STRING THEORY DUALITIES

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The past year has seen enormous progress in string theory. It has become clear that all of the different string theories are different limits of a single theory. Moreover, in certain limits, one obtains a new, eleven-dimensional structure known as M-theory. Strings with unusual boundary conditions, known as D-branes, turn out to be soliton solutions of string theory. These have provided a powerful tool to probe the structure of these theories. Most dramatically, they have yielded a partial understanding of the thermodynamics of black holes in a consistent quantum mechanical framework. In this brief talk, I attempt to give some flavor of these developments.

1 Introduction

The past two years have been an extraordinary period for those working on string theory. Under the rubric of duality, we have acquired many new insights into the theory.[1] Many cherished assumptions have proven incorrect. Seemingly important principles have turned out to be technical niceties, and structures long believed irrelevant have turned out to play a pivotal role. Many extraordinary connections have been discovered, and new mysteries have arisen. For phenomenology, we have learned that weakly coupled strings may be a very poor approximation to the real world, and that a better approximation may be provided by an 11-dimensional theory called M theory. But perhaps among the most exciting developments, string theory for the first time has begun to yield fundamental insights about quantum gravity.

Before reviewing these important developments, it is worthwhile recalling why string theory is of interest in the first place. There are parallels in our current situation vis a vis string theory and the situation 30 years ago in the theory of weak interactions.[5] For many years, it was clear that the four fermi interaction was not renormalizable, and that this was the signal of some new physics at energy scales below a TeV. Moreover, it was widely believed that this new physics involved the exchange of a vector boson. Gradually, it became clear that only in theories with non-abelian gauge bosons are such exchanges consistent. The requirement of consistency, in other words, strongly constrained the nature of the microscopic theory. It only was necessary to determine the gauge group, and this could be done by studying experimentally and theoretically the interactions of the light states of the theory at low

energies. Of course, until the discovery of the W and Z, one might have wondered whether we had missed something, whether there might be some other structure which permitted massive vector fields, but in the end, this was not the case.

The current situation with respect to gravitational interactions is similar. General relativity cannot be a consistent theory up to arbitrarily high energies. We know only one microscopic structure which can consistently incorporate gravity and gauge interactions: string theory. We do not know for certain that there are not other structures, but – as we will in some sense see in this talk – it is quite possible there are not. Of course, it will be a long time before we can do direct experiments on Planck scale physics, but we might hope to elucidate many features of the microscopic theory by studying its low energy phenomenology.

It is remarkable that postulating that the fundamental entities in nature are strings rather than point particles *automatically* gives

- Gravity.

- Gauge interactions, with a gauge group $\mathcal{G}$ which is large enough to contain the $SU(3) \times SU(2) \times U(1)$ group of the standard model.

- Finite theories with good high energy behavior.

More evidence that the theory might be true comes from studying classical solutions. Particular solutions give:

- Repetitive generations (sometimes 3).

- Light Higgs: An essential piece of hierarchy problem.

- Axions: The theory automatically has Peccei Quinn symmetries which hold to a sufficiently good approximation to solve the strong CP problem.

- N=1 Supersymmetry: String theory doesn't make sense without supersymmetry. In fact, we don't really know how to make sense of the theory unless supersymmetry survives to low energy. So it is probably fair to call low energy supersymmetry a *prediction* of string theory.

These are extraordinary achievements for a theory, and make it quite plausible that string theory might be some sort of ultimate description of nature. There are, however, serious difficulties which must be overcome before the case can be convincing:

- We don't really know what the theory is. It is as if we had Feynman rules for a theory like QCD, but don't know the field theory. There are some questions we can address, but many more which we cannot. The recent developments are providing new insights, but we still seem a long way from a complete answer.

- Too many vacua. Among these, there are a large number of discrete choices as well as continuous choices. Recent developments have given us some insight into the meaning of the discrete choices.

- At weak coupling, which is the only regime in which one can make real calculations, one inevitably makes some predictions which are qualitatively wrong. In particular, one predicts that the vacuum is unstable.

