Some Issues in Gauge Mediation

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After briefly reviewing some arguments in favor of high scale and low scale supersymmetry breaking, I discuss a possible solution of the $\mu$-problem of gauge mediated models.

1. Introduction: High Scale Vs. Low Scale Supersymmetry Breaking

In the absence of any experimental evidence one way or the other, a good case can be made for either high scale or low scale breaking. Perhaps the most compelling argument in favor of high scale breaking is that such breaking seems natural in string theory. For example, gaugino condensation generates a superpotential for moduli,

$$W = e^{-S/M_p} M_p^3 \sim M_3/2 M_p^2.$$ (1)

with corresponding potential

$$V = e^K \left( \frac{\partial W}{\partial \phi} + \frac{\partial K}{\partial \phi} |W|^2 - 3|W|^2 \right).$$ (2)

If all of the moduli vary on the scale $M_p$, then all terms here are of the same order. $O(1)$ effects in the Kahler potential can stabilize the moduli, while the $-3|W|^2$ term can cancel the cosmological constant[1].

This is not the case if one has a low scale of supersymmetry breaking. The term $-3|W|^2$ above is then simply too small to cancel the cosmological constant, unless there are some other interactions with a much larger scale, introduced just for this purpose. One can shrug off this distinction, saying that we don’t understand the cosmological constant. After all, while for high scale breaking all of the terms are of the same order, they still must cancel to an extraordinary degree of precision. Still, it is troubling that a new scale of interactions seems to be required in the low scale case.

There are serious problems with the high scale scenario as well. One needs some mechanism to understand squark and slepton degeneracy. The various proposals to understand this are by now quite familiar: flavor symmetries, luck, or some sort of string miracle?

It is perhaps worth stating that if string theory were to provide a theory of soft breakings (especially prior to the discovery of supersymmetry), string theory could find itself in a position similar to that of weak interactions prior to the discovery of the W boson. The Weinberg-Salam theory was well established long before the discovery of the W and Z because it reproduced existing low energy data and because it successfully predicted the results of many new low energy measurements. Similarly, string theory could be convincingly confirmed if it provided a theory of soft breakings, even though experimental study of Planck scale physics would be out of reach.

Low energy breaking has certain clear theoretical advantages over high energy breaking[2]:

- It is more predictive. One doesn’t need to solve string theory to determine the 106 parameters of the MSSM.
- Flavor changing processes are automatically suppressed.
- The scale of supersymmetry breaking is perhaps not too far removed from conceivable experiments.
- Interesting new phenomena are possible, such as $\gamma$’s plus missing energy if the scale is low enough[6][7].
There exist real models which make predictions.

Apart from the question of scales mentioned above, there are a number of troubling features of existing approaches to model building. Many of these are cosmological. For example, in the framework of string theory, one expects extremely light moduli. This problem seems quite serious. It might be solved if our vacuum lies at or near a point of enhanced symmetry, where all of the moduli are charged under some group. It might also be solved by a period of very late inflation[8]. Each of these ideas has difficulties (but then so do ideas for solving the more conventional cosmological moduli problem).

More immediate are a set of problems on which there has been some progress in the last year:

- **Complexity:** The original models were quite complex, with several layers of interaction. During the past year, there has been substantial improvement in this picture, especially if one is willing to give up the idea that the SUSY breaking scale is “nearby.”[9]

- **Fine Tuning:** In minimal gauge mediation (MGM), the negative correction to the Higgs mass from loops containing the top squark is given by:

\[
\delta m_{H_U}^2 \approx -\frac{12y_t^2}{16\pi^2} \ln(\alpha_s/\pi) \tilde{m}_t^2.
\]  

(3)

Numerically, this is about 8 times the positive contribution from gauge loops, and about 70 times the contribution to the right handed sleptons. But we know that the masses of these latter particles are greater than about 85 GeV. So obtaining the correct breaking of \(SU(2) \times U(1)\) requires fine tuning. This problem can perhaps be ameliorated by relaxing some of the assumptions of minimal gauge mediation[10][11]. For example, in the models we discuss below, the messengers lie in a \(10 + \bar{10}\) representation of \(SU(5)\), and couple to several singlets. The squark and slepton spectrum in such models is different than in the case of minimal gauge mediation, and it is possible for the singlet slepton masses to be close to those of the electroweak doublets and color triplets.

- **Light Moduli:** We have already alluded to the fact that in the framework of string theory, low energy breaking implies very light moduli. These pose cosmological challenges; they also can be interesting, in that they mediate long range forces[12]. The cosmological problems might be solved if the minimum of the moduli potential is at or very near a point of enhanced symmetry. Alternatively, it might be solved within various inflationary scenarios.

