Is there a Peccei–Quinn phase transition?

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Abstract. The nature of axion cosmology is usually said to depend on whether the Peccei–Quinn (PQ) symmetry breaks before or after inflation. The PQ symmetry itself is believed to be an accident, so there is not necessarily a symmetry during inflation at all. We explore these issues in some simple models, which provide examples of symmetry breaking before and after inflation, or in which there is no symmetry during inflation and no phase transition at all. One effect of these observations is to relax the constraints from isocurvature fluctuations due to the axion during inflation. We also observe new possibilities for evading the constraints due to cosmic strings and domain walls, but they seem less generic.

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Contents

1. Introduction

Traditional discussions of the axion in cosmology begin with the assumption that the Peccei–Quinn symmetry is exact at all times in cosmic history, and that the universe was in thermal equilibrium at the time of the Peccei–Quinn (PQ) phase transition and thereafter. This leads in a rather straightforward way to an upper limit on the axion decay constant (and a lower limit on the axion mass) [1]. In non-supersymmetric models, such a scenario may seem plausible. However, the low axion scale is likely to require fine-tuning, or, more plausibly, some new strong dynamics which might modify the naive picture.

But in many contexts these assumptions do not seem reasonable. If nature is supersymmetric, they are unlikely to hold. Many authors have discussed the cosmology of a possible light scalar field (called the saxion [2]) which accompanies the axion [3]. In [4], it was stressed that this particle is necessarily a modulus or approximate modulus. It was noted that the coherent motion of this modulus is likely to dominate the energy density of the universe long before the QCD phase transition and that string theory suggests that there might, in fact, be many such moduli. The cosmological problems associated with these additional moduli are parametrically much more severe than those associated with the axion, so it does not make sense to discuss axion cosmology without first resolving these. Some proposed solutions to these problems significantly relax the usual axion limit. In particular, decays of the moduli dilute the axion, permitting decay constants as large as 10^{15} GeV.

The discussion of [4] assumed that there was a stage of inflation before PQ symmetry breaking. There is a widely held prejudice that this is unlikely. This prejudice is based on the plausible assumption that the PQ transition occurs when the temperature of the universe is comparable to the PQ scale. If the scale is, say, 10^{11} GeV, then this might be expected to be well after inflation.

Again, though, in the context of supersymmetry, this reasoning is suspect. Our arguments above suggests that the universe might not be radiation dominated until rather late in its history. In supersymmetry, in addition, the gravitino problem indicates that after inflation the universe did not reheat to a temperature as high as the PQ scale. The aim of this paper is to enumerate some of the possible PQ phase transitions in supersymmetric theories and their implications for axion cosmology. In the next section, we will review a class of field theory models (sometimes called 'flatton models') in which the PQ symmetry arises as an accident of discrete symmetries of the low energy effective theory. In these models, we will see that, depending on the transformation properties of the inflaton(s) and the signs of certain terms in the effective action, the PQ transition can[4] occur readily *either* during or after inflation, or there may be no transition at all. If the transition occurs before (or during) inflation, there is a potential problem with isocurvature fluctuations [9]. As we will discuss, the decay constant of the axion initially takes a value generally quite different to its value at late times. This has possible implications for isocurvature fluctuations (some aspects of this problem were discussed in [5, 11]).

Potentially more important is the fact that there is quite possibly no phase transition at all. More precisely, during inflation, there may be no Peccei–Quinn symmetry. Some time after inflation ends, there would be an approximate symmetry, which would get better and better with the passage of time. By the time the symmetry can be thought of as a good symmetry, it is non-linearly realized, i.e. it is already spontaneously broken.

In section 3, we will turn to the case where the phase transition occurs after inflation. The usual picture of this transition is that in different domains, the axion field is essentially a random variable. This leads to the formation of axion cosmic strings and domain walls, as well as an alignment problem. But we will see that, generically, there is a period after the PQ phase transition in which the PQ symmetry is badly broken. Indeed, for a rather long time, one can have $m_{\mathrm{a}} \gg H$. As a result, the axion can be driven to a particular point. One might then hope to avoid domain walls and strings, and perhaps solve the alignment problem, as in the first case, through dilution. We will see, however, that while the symmetry is broken, the system is typically driven not to one but to several points. This leads, again, to the domain wall problem. We will discuss these issues, and a subclass of models which might evade these problems.

