-i\alpha}\theta), \text{ etc.}. \tag{87}$$

(Also $Z_2$ symmetry, $Y \to -Y, A \to -A$ forbids $YA^2$). SUSY broken; equations:

$$\frac{\partial W}{\partial X} = \frac{\partial W}{\partial Y} = 0 \tag{88}$$

are not compatible.
Features: If $m^2 > \mu^2$, $\langle A \rangle = 0 = \langle Y \rangle$; $X$ undetermined.

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Potential for $X$ at one loop (Coleman-Weinberg); $\langle X \rangle = 0$. $X$ lighter than other fields (by a loop factor). Scalar components – light pseudomodulus. Spinor is Goldstino.

$$\langle F_X \rangle = \lambda \mu^2 \tag{89}$$

is the decay constant of the Goldstino.

Basic idea of Coleman Weinberg calculations is simple. Calculate masses of particles as functions of the pseudomodulus. From these, compute the vacuum energy:

$$\sum (-1)^F \int \frac{d^3 k}{(2\pi)^3} \sqrt{k^2 + m_i^2} \qquad (90)$$

$$= \sum (-1)^f \left( \Lambda^4 + m_i^2 \Lambda^2 + \frac{1}{(16\pi^2)} m_i^4 \ln(m_i)^4 \right).$$

(E.g. evaluate using formulae of dimensional regularization; poles cancel due to sum rules). The first two terms vanish because of features of supersymmetry. The last must be evaluated, when supersymmetry is broken.

For large $X$, the potential grows logarithmically with $|X|$. For smaller $X$, need to be more careful.

One finds (Shih) that if all fields have $R$ charge 0 or 2, then the $R$ symmetry is unbroken [exceptions, hidden assumptions, e.g. Shadmi]. Shih constructed models for which this is not the case. One of the simplest:

$$W = X_2(\phi_1 \phi_{-1} - \mu^2) + m_1 \phi_1 \phi_1 + m_2 \phi_3 \phi_{-1}. \tag{91}$$

The continuous symmetry of the OR model might arise as an accidental consequence of a discrete, $Z_N$ R symmetry. E.g.

$$X \rightarrow e^{\frac{4\pi i}{N}} X; \ Y \rightarrow e^{\frac{4\pi i}{N}} Y \tag{92}$$

corresponding to $\alpha = \frac{2\pi}{N}$ in the continuous $R$ transformation. Suppose, for example, $N = 5$. The discrete symmetry now allows couplings such as

$$\delta \mathcal{L} = \frac{1}{M^3} \left( aX^6 + bY^6 + cX^4Y^2 + dX^2Y^4 + \dots \right). \tag{93}$$

Note that $W \rightarrow e^{\frac{4\pi i}{N}} W$.

The theory now has $N$ supersymmetric minima, with

$$X \sim \left( \mu^2 M^3 \right)^{1/5} \alpha^k \tag{94}$$

where $\alpha = e^{\frac{2\pi i}{5}}$, $k = 1, \dots, 5$.

Need a separate lecture to discuss tunneling in quantum field theory. Suffice it to say that in models such as those introduced above, the metastable supersymmetric state can be extremely long lived. In particular, the system has to tunnel a "long way" (compared with characteristic energy scales) to reach the "true" vacuum. Thinking (correctly) by analogy to WKB, the amplitude is exponentially suppressed by a (large) power of the ratio of these scales.

*W* transforms under any *R* symmetry; an order parameter for *R* breaking.

Gaugino condensation: $\langle\lambda\lambda\rangle \equiv \langle W\rangle$ breaks discrete *R* without breaking supersymmetry.

Readily generalized (J. Kehayias, M.D.) to include order parameters of dimension one.

E.g. $N_f$ flavors, $N$ colors, $N_f < N$:

$$W = yS\bar{Q}_f Q_{f'} + \lambda S^3 \tag{95}$$

exhibits a $Z_{2(3N-N_f)}$ symmetry, spontaneously broken by $\langle S\rangle$; $\langle\bar{Q}Q\rangle$; $\langle W\rangle$.