- **The \(\mu\) problem:** The \(\mu\) problem has several aspects. First, one can ask why \(\mu\) isn’t simply order \(M_P\). This might be a consequence of symmetries. Alternatively, in string theory, it has been known for a long time that \(\mu\) may simply be small, at the classical level, more or less by accident, and remain small as a consequence of non-renormalization theorems. \(\mu\), however, must not be zero but instead must be of order the weak scale. Similarly, \(B_\mu\), the coefficient of \(H_UH_D\) in the potential should also be of order the weak scale. In the context of high scale breaking, this can be arranged automatically. This is not so easily arranged in the framework of gauge mediation. For example, in models of gauge mediation there is usually a singlet, \(S\), whose scalar and \(F\) components have vev’s, and which couples to the messenger fields. One can simply suppose that this field couples to the Higgs fields as well, with coupling \(\lambda H_UH_D\). One needs, however, that \(\lambda S \sim 100\text{GeV}\). But then \(B_\mu\) is

\[
B_\mu = \lambda F_S = \lambda S \frac{F_S}{S} \sim 10^7 \text{GeV}^2 (\frac{S}{30\text{TeV}})^2.
\]  

(4)

Various solutions to this problem have been proposed[13,5].
2. Large $m_{H_D}^2$ and the $\mu$ Problem

This last problem represents another rather severe fine tuning, and will be the focus of the rest of this talk. I want to consider here another solution, inspired by ref. [14], which has been developed with Scott Thomas and Jim Wells[11]. One of the features of this work is a large value for $m_{H_D}^2$. These authors made this proposal to solve a number of fine-tuning difficulties. Here, we exploit the fact that a large value of $m_{H_D}^2$ can naturally solve the $\mu$ problem in the form described above. The main point is quite simple, and can be seen by inspection of the quadratic terms in the general Higgs potential:

$$V(H^+, H^-) = m_{H_D}^2 |H_D|^2 - m_{H_u}^2 |H_u|^2 + B_H H_u H_D.$$  

If $m_{H_D}^2$ is large, then

$$\langle H_D \rangle = \frac{B_H (H_u)}{m_{H_D}^2}.$$  

Note that if $m_{H_D}^2 \gg B_H$, one has large $\tan(\beta)$.

How does one get large $m_{H_D}^2$? One approach is to give up the rigid philosophy of the gauge mediated models, in which there is a complete separation of the visible and messenger sectors. For example, in the MGM, one has a singlet coupled to a 5 and 5 of messengers:

$$W_{m_{g_{mg}}} = S q q + S \ell \ell.$$  

In such a model, one can have, in principle, couplings such as $H_D \ell \ell$ and $H_D Q \bar{q}$. Ordinarily, one assumes that these are forbidden by discrete symmetries. However, most Yukawa couplings in nature are very small, for reasons we can only guess, and perhaps these undesirable couplings are small as well. If most of them are small enough (where $10^{-2}$ is small enough) then one will still have no problem with flavor changing neutral currents or with the unitarity of the KM matrix. For this talk, I will adopt this point of view, and assume that, in addition to the Yukawas above, only a few others are appreciable, such as $H_D \bar{e} \bar{\tau}$[15]. In this case, in addition to the usual two loop gauge contributions familiar in gauge mediation, there are additional one-loop contributions to the masses of $H_D$ and $\bar{\tau}$. These were evaluated in [15]. One obtains:

$$\delta m_{H_D}^2 = -\frac{b^2}{24 \times 16\pi^2} \frac{|F|^4}{M^6},$$  

where $b$ is the appropriate coupling constant from the superpotential and $M$ is the mean mass of the multiplet.

So this model does produce a one loop mass for $H_D$, but unfortunately of the wrong sign. We have tried a variety of approaches to obtaining a positive sign, and only one seems to succeed without leading to other undesirable consequences.

- Larger $F_S$: If $F_S$ is large, then one cannot simply expand in powers of $F_S$. However, it turns out that for all physically acceptable $F_S$, $m_{H_D}^2$ is negative.

- More $5 + 5'$s: If there are no additional singlets, this just leads to copies of the computation above.

- More singlets: Now one can obtain a positive contribution. But there are other problems. The most serious is the appearance of a Fayet-Iliopoulos D-term for hypercharge. In the MGM, there is an accidental left-right symmetry of the messenger sector, which forbids such a term. This is not the case in more general models.

