Physics 101B. Introductory Modern Physics. Professor Dine

Winter, 2005. Homework Set 3. Due Monday, Jan. 31.

Problem numbers refer to your textbook.

1. The "central field approximation" is the basis for our understanding of the periodic table. In this approximation, it is assumed that the Hamiltonian (energy operator) for an atom with Z electrons looks like:
$$H = \sum_{i=1}^{Z} \left(\frac{\vec{p}_i^{\,2}}{2m} + V(r_i) \right) \tag{1}$$
where r_i is the distance of the i'th electron from the nucleus. Write the Schrodinger equation for this system. Separate variables by writing the wave function for the z electrons, $\Psi(\vec{x}_1, \ldots \vec{x}_Z)$ as
$$\Psi = \psi_1(\vec{x}_1) \ldots \psi_Z(\vec{x}_Z) \tag{2}$$
Show that ψ_i satisfies:
$$(\frac{-\hbar^2 \vec{\nabla}^2}{2m} + V(r_i))\psi_i(\vec{x}_i) = E_i \psi_i(\vec{x}_i) \tag{3}$$
where $E = \sum_{i=1}^{Z} E_i$ is the total energy.

2. A spin-1/2 particle moves in a magnetic field in the y direction. For this system the Hamiltonian (Energy) operator is:
$$H = \frac{e S_y B}{M} \tag{4}$$
where
$$S_y = \frac{\hbar}{2} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}. \tag{5}$$
Shown that the eigenvectors of S_y are:
$$\psi_{+y} = \begin{pmatrix} 1 \\ i \end{pmatrix} \quad \psi_{-y} \begin{pmatrix} 1 \\ -i \end{pmatrix} \tag{6}$$
What are the corresponding eigenvalues? What are the energy eigenvalues?

Now suppose that at $t = 0$ the system is prepared in a Stern Gerlach experiment with spin up in the z direction, i.e.
$$\Psi(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{7}$$
Find the probability that the system has spin down in the z direction at time t. (Hint:
$$\Psi(0) = \frac{1}{2}(\psi_{+y} + \psi_{-y}). \tag{8}$$
You'll want to look at the spin handout.)

3. 8-2

4. 8-5

5. 8-8
6. 8-11
7. 8-17