In section 4, we briefly discuss string theory models. It is logically possible that string models could reproduce any of the phenomena of the field theory models. But our current understanding of PQ symmetry breaking in M theory is too rudimentary to allow us to say much more. In section 5, we conclude with an overview of recent results on axion cosmology, and assess the significance of experimental limits on axion production. In particular, we stress that without further knowledge of microscopic physics, it is difficult to place cosmological constraints on axions.

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[4] This point discussed below, that the sign of an effective mass term can take either value during inflation, and determines the nature of the transition, has been made by a number of authors [5, 6]. That this would be relevant to the question of isocurvature perturbations was already discussed in Linde's paper. More recently, some of the effects of this term during inflation have been discussed in [7, 8], with particular attention given to the possibility that the axion itself is the source of the observed fluctuations.

2. Field theory models and their possible cosmologies

In four-dimensional field theories, one way to obtain PQ symmetries is as accidental consequences of discrete symmetries [12]. In supersymmetric theories, this idea is readily implemented [13]. Models of this type suggest a very specific picture for the PQ phase transition

2.1. A simple model

As a simple example, consider a theory with fields S and S' (one can consider models with a single scalar field, but in this case the PQ symmetry is an R symmetry, and it is difficult to reconcile with the various constraints on soft breaking terms). Suppose that the model possesses a Z_N symmetry under which

$$S \to \alpha S \qquad S' \to \alpha^{N-n} S'. \tag{2.1}$$

Then the leading allowed terms in the superpotential are

$$W = \frac{1}{M^{n-2}} S^n S' + \frac{1}{M^{N-3}} S^N + \cdots. \tag{2.2}$$

Here M denotes the (reduced) Planck scale and the dots indicate terms with more than one power of S' as well as higher powers of S. (Our arguments can be reworked if scales other than the Planck scale are important.)

Now we expect the soft breaking terms to include

$$\mathcal{L}_{\text{soft}} = -m_{3/2}^2 |S^2| + m_{3/2}^2 |S'|^2 + A \frac{m_{3/2}}{M^{n-2}} S^n S' + \cdots. \tag{2.3}$$

The potential has an approximate PQ symmetry under which

$$S \to S e^{i\omega}. \tag{2.4}$$

The leading symmetry-breaking operators are the terms in the superpotential indicated above of the form $(1/M^{N-3})S^N$ as well as soft breaking terms,

$$\frac{m_{3/2}}{M^{N-3}} S^N. \tag{2.5}$$

The potential has a minimum with

$$S = (m_{3/2} M^{n-2})^{1/(n-1)}. \tag{2.6}$$

This spontaneously breaks the PQ symmetry, giving rise to an axion. Note that for $n = 3$, $S = f_A \sim 10^{11}$. For $N \geq 11$, the explicit symmetry breaking is small enough that this axion can solve the strong CP problem. More elegant models can be constructed with products of smaller discrete symmetries, such as frequently arise in string theory.

2.2. The crucial question: does the inflaton transform under the discrete symmetries?

Now consider the early universe. In the supersymmetric case, we suppose that we have an inflaton, I, with a non-vanishing F-component. The Hubble constant during inflation is then

$$H^2 \approx \left(\frac{F_I^2}{M^2}, \frac{W^2}{M^4} \right). \tag{2.7}$$

For definiteness, we will assume that the Hubble constant during inflation is at least as large as the weak scale, and that the expectation value of the inflaton is of order the Planck scale, M. It is straightforward to modify our discussion for other possibilities. After inflation, we will assume that the inflaton undergoes a period of oscillation, during which the universe is matter dominated. Now there are several possible phase structures, depending on the transformation properties of the inflaton under the discrete symmetry. The simplest case to consider is that where the inflaton is neutral. Then the effect of the couplings of the inflaton to the field S is mainly to give supersymmetry breaking of order H^2 [14]. The effective potential now includes soft breaking terms of the form of equation (2.3), but with $m_{3/2}$ replaced by H:

$$\mathcal{L}_{\text{soft}} = (a\, H^2 - m_{3/2}^2)|S^2| + (b\, H^2 + m_{3/2}^2)|S'|^2 + \cdots. \tag{2.8}$$

Depending on the sign of these terms the transition between the broken and unbroken phase occurs in quite different epochs of the universe. If, during inflation, the constant a is negative, the phase transition will occur at the Hubble scale of inflation, $H = H_{\text{inf}}$. In many models of inflation, this is long before $H = m_{3/2}$. In theories of hybrid inflation, along the lines of [15], the scale can be comparable to $m_{3/2}$. The axion field will be homogeneous after inflation, and there will be no issues of cosmic strings or domain walls. One will have a problem of alignment. If a is positive the transition occurs after inflation, once $H \sim m_{3/2}$ (in the case of hybrid inflation, it occurs after inflation in the era when the additional modulus is settling to its minimum).