- $10, \overline{10}$ messengers: Now with one singlet, one has the couplings

$$W_{m_{g_{m_{10}}}} = \lambda_1 S Q \bar{Q} + \lambda_2 S U \bar{U}$$  

$$+ \lambda_3 S E \bar{E} + y H_D \bar{Q} U.$$  

Now the required computation is different, but one still finds a negative mass-squared, at order $|F|^4/M^4$. Again, this persists to all orders in $F$. But now one can add more singlets to the model without destroying the left-right symmetry of the model and generating a one-loop Fayet-Iliopoulos term. For a range of parameters, one finds that the masses are positive! For example, if all
of the supersymmetric masses are identical and equal to $M$, and if one has a susy-breaking potential, 

$$\delta V = \mu^2_G \bar{Q}Q + \mu_U^2 U\bar{U},$$  \hspace{1cm} (10)$$

then one finds 

$$m^2_{H_D} = \frac{1}{2} \frac{2\pi^2}{16} \frac{(\mu^2_G - \mu_U^2)}{M^2},$$ \hspace{1cm} (11)$$

where $p$ is a combination of Yukawas in the superpotential. So at least in this case, there is a positive one loop mass.

The model with the 10 and $\overline{10}$, then, provides at least an existence proof that one can obtain a large mass for $H_D$. It is interesting to take the model seriously, and note a number of differences with the MGM. First, the gauge contributions to the low energy soft breakings are characterized not by one parameter, $\Lambda = F_S/S$, but three, which we can call $\Lambda_Q$, $\Lambda_U$ and $\Lambda_E$. In particular, if $\Lambda_Q > \Lambda_U, \Lambda_E$, the ratio $m^2_\mu/m^2_\mu$ is reduced, by a factor of $2/3$, improving the situation with regards to fine tuning. Even better is the situation where $\Lambda_E \gg \Lambda_Q, \Lambda_U$.

3. A model for $\mu$

In light of these observations, let us reexamine the $\mu$-problem. As we said in the introduction, we can generate $\mu$ through a coupling $\lambda_i S_i H_U H_D$, where $S_i$ are the singlets which couple to the messengers. The couplings $\lambda_i$ must satisfy 

$$\lambda_i \approx \frac{\alpha_w}{4\pi},$$ \hspace{1cm} (12)$$

if $S \sim 100$TeV, in order that the $\mu$ term be of order the scale of electroweak symmetry breaking. This requires, if not an appreciable fine tuning, a certain degree of good luck. One of the virtues of gauge mediation is that gaugino masses are automatically one loop, while squark and slepton masses-squared are automatically two loop effects. We can try and go further in the present context and construct a model in which the $\mu$ term is automatically a one loop effect, while $B_\mu$ is automatically a two loop effect[11].

Suppose that none of the singlets, $S_i$, couples to $H_U H_D$, but that there is another singlet, $S$, with couplings 

$$W_S = \lambda S H_U H_D + S^3.$$ \hspace{1cm} (13)$$

Now at one loop there is a contribution to $m^2_S$ from loops containing $H_D$. This is similar to the usual negative contribution to the Higgs from stop loops. It automatically gives a negative mass to $S$, which, since it is proportional to $m^2_{H_D}$, is necessarily a two loop effect. So $\mu$ is automatically of one loop order, while $B_\mu$ is necessarily a two loop effect.

While this is quite appealing, it is not immediately clear that such a model can be realistic. There are a number of issues which must be faced:

- $\tan(\beta)$: In this framework, $\tan(\beta)$ is necessarily large, 

$$\tan(\beta) \approx \frac{m^2_{H_D}}{B_\mu} \approx \frac{16\pi^2}{\ln(M^2/M^2_{H_D}) \times \text{couplings}}.$$ \hspace{1cm} (14)$$

There is a danger that $\tan(\beta)$ will be so large that the bottom Yukawa coupling will become non-perturbative.

- Other couplings: $\tan(\beta)$ and other potentially problematic effects get larger as the couplings $\lambda$ and $\lambda'$ become smaller. But these couplings are bounded above by numbers of order 0.5 if one requires that they remain perturbative up to the unification scale.

- $m^2_\tau$: This mass receives a large negative contribution from $H_D$ loops when $\tan(\beta)$ and $m^2_{H_D}$ are large.

- Light pseudoscalars: The model contains a relatively light pseudoscalar (due to an approximate Peccei-Quinn symmetry, broken by three loop effects). The couplings of this particle to the $Z$, however, are suppressed by powers of $\tan(\beta)$, and this suppression seems adequate to explain why the particle has not yet been observed.

A survey of the parameter space indicates that these (and other) constraints can be satisfied[11].
For large $m^2_{F_D}$, this is most easily achieved if the messenger scale is large; this is perhaps appealing, given that many new models prefer such a large scale.

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REFERENCES

2. The possibility that supersymmetry is broken at low energies with gauge interactions serving as messengers was considered by a number of authors in the early 1980's. These models are reviewed in [3]. More recently, models with dynamical breaking of supersymmetry at low energies have been proposed[15,5].