- Unification of couplings: It is well known that low energy supersymmetry leads to successful unification of couplings, with a unification scale of order $2 - 3 \times 10^{16}$GeV. In weakly coupled string theory, one expects unification, but at a scale of order 30 times higher. The argument is quite simple. The dimensionless coupling constant of string theory is related to the unified coupling, α_{GUT}, the compactification radius, $R \approx M_{GUT}$, and the tension, T by

$$\alpha_{st} = \alpha_{GUT} R^6 T^3. \tag{1}$$

  If we require $\alpha_{st} < 1$, then $R^2 \approx T^{-1} \approx 6 \times 10^{17}$GeV.[6] We will see that the recent developments in string duality suggest a solution to this problem.

Almost all of these points could have been made years ago.[7] In the last two years, however, there have been striking developments which bear on each of the points 1-4, associated with "Duality." Duality is a term which is used, loosely, to define equivalences between different physical theories. The recent explosion of activity involves many kinds of dualities: equivalence between different string theories, between theories with different values of couplings, compactification radii, and, perhaps most surprisingly, equivalences between string theories and certain – as yet poorly understood – theories in eleven dimensions. If there has been a general theme underlying these efforts, it has been to exploit the huge degeneracy of vacua and the symmetries of the theory(ies) to gain insight.

This work has already taught us *many* lessons, but there are two which are particularly striking:

- There is only one string theory; all of the previously known theories are equivalent. The fact that all of the theories of gravity we previously

knew are equivalent suggests that there is only one consistent theory of gravity!

- If "string theory" does describe the real world, weakly coupled strings are likely to be a very poor approximation. A better description can be obtained in terms of an 11-dimensional theory, only some of whose features we know. This theory is referred to as "M-theory."

These developments are also finding an application: for the first time, we are making a controlled attack on one of the fundamental problems of general relativity: the thermodynamics of black holes and the problem of information loss. One aspect of this problem is that black holes behave as if they possess an entropy, the famed Beckenstein-Hawking entropy:

$$ S = \frac{A}{4G}. \tag{2} $$

Up to now, the significance of this entropy has been obscure. However, for certain black holes in string theory, it has been possible to count the degenerate ground states, and show that one obtains eqn. 2.

In the rest of this talk, I will give a brief overview of these developments. Progress has been extraordinary; one could easily write several books on the subject. In 30 minutes, I must, of course, be highly selective. Indeed, the usual apology that one can't cover everything is more heartfelt here than usual. For string duality, at the moment, represents a large collection of beautiful observations, but the big lesson they are teaching us is not entirely clear. It is quite possible that the most important lessons may lie in things which I omit. That said, I will first briefly remind you about electric-magnetic duality. I say "remind" because this is a topic you can find covered nicely in Jackson.[8] I will then turn to the interconnection of different string theories. I will discuss the large degeneracy of string vacua (associated with "moduli"), and describe how, as one moves around this "moduli space," one encounters all of the different string theories. I will then explain the connection between ten and eleven dimensions, and discuss some phenomenology of M theory. Towards the end, I will discuss a new tool for studying non-perturbative questions in string theory: "D(irichlet)-branes," and the application of this tool to the black hole information problem. I will conclude with a listing of some recent developments, and deep questions.

2 Electric-Magnetic Duality

We have all stared at Maxwell's equations and wondered whether there might be magnetic charges. In the presence of magnetic charges and currents, one

has[8]

$$\vec{\nabla} \cdot \vec{E} = \rho_e \qquad \vec{\nabla} \cdot \vec{B} = \rho_m \tag{3}$$

$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J}_e \qquad \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = -\vec{J}_m. \tag{4}$$

These equations possess a symmetry, under the replacements

$$\vec{E} \to \vec{B} \qquad \vec{B} \to -\vec{E} \qquad \rho_e \to \rho_m \qquad \vec{J}_e \to \vec{J}_m \qquad \vec{J}_m \to -\vec{J}_e. \tag{5}$$