The dynamics responsible for this breaking can be understood using familiar facts about supersymmetric gauge dynamics. Suppose, for example, that $\lambda \ll y$. Then we might guess that $S$ will acquire a large vev, giving large masses to the quarks. In this case, one can integrate out the quarks, leaving a pure $SU(N)$ gauge theory, and the singlet $S$. The singlet superpotential follows by noting that the scale, $\Lambda$, of the low energy gauge theory depends on the masses of the quarks, which in turn depend on $S$. So

$$W(S) = \lambda S^3 + \langle \lambda \lambda \rangle_S. \tag{96}$$

$$\langle \lambda \lambda \rangle = \mu^3 e^{-3 \frac{8\pi^2}{b_{LE} g^2(\mu)}} \tag{97}$$

$$= \mu^3 e^{-3 \frac{8\pi^2}{g_{LE} g^2(M)} + 3 \frac{b_0}{b_{LE}} \ln(\mu/M)}$$

$$b_0 = 3N - N_F; \; b_{LE} = 3N \tag{98}$$

So

$$\langle \lambda\lambda \rangle = M^{\frac{3N-N_f}{N}} e^{-\frac{8\pi^2}{Ng^2(M)}} \mu^{\frac{N_f}{N}}. \tag{99}$$

In our case, $\mu = yS$, so the effective superpotential has the form

$$W(S) = \lambda S^3 + (yS)^{N_f/N} \Lambda^{3-N_f/N}. \tag{100}$$

This has roots

$$S = \Lambda \left( \frac{y^{N_f/N}}{\lambda} \right)^{\frac{N}{3N-N_F}} \tag{101}$$

times a $Z_{3N-N_F}$ phase.

Consistent with our original argument that $S$ large for small $\lambda$.

Alternative descriptions of the dynamics in other ranges of coupling.

Retrofitted Models (Feng, Silverstein, M.D.): OR parameter $f$ from coupling

$$X(A^2 - \mu^2) + mAY \rightarrow \quad (102)$$

$$\frac{XW_\alpha^2}{M_p} + \gamma SAY.$$

Need $\langle W \rangle = fM_p = \Lambda^3$, $\langle S \rangle \sim \Lambda$, for example.

$$m^2 \gg \mu^2$$

SUSY breaking is metastable (supersymmetric vacuum far away).

Gauge mediation: traditional objection: c.c. requires large constant in $W$, unrelated to anything else.

Retrofitted models: scales consistent with our requirements for canceling c.c. Makes retrofitting, or something like it, inevitable in gauge mediation.

Other small mass parameters: $m$, $\mu$-term, arise from dynamical breaking of discrete $R$ symmetry. E.g.

$$W_\mu = \frac{S^2}{M_p} H_U H_D. \tag{103}$$

Readily build realistic models of gauge mediation/dynamical supersymmetry breaking with all scales dynamical, no $\mu$ problem, and prediction of a large $\tan \beta$.

# RETHINKING NATURALNESS

# Sources of Pessimism – and Optimism

Already with the end of the LEP program, there were serious reasons for skepticism about supersymmetry The most natural scale for low energy supersymmetry would seem to be $M_Z$. The absence of any direct signal, the failure to discover the Higgs, the problem of CP violation, the absence of deviations from the Standard Model in $b \rightarrow s + \gamma$, the non-observation of proton decay, all suggested that supersymmetry, if present, was working hard to hide itself.

The absence of a natural explanation for the observed dark energy, and the emergence of the landscape as a plausible concept, sharpened these concerns.

In the last year, these concerns have been sharpened. The LHC has quickly excluded broad swaths of the SUSY parameter space; near TeV limits are common.

As Michael Peskin said in Mumbai, "No reasonable person could view [the SUSY exclusions] without concluding that we need to change our perspective." He added the question: "What new perspective is called for?"

I am certainly no wiser than Michael, so I won't claim to have any answer he doesn't. But I hope to provide some guidance for thinking about these issues.

Assuming that supersymmetry exists at some scale well below the Planck scale:

1. Conventional ideas are correct. Within some class of models, the weak scale arises without appreciable fine tuning of parameters.

2. There is some modest level of fine tuning. We will discover – or just fail to discover – supersymmetry, more or less in some form we imagined, with fine tuning of, say, a part in 1000.

3. There is lots of tuning. We will see a relatively light Higgs. Nothing else.

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## Naturalness: three logical possibilities

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Why would we doubt?

Hierarchies in nature for which we have possible symmetry or dynamical explanations:

1. Weak/Planck hierarchy
2. Yukawa hierarchies

### Hierarchies for which we don't:

1. The cosmological constant (huge elephant) (part in $10^{68} - 10^{120}$).

2. Inflation (part in 100?)

3. Hypothetical: $\theta_{qcd} \rightarrow \text{axion} - f_a/M_p$

4. Hypothetical: dark matter (see (3), or new light state tuned for thermal production).

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All of these problems are substantially ameliorated by supersymmetry, but the first two are not resolved in any framework I know.