It is quite likely that the inflaton itself transforms under the discrete symmetry. In this case, there is a richer set of possible behaviours. First, if $I \sim M$, then S, and S and/or S' will remain light only if they are protected by symmetries unbroken by the inflaton. If they are both heavy, the transition can occur only well after inflation has ended. In other words, if the PQ symmetry is to occur during inflation, it is necessary that the inflaton be neutral at least under a subset of the discrete symmetries. Even if S, S' are light, their couplings to the inflaton will typically 'break' the discrete symmetry, i.e. there will be no approximate PQ symmetry during this early era. We will explore some of the possible phenomena in this case in the next section.

2.3. PQ breaking during inflation

Even if the PQ symmetry breaking occurs during inflation, the axion cosmology is different to that usually assumed. Consider, first, the case where the inflaton is neutral under all of the discrete symmetries. Then there is still a Peccei–Quinn symmetry during inflation, but the axion decay constant during inflation is significantly larger than at later times. This already weakens the constraints coming from the absence of isocurvature fluctuations

in the spectrum of the CMBR, permitting rather large values of H_{inf}. If the mass of S is negative during inflation while that of S' is positive, then

$$f_{\text{a}} \sim S = (H_{\text{inf}} M^{n-2})^{1/(n-1)}. \tag{2.9}$$

If both S and S' have negative masses, f_{a} will be substantially larger. Indeed, if we only keep the operator of equation (2.2), then the S' vev can run off to ∞. So the precise value of f_{a} depends on the details of the model, and it can be far larger than even that suggested by equation (2.9) above. For example, if $n = 5$, so that $f_{\text{a}} \sim 10^{15}$ today, it will be closer to 10^{17} GeV during inflation if the S mass is negative and the S' mass positive. If the S', mass is negative, then the value of f_{a} depends on the model. In the Z_{11} example, the leading operator which stabilizes the S' vev is $S'^{\,11}$, giving $f_{\text{a}} \sim 10^{17.5}$.

If the inflaton transforms under some of the discrete symmetry, as noted above, it is necessary that enough symmetry survive that at least one of the fields S or S' does not acquire a large mass (here we are assuming $I \sim M$, so that operators involving arbitrarily large powers of I are not suppressed). For example, S' might acquire a large mass, but the leading operator giving rise to a superpotential for S might have the form $I^n S^{m+3}/M^{m+n}$. In this case, assuming $I \approx M$, the S vev is of order

$$S \approx (M^m H_{\text{inf}})^{1/(m+1)}. \tag{2.10}$$

This can be quite different, again, to the value today.

But what is striking in this case is that there may be additional effects which explicitly break the Peccei–Quinn symmetry. For example, soft breaking terms of the form

$$V_{\text{soft}} = H_{\text{inf}} S^{m+3} \tag{2.11}$$

break the Peccei–Quinn symmetry, giving rise to a mass for the axion—like that of the real part of S—of order H_{inf}. In other words, while we are speaking here of PQ symmetry breaking during inflation, there is really no sense in which there is a PQ symmetry at all! Instead, after inflation ends, the explicit breaking of the symmetry decreases with time. There is no real phase transition.

To summarize, the PQ transition could well occur during inflation. There may be a sense in which there is an approximate PQ symmetry at this time, or there may not. What are the implications of these various possibilities?

If the PQ transition occurs during inflation and the axion field is light, it is well known that there are significant constraints from the lack of observed isocurvature fluctuations. In order that isocurvature fluctuations be small enough to be consistent with recent CMBR measurements, one requires at least

$$\frac{\delta\theta}{\theta} < 10^{-5}. \tag{2.12}$$

As has recently been stressed by [19], observation of gravitational waves from inflation would place $H \geq 10^{13}$, and significantly constrain axion models. With much lower scales of inflation, as can arise in hybrid inflation, isocurvature perturbations are not a serious issue.