What sort of symmetry is this? In field theory, monopoles arise as solitons. Their masses behave as

$$M_m = \frac{1}{e^2} v. \tag{6}$$

They are big fat objects, which obey the Dirac quantization condition, $eg = 2\pi n$, where g is the magnetic charge. If duality is to be a symmetry, it must somehow interchange "fundamental" particles and solitons. This is not completely crazy, since it also must interchange $e \leftrightarrow 2\pi/e$, i.e. electric-magnetic duality is weak-strong coupling duality. On the other hand, because of this, it is hard to see how duality can be more than a speculation.

It turns out, however, that in theories with enough supersymmetry, one can check the duality conjecture. In such theories, the supersymmetry algebra takes the form,

$$\{Q_\alpha^I, Q_\beta^J\} = P_\mu \gamma_{\alpha\beta}^\mu \delta^{IJ} + \epsilon_{\alpha\beta} Z^{IJ} \qquad I, J = 1, \ldots, N. \tag{7}$$

Here the Z's are some set of charges (e.g. the electric and magnetic charges) which are referred to as "central charges." If one has a soliton which is invariant under some of the Q's, one can prove exact formulas for the mass, called BPS formulas. The basic point is quite simple. Schematically,

$$\langle \{Q_\alpha Q_\beta\} \rangle = 0 = \langle H \rangle + q. \tag{8}$$

$\langle H \rangle$ is just the mass and q is a charge, so the mass is related to a charge. Now one can determine the mass at weak coupling, interpolate to strong coupling, and verify the duality conjecture. This type of analysis, first performed in field theory, can be extended to string theory.

So there are theories in which electric-magnetic duality holds. Solitons are mapped to "fundamental" particles under these transformations. The duality symmetry can be thought of as a spontaneously broken symmetry. It is restored if $e = \sqrt{2\pi}$.

3 Moduli Spaces and String Equivalences

In string theory, there are a variety of dynamical fields, referred to as moduli, whose expectation values determine the parameters of the theory. They have the property that in some lowest order approximation, they have no potential; in many cases (particularly if there is a high degree of supersymmetry), one can argue that they have no potential exactly, i.e. even when all non-perturbative effects are taken into account. This phenomenon is not familiar in conventional, non-supersymmetric field theories, so it is perhaps best to illustrate with some examples:

- The dilaton. In string theory, there is a field, usually denoted by ϕ, called the dilaton. The expectation value of this field determines the dimensionless coupling of the theory, through an equation of the form

$$\langle e^\phi \rangle = g_s. \tag{9}$$

- When one compactifies string theories, the size and shape of the compact spaces are determined by additional moduli fields.

The second phenomenon is nicely illustrated by compactification of a 10 dimensional string on a circle of radius R. From the perspective of a nine-dimensional physicist, there are a number of massless states. For example, for the components of the ten-dimensional metric one has the decomposition:

$$g_{MN}(x,\theta) \to g_{\mu\nu}(x) \qquad g_{\mu 9}(x) = A_\mu(x) \qquad g_{99} = R^2(x). \tag{10}$$

The radius, R, can be thought of as the expectation value of the field, $R^2(x)$, i.e. the radius is dynamical. Also it can take any value, so there is no potential for this field.

There are dualities associated with R. These dualities are easy to establish, since (unlike weak-strong duality) they are already visible in perturbation theory. If we compactify on a circle, we have momenta, $p^9 = n/R$. From the perspective of a nine dimensional observer, a *ten* dimensional massless field (such as the metric) with momentum p_9 has mass p_9^2. There are also windings, corresponding to the fact that the string can wind m times around the circle. The mass spectrum is given by

$$M^2 = T(\frac{n^2}{R^2} + m^2 R^2). \tag{11}$$

This spectrum is symmetric under $R \to 1/R$. Compactifying more dimensions yields more elaborate dualities; these are generically referred to as T-dualities.