So logically we have to acknowledge, even before proposing an underlying explanation for these puzzles, that in imposing notions of naturalness we are on shaky grounds.

SUSY, of course, has other attractive features:

1. Unification
2. Dark matter

I will try to convince you that there are more.

# The Landscape As a Setting for Questions of Naturalness

The *landscape* has been the Damocles sword hanging over our (SUSY's) head. It is, for better or worse, the most compelling explanation we have of the observed dark energy.

Without worrying how the landscape comes about, can embody the basic idea in the statement:

The laws of nature we observe (degrees of freedom, lagrangian parameters) are selected from a large ensemble of possibilities.

The probability distribution associated with this ensemble depends on the underlying microphysics (string theory? some larger structure incorporating gravity?), cosmology, other unknown features.

Ignoring tuning, we might expect the Higgs coupling at the high scale to be large, if selected from a distribution ($\lambda = \pi^2$?). Then evolve down. If Higgs parameters are selected anthropically, then certainly must be large enough that the universe has not already decayed. Leaves an interesting band:
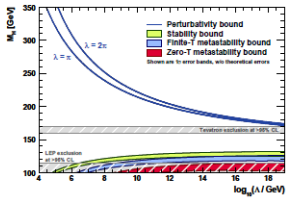
From this perspective, a *model* is a choice of probability distribution for d.o.f, symmetries, parameters. In making a selection from the distribution, we impose certain prior constraints; these may be anthropic (as in the prediction of the dark energy) or simply viewed as observational. Predictions arise if some outcome is strongly favored.

Models can fail! ["Falsifiable"]

Within such a framework, *naturalness is a precise notion*. We can ask the relative likelihood, say, of a light Higgs given supersymmetry or not.

Question of low energy susy is, then, how common, in the landscape, is dynamical susy breaking, vs. non-dynamical or total absence of supersymmetry.

The answer to this question is not known within, e.g., any well-understood string model.

# Models and Their Implications for Low Energy Supersymmetry

**Model A** No SUSY below Planck scale (would seem generic). Low Higgs mass selected by anthropic criteria.

**Model B**: Assume (motivated by studies of IIB flux vacua) non-dynamical breaking of supersymmetry, superpotential parameters distributed uniformly as complex numbers: high (Planck) scale susy favored even by small Higgs mass, cosmological constant. (Douglas/Susskind)

**Model C**: Dynamical breaking favors lower breaking of SUSY (Gorbatov, Thomas, M.D.).

**Model D**: Dynamical breaking and discrete $R$ symmetries: very low scales (as in gauge mediation).

So in landscape, question of low energy susy is one of relative probability of dynamical susy vs. non-susy or non-dynamical susy.

Not enough known about landscapes from any underlying theory to settle these questions from "top down".

# A cosmological argument for low scale susy in the landscape:

One attempt at a "top down" argument:
The prototypical flux landscape models generate a large class of effective actions, and one counts vacua by counting stationary points. Typically these will be non-supersymmetric or exhibit large supersymmetry breaking. But a typical low cosmological constant state found this way will have *many* neighbors with negative cosmological constant. Typically decay will be very rapid.

Large volume, weak coupling typically are not sufficient to account for generic stability. But Supersymmetry is!

For a broad class of models (Festuccia, Morisse, M.D.):

$$\Gamma \propto e^{-2\pi^2 \left( \frac{M_p^2}{m_{3/2}^2} \right)} \tag{104}$$

A related question: Does one expect symmetries (pointing to low scale breaking, as needed to suppress proton decay, etc.?). Naive landscape counting in flux models: no! Only an exponentially small fraction of fluxes allow symmetry (Z. Sun, M.D.).

Challenges accepted wisdom that symmetries are natural.

But perhaps *too* naive. (Festuccia, Morisse, M.D.)

In such a framework, notions of naturalness, we see, can hold. If there is low energy susy, might one still encounter a little hierarchy, or do strict notions of naturalness hold?

E.g. inflation, with SUSY, typically requires $1/100$ fine tuning. Without SUSY generally *much* more severe. If the dynamics of inflation are tied to those of supersymmetry breaking, there might be a tension between the two (higher scales more natural for inflation, lower scales for Higgs mass). The result could be a "compromise". Dark matter might also lead to such a tension.

Models relating supersymmetry to inflation can give little or "medium size" hierarchies.