To assess the constraints on H and f_{a} during inflation, recall that a light axion generates isocurvature fluctuations with the power spectrum of a massless field in de Sitter

spacetime

$$\mathcal{P}_a(k) = \frac{k^3}{2\pi^2}|a_k|^2 = \left(\frac{H_{\text{inf}}}{2\pi}\right)^2,$$

(2.13)

thus yielding for $\theta_a = a/f_a$

$$\mathcal{P}_{\theta_a}(k) = \left(\frac{H_{\text{inf}}}{2\pi f_a}\right)^2.$$

(2.14)

The expression (2.14) is correct provided that the axion mass is small compared to H_{inf} during inflation.

While during the radiation dominated period isocurvature fluctuations do not generate any curvature perturbations, during the matter dominated period they do, due to the Sachs–Wolfe effect: $\mathcal{R}_a = (1/3)S_a$[5] (see, for example, [16]). For scales larger than horizon size at matter–radiation equality (i.e. for low multipoles, $l < 200$) the cmb temperature anisotropy coming from isocurvature fluctuations is a combination of the Sachs–Wolfe contribution and a contribution generated by entropy fluctuations [17]

$$\left(\frac{\delta T}{T}\right)_{\text{iso}} = -\frac{1}{5}\mathcal{R} - \frac{1}{3}S = -\frac{6}{15}\left(\frac{H_{\text{inf}}}{\pi f_a \theta_0}\right).$$

(2.15)

From the recent WMAP data the limit on CDM isocurvature fraction in the CMBR spectrum at low multipoles is [18]

$$\left(\frac{\delta T}{T}\right)_{\text{iso}}^2 < \frac{\alpha}{1-\alpha}\left(\frac{\delta T}{T}\right)_{\text{ad}}^2,$$

(2.16)

with $\alpha = 0.31$. Taking all of the above into account[6], we can put a constraint on the ratio of H_{inf} to f_a:

$$\frac{H_{\text{inf}}}{f_a} < 5.8 \times 10^{-5}\theta_0\Omega_a^{-1},$$

(2.17)

where Ω_a is the axion fraction in the total matter density in the universe. If we assume that $\theta_0 \sim \Omega_a^{-1} \sim 1$, $H_{\text{inf}} \sim 10^{13}$ GeV, this constrains f_a to be larger than 10^{17} GeV during inflation, so that isocurvature fluctuations may be at a barely acceptable level.

A much more important suppression of axion isocurvature fluctuations may come from the fact that during inflation the axion may acquire a large mass due to its coupling to the inflaton. For example, there could be a superpotential term

$$W = \frac{1}{M_p^{m-1}}S^{m+1}I.$$

(2.18)

Taking $S = S_0 \exp(ia/S_0)$ with $S_0 \sim (H_{\text{inf}}M^n)^{1/(n+1)}$ gives rise to a mass for the axion of

$$m_a^2 \sim H_{\text{inf}}^2\left(\frac{H_{\text{inf}}}{M}\right)^{((m-1)/(n+1))-1}.$$

(2.19)

For $m < n + 2$ the axion mass is parametrically larger than H_{inf}.

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[5] Here S_a is the entropy fluctuation, generated by axion field $S_a = (\delta\rho_a/\rho_a) = 2(\delta\theta_a/\theta_0) = H_{\text{inf}}/(\theta_0\pi f_a)$, where θ_0 is the initial value of the misalignment angle.

[6] For $(\delta T/T)_{\text{ad}}$ we took for simplicity the COBE result $(\delta T/T)|_{10^0} = 1.1 \times 10^{-5}$.

If this occurs, the power spectrum of axion fluctuations is no longer flat but rather strongly suppressed on large scales by a factor of $(H_{\text{inf}}^2/m_{\text{a}}^2)(k/aH)^3$, thus giving no contribution to the observed CMBR.

3. PQ breaking after inflation

If the constant a is positive, or if the superpotential is such that it gives a large mass to S and S', the phase transition will not occur until $H \approx m_{3/2}$ or later. This may be long after inflation. The usual assumption in this case is that the PQ field is a random variable. This leads to the production of domain walls and cosmic strings, both of which are potentially problematic [10]. As discussed in [11], in this situation there is likely to be a period of thermal inflation (see, for example [6]). But we will see that, as in the previous section, the behaviour of the system is very sensitive to the transformation properties of the inflaton under the discrete symmetry responsible for the PQ symmetry. For a broad range of possible transformation laws, the PQ symmetry is explicitly, and significantly, broken. The axion acquires a mass which, for a significant period, is large compared to the Hubble parameter.