(Strong-weak coupling dualities are called S-dualities; duality transformations which mix coupling and moduli are called U-dualities).

With these preliminaries, we are in a position to discuss the equivalence of the various string theories. In textbooks, one learns that there are five types of string theory: heterotic $E_8 \times E_8$, heterotic O(32) , Type IIA and IIB (all theories of oriented closed strings), and the O(32) Type I theory (a theory of open and unoriented closed strings). For some time, it has been known that the $E_8 \times E_8$ and heterotic O(32), as well as the IIA and IIB theories, are related by T-dualities. For example, compactifying the IIA theory on a circle of radius R gives the same theory as the IIB compactified on a circle of radius $1/R$. More recently it has been realized that the strong coupling limit of the O(32) heterotic theory is the weakly coupled Type I theory. This is rather amazing, since these theories are formulated in terms of quite different objects (closed vs. open strings, oriented vs. unoriented). Similarly, suitable compactifications of the $E_8 \times E_8$ theory at weak coupling are equivalent to different compactifications of the type II theories. But perhaps most surprising of all, is that the ten-dimensional Type IIA and the heterotic $E_8 \times E_8$ theories become, in the strong coupling limit, eleven dimensional!

There is not time here to explore all of these connections, but I would like to describe some of the features of the 10-11 dimensional duality. The point, again, has to do with the solitons of the theory. In the IIA theory, at weak coupling, there is a tower of solitons with mass

$$M = \frac{n}{g} \tag{12}$$

for integer n. These states are BPS states, so the mass formula is exact and holds even as $g \to \infty$. Equation 12 is similar to the formula for the momentum states in Kaluza-Klein compactification, with $M = n/R$. So $g \to \infty$ is similar to $R \to \infty$. So the IIA theory resembles an 11 dimensional theory compactified on a circle. By more careful study, one can show that this eleven dimensional theory is 11-dimensional supergravity (the only supersymmetric theory in 11 dimensions). Note that now the dilaton and the radius which we described before are more or less on the same footing. It turns out that the radius of the eleventh dimension, R_{11}, goes roughly as $R_{11} \sim g^3$. In fact, this connection has been exploited to provide a deeper understanding of various duality symmetries.

For the heterotic string, the story is more intricate. Again, one finds that the large coupling limit of the theory is an eleven dimensional theory. However, the relevant eleven dimensional world now has two walls, separated by a distance R_{11}. The graviton, metric and other fields of eleven dimensional

supergravity now propagate throughout the eleven dimensional space, but the gauge fields and gauginos live on the walls. This picture is established by considering space-time anomalies, the low energy spectrum and the low energy effective action.

What is really going on microscopically, from the eleven dimensional perspective, is still not known. Eleven dimensional supergravity is not a renormalizable, much less finite theory, so it is presumably the low energy limit of some other structure. This structure has been called M-theory. One might wonder, for example, whether the walls described above are real walls, and the gauge fields are states bound to them.

In any case, in this framework, we can solve the problem of string unification. Before we argued that if M_{GUT} was of order 10^{16} GeV, the string coupling was enormous (of order 10^7). But from our present perspective, this suggests that we should consider the problem from the point of view of M-theory. The precise relations between ten-dimensional and four dimensional quantities are:

$$R_{11}^2 = \frac{\alpha_{GUT}^3 V}{512\pi^4 G_N^2}, \tag{13}$$

and

$$M_{11} = R^{-1} \left(2(4\pi)^{-2/3}\alpha_{GUT}\right)^{-1/6}. \tag{14}$$

Plugging in reasonable numbers gives that $M_{11}R \approx 2$, while $M_{11}R_{11} \approx 70$. Taking these formulas at face value, the universe is approximately five dimensional, and the eleven dimensional supergravity approximation should perhaps not be so bad!

Even with this starting point, it is not so easy to develop a detailed phenomenology, but there are at least two immediate implications.

- The fundamental scale of the theory is of order M_{GUT}, not M_p. This raises issues for baryon number violation and other effects mediated by high dimension operators.