As a warm-up, consider first the case where I is neutral under the discrete symmetries. Then the question is just that of the sign of the constant a in equation (2.8). If a is positive, the transition occurs after inflation, and completes once H is somewhat less than $m_{3/2}$. In this case, we have the usual problems of misalignment, cosmic strings and domain walls.

Now suppose that I does transform under the discrete symmetry. Clearly there are now myriad possibilities. We will discuss a few for purposes of illustration. Consider a superpotential which includes inflaton couplings to the fields S and S':

$$W = \frac{S'S^{n+2}}{M^n} + I^2 \frac{S^{m+1}}{M^m}. \tag{3.1}$$

Our working assumption will be that $I \sim M$ immediately after inflation. (In particular, if I transforms under the discrete symmetry, then after inflation, it has a Planck scale vev, but this decreases to zero over time, restoring the symmetry.)

This model, for $I = 0$, has an approximate PQ symmetry:

$$S \to e^{i\alpha}S \qquad S' \to e^{-i(n+2)\alpha}S'. \tag{3.2}$$

As usual, we will assume that, as a result of discrete symmetries, the leading symmetry-violating terms (for $I = 0$) are of very high dimension.

We will also assume that there is a soft breaking term:

$$V_{\text{soft}} = m_{3/2}I^2 \frac{S^{m+1}}{M^n} + \cdots. \tag{3.3}$$

(where the dots denote other soft breaking terms which are either highly suppressed or do not break the PQ symmetry). Note that I^2 will average, in general, to a complex number; had we taken a linear term, it would average to zero.

We will suppose that $m < n + 2$. Also we assume that, immediately at the end of inflation, $I \sim M$. So the value of S (and S') immediately after inflation is much less than its value at late times. Initially, the first term in the superpotential of equation (3.1) dominates. It respects its own global symmetry, which is broken by the second term.

As time evolves, however, I^2 decreases (like $H^2 e^{-\Gamma_I t}$) and the relative importance of the second term in equation (3.1) grows. Particularly interesting is the time when the two terms are comparable. At this time,

$$|S^{n+1}| \approx M^n m_{3/2} \qquad |S'| \approx \frac{|S^{*n+2}|}{m_{3/2} M^n} \qquad \langle I^2 \rangle \approx \frac{m_{3/2} M^m}{S^{m-1}} e^{i\delta}. \qquad (3.4)$$

This leads to a potential for the phase of S, $S = S_0 e^{ia/S_0}$:

$$V(a) \approx m_{3/2}^2 S^2 \cos((m+1)a/S + \delta). \qquad (3.5)$$

This corresponds to an axion mass of order $m_{3/2}^2$, at a time when the Hubble constant is much smaller.

We have studied the behaviour of a number of such systems numerically. We find that the modulus of the field does indeed track the minimum of the potential, and that the axion also settles into its minimum, damping at a rate somewhat slower than $1/t$.

As it stands, however, the model generates domain walls. The origin of the problem is the $m + 1$ within the cosine. This arises because, in the approximation in which we keep just these two terms in the superpotential, the model has a Z_{m+1} symmetry, which is spontaneously broken by the expectation value of S. So while the axion has mass which becomes large compared to the Hubble constant, it has $m + 1$ minima, and thus domains form in this model. Were it not for the domains, the axion field would be homogeneous (it is homogeneous on the Hubble scale), and domain walls would not form at the QCD phase transition.

This problem would be avoided if:

(1) There are additional fields involved in inflation (or the inflaton is a linear combination of fields with different Z_N charges). In this case, there could be terms like

$$\frac{I'^2 S' S^m}{M^m}. \qquad (3.6)$$

These give a comparable contribution to the axion potential, with a different periodicity.

(2) There could be terms of slightly higher dimension. These lift the degeneracy between the states. The domain walls tend, then, to collapse. However, at best, only one additional power of S (S') permits domain wall collapse within a Hubble time.

In the first case, the discrete symmetry is explicitly broken, and there is no corresponding degeneracy among states. However, there is still a potential problem. These complicated potentials may well have several local minima, and it is possible that in some regions, the system will become trapped in these. Whether or not this occurs requires a painstaking, case by case analysis, which we have not performed. However, the general lesson seems to be that despite the substantial breaking of the PQ symmetry, for most choices of discrete charge assignments, there remains a domain wall problem.

This analysis indicates that, quite generally, the PQ symmetry is badly broken immediately after inflation. It also makes clear that the implications of this breaking for the formation of topological defects depend on the details of the underlying theory. It appears that, if the Peccei–Quinn symmetry is broken during inflation (either spontaneously, explicitly or both), the usual constraints from isocurvature fluctuations

are ameliorated or even eliminated. In the case where the transition occurs later, this breaking can provide a solution of the domain wall problem, but only in rather special circumstances.

4. String models

Axions arise very naturally in string theory. They can often be thought of as accidental consequences of higher dimensional gauge symmetries associated with antisymmetric tensor fields, but this is not always the case. These symmetries are good symmetries of perturbation theory. But, in general, the size of non-perturbative violations is not known. In particular, while one can use discrete shift symmetries to place constraints on the size of superpotential terms which break the symmetry, there are, *a priori*, a wide variety of possible Kahler potential corrections, and their parametric dependence on the coupling constants is not known [20].

In weakly coupled string phenomenology, the string and Planck scale are not significantly different. Then the axion decay constant is large, of order $\alpha_{\mathrm{gut}} \times M$ [21]. The axion field is part of a supermultiplet with other moduli. The potential for the real part of the modulus, ϕ, is assumed to take the form

$$V = m_{3/2}^2 f(\phi/M). \tag{4.1}$$

The behaviour of the imaginary part is more complicated and highly model dependent. It has been discussed in [4].

At early times, one expects a structure

$$V = H^2 M^2 g(\phi/M). \tag{4.2}$$

The function g is in general unrelated to the function f; its minima, if any, are not expected to lie near those of f. As a result, during inflation, couplings can easily differ by an amount of order one from their values now. The axion decay constant can also differ by an amount of order one from its value now; in some scenarios, it might differ by several orders of magnitude. The axion potential can easily differ by orders of magnitude from its current value.

We have little general understanding of the non-perturbative breaking of PQ symmetry string theory. For example, we do not know whether there might be Kahler potential effects of order $\mathrm{e}^{-1/g}$, or whether all effects are of order e^{-a/g^2}. So we can only speculate. We might imagine that the possibilities are similar to those we described in effective field theories with discrete symmetries. For example, it is possible that the couplings were much larger during inflation, and so the explicit breaking of the PQ symmetry was large.

The recent suggestion that all moduli might be fixed in flux compactifications [22] provides another interesting framework in which one might hope to address these questions. But at present, it is not clear how axions would arise in this framework.

Axions might also arise as accidents of discrete symmetries, as in our field theory models. Because of all of these uncertainties, the only sensible approach at the moment would seem to be to view the field theory models as indicating the range of possibilities.

5. Conclusions

The main lesson of this work is that the traditional analyses of axion cosmology are not robust. Already, in [4], it was stressed that in supersymmetric models, inevitably the constraints on the axion decay constant from axion misalignment are altered. In this paper, we have seen that the underlying phenomena which give rise to the PQ symmetry, as well as the mechanism of inflation, strongly affect the PQ phase transition. This, we have seen, has implications for inflationary fluctuations and the formation of defects after inflation. The PQ symmetry is most probably an accidental consequence of some deeper, underlying symmetry, perhaps a gauged discrete symmetry or a higher dimensional continuous gauge invariance. In either case, the fate of the axion in the early universe depends on how the inflaton (or whatever drives inflation) influences this accident. If the inflaton transforms, for example, under the discrete symmetry, we have seen that there are a host of possibilities.

It is perhaps worth closing by summarizing the possible axion cosmologies we have enumerated here:

(1) The PQ symmetry may already be broken during inflation. This breaking may be spontaneous, explicit or both. In all cases, the constraints from isocurvature fluctuations are altered and, in the explicit case, they may be eliminated altogether.

(2) The PQ transition may occur after inflation. The usual picture, with formation of cosmic strings and domain walls, may be correct. Alternatively, for an epoch after inflation ends, when the universe is dominated by the oscillating inflaton and/or moduli, the PQ symmetry may be explicitly—and badly—broken by inflaton dynamics. In this case, the axion may be aligned. Depending on the details of the underlying theory, this may eliminate the problems of domain walls, or it may not. One might imagine that states without such problems might be selected by rather mild anthropic considerations.

All of this suggests that the breaking of the PQ symmetry during inflation may be the more likely possibility and, perhaps disappointingly, it may have little consequence for cosmological or astrophysical observations.

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