- Axions: The presence of axions is one of the virtues of string theory. However, most of the Peccei-Quinn symmetries are violated by effects of order $e^{-R^2 T}$, and one usually says that this is of order one. Now that we think of R as large, this is not the case, and these axions are viable. They have decay constants of order M_p, which presents problems for conventional cosmology, but because of the presence of the moduli, string cosmology is likely to be unconventional, and it is not clear that these problems are so serious.

Eventually, we would like to understand why the scales are what they are, i.e. what dynamics determines the moduli expectation values. There are hints of a possible mechanism in recent work of Witten. He showed that if one holds the compactification volume and the coupling of one of the gauge groups fixed, that of the other grows with R_{11}, blowing up when

$$m_p = c \frac{M_{GUT}}{\alpha_{GUT}^{2/3}},$$

(15)

with c a number of order one. This is not an not unreasonable value. Still, we are far from a complete picture.

4 D-Branes: A New Tool

One of the lessons of the recent developments is that features of theories which appeared fundamental, such as whether a theory contained closed or open strings, are of no invariant significance. Similarly, we have seen that solitons in one description are fundamental entities in another. Indeed, Polchinski has observed that certain classical solutions of the string equations can actually be described as fundamental strings with unusual boundary conditions.[9]

One can understand the appearance of D-branes by considering open strings. The usual free string action is

$$S = \frac{T}{2} \int d^2\sigma \partial_\alpha X^\mu \partial_\alpha X_\mu d^2\sigma.$$

(16)

There are two possible boundary conditions at the endpoints:

$$\partial_\sigma X^\mu = 0$$

(17)

$$X^\mu = Y^\mu (constant).$$

(18)

The first of these are the usual Neumann conditions. The second, Dirichlet, condition, is usually discounted because it violates translation invariance. But for the description of solitons, this is fine. The Y^μ correspond to the locations of the soliton. One can imagine that the time and p of the space components obey Neumann conditions, while the remaining coordinates obey Dirichlet conditions. The resulting object is called a Dirichlet p-brane (a $D0$ brane is a particle, a $D1$-brane a string, a $D2$ brane a membrane, and so on). What is striking here is the simplicity of the D-brane description. Complicated solitons – and the quantum fluctuations about them – are described in terms of simple two dimensional field theories.

This D-brane technology has found many applications. Among the most interesting are to black hole physics. There is a class of puzzles associated with the fact that black holes behave, in many ways, as thermodynamic objects. One can associate with them an entropy which obeys the usual laws of thermodynamics. They emit particles like black bodies at that temperature. This raises the possibility, however, that an initially pure (albeit quite complicated) state which formed a black hole might evolve into a thermal, mixed state. The puzzle is quite serious. Simple explanations, such as the possibility that the information is encoded in subtle correlations among the emitted particles, run afoul of principles of field theory such as locality and causality.

Various scenarios have been offered for how this problem might be resolved in string theory, but they generally involve strong coupling, and are difficult to discuss concretely. With the recent progress in string theory, however, some real steps have been taken towards addressing these questions. In several instances, the Beckenstein-Hawking entropy, eqn. 2, has been shown to be equal to the logarithm of the number of microstates.

The strategy in these calculations is not terribly complicated. One identifies soliton black holes in the theory with certain configurations of D branes. The D branes are described by a free two dimensional field theory, so the counting of degenerate ground states is a reasonably straightforward problem. One difficulty is that the calculation is only valid when the coupling is weak, which turns out to correspond to a Schwarschild radius much less than the string length. In the interesting limit, the soliton picture is not valid. However, we are rescued, again, by the fact that the D-brane states are BPS states, so their masses are correctly given, even at strong coupling, by their weak coupling expressions. Presently, many workers are seeking to go beyond the BPS limit in order to get a clearer physical picture.

The problems of black hole physics are important, not so much in themselves, as for the challenge they provide to our understanding of physics at very short distances. Hawking has long advocated the view that they signal that quantum mechanics itself must be modified in some drastic way. In the framework of conventional field theory, many workers have tried, and failed, to address this challenge. The fact that string theory is likely to resolve these questions – in a conventional quantum mechanical framework – is strong support for the idea that string theory is the correct, underlying theory of gravity.

5   Other Developments

There have been a long list of additional beautiful results, some of which may have profound significance. Limitations of space (and of my knowledge)

prevent making any sort of complete list, but let me mention a few (chosen largely because I want to learn about them and understand them better):

- There have been a number of applications of duality to theories with $N = 1$ supersymmetry in four dimensions. Dual pairs have been uncovered. Mysterious phenomena in one picture (for example, intricate cancellations) appear simple in another.

- New phases of theories have been uncovered. Singularities in low energy effective actions are usually associated with the appearance of new light states. Examples have been exhibited where, for example, the topology of space-time appears to change, and/or where monopoles or black holes become massless. One also has examples where an entire string-like tower of states becomes massless (tensionless strings).

- Related to this, it has become clear that one can have much larger gauge symmetries than are possible in weak coupling strings.

- One has obtained some insight into the meaning of gauge symmetries. Perhaps the most outstanding example of this is due to Seiberg, who has exhibited field theories with massless composite gauge bosons.

- Evidence for a new scale? Perturbative string amplitudes are very soft for momenta and distances of order $\sqrt{T}$. This is small compared to the Planck scale, $M_p = \sqrt{T}/g$. It is usually said that it does not make sense to probe shorter distances, and that the notion of space-time ceases to make sense at this scale. But duality has obscured the significance of T, and there is evidence for a harder, short distance component.

6 Conclusions and Forecast

Much has been learned in the last two years. We know that there is only one string theory. We have greater insight into the moduli space of string vacua. We have understood certain non-perturbative phenomena in the theory. String theories seem poised to meet the challenge of black hole physics. Yet we still feel like the proverbial blind persons faced with the elephant. While we finally know that we are studying one creature, not several, we still don't understand quite what it is we have gotten a hold of.

Let me close, in the spirit of the times, with a forecast. Over the next year, I look for:

- Further beautiful verification of string dualities.

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- Persuasive resolution of the black hole information loss problem.

- Perhaps something even more spectacular, yielding greater insight into what exactly these theories are.

But there are some questions which I am less optimistic we will answer very soon. The progress in duality involves reformulating interesting strong coupling problems as weak coupling problems. Unfortunately, general arguments suggest that if string theory describes nature, *no* weak coupling analysis can be valid. So I don't expect to see, for example, a calculation of m_e/m_μ in the coming year.

Acknowledgments

1. A complete list of references on the subject of string duality would run many pages. J. Polchinski [2] and John Schwarz [3] have written excellent reviews geared to non-experts which collect most of the needed references. Witten has written an article which summarizes the major ideas.[4] As a result, I will only make selected references in the text, either to reviews which I believe may be instructive or to highlight particular points.
2. J. Polchinski, "String Duality: A Colloquium," NSF-ITP-96-60, hep-th/9607050.
3. J.H. Schwarz, "The Second Superstring Revolution," hep-th/9607067.
4. E. Witten, "Reflections on the Fate of Spacetime," Physics Today **49** (1996) 24.
5. This particular analogy is an elaboration of one given by Polchinski in [2].
6. K. Dienes, "String Theory and the Path to Unification: A Review of Recent Developments," IASSNS-HEP-95-97, hep-th/9602045.
7. M. Dine, "String Theory in 1988: An Idiosyncratic View," CCNY-HEP-88/17. Plenary talk given at 1988 Mtg. of Div. of Particles and Fields of the APS, Storrs, CT, Aug 15-18, 1988.
8. J.D. Jackson, *Classical Electrodynamics*, Wiley (1975) New York. Note that this discussion appears only in the second edition.
9. J. Polchinski, S. Chaudhuri and C.V. Johnson, "Notes on D-Branes," hep-th/9602052.