7.2 $E_{n_1 n_2 n_3} = n_1^2 E_1^{(1)} + n_2^2 E_1^{(2)} + n_3^2 E_1^{(3)}$

WHERE $E_1^{(2)} = 1/4\, E_1^{(1)}$

$E_1^{(3)} = 1/9\, E_1^{(1)}$

$\Rightarrow E_{n_1 n_2 n_3} = (n_1^2 + n_2^2/4 + n_3^2/9)\, E_1^{(1)}$ $\qquad E_1^{(1)} = \pi^2\hbar^2/2mL^2$

① Lowest value given by $n_1 = n_2 = n_3 = 1$ $\Rightarrow (1^2 + (1/4)^2 + (1/9)) = 1.361$
$\Rightarrow E_{111} = 1.361 E_1^{(1)}$

② Raise n_3 first because it's the smallest contribution:
$E_{112} = (1 + 1/4 + 4/9) E_1^{(1)} = 1.694 E_1^{(1)}$

③ Either E_{113} OR E_{121} must be next:
$E_{113} = 2.25 E_1^{(1)}$, $E_{121} = 2.111 E_1^{(1)}$ so next lowest is E_{121}

④ REPEATING THE PROCESS GIVES $E_{113} = 2.25 E_1^{(1)}$ AS next lowest

⑤ $E_{122} = 2.444\, E_1^{(1)}$

⑥ $E_{123} = 3.000\, E_1^{(1)}$

⑦ $E_{114} = 3.028\, E_1^{(1)}$

⑧ $E_{131} = 3.360\, E_1^{(1)}$

⑨ $E_{132} = 3.472\, E_1^{(1)}$

⑩ $E_{124} = 3.778\, E_1^{(1)}$

7.3 (a) $\psi_{n_x n_y n_z} = \psi_{n_x}(x)\, \psi_{n_y}(y)\, \psi_{n_z}(z)$

FOR 1D CASE, $\psi_n(x) = \sqrt{\frac{2}{L}} \cos \frac{n\pi x}{L}$ (odd n) for $-\frac{L}{2} < x < \frac{L}{2}$

$= \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$ (even n)

AND $\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$ for $-L < x < L$.

Now apply the 1ST FORM for $\psi_{n_x}(x)$ AND the 2ND to $\psi_{n_y}(y), \psi_{n_z}(z)$:

$$\Rightarrow \psi_{n_x n_y n_z}(x,y,z) = \left(\frac{2}{L}\right)^{3/2} \left\{ \begin{array}{l} \cos n_x\pi x/L \ \text{(n odd)} \\ \sin n_x \pi x/L \ \text{(n even)} \end{array} \right\} \sin \frac{n_y \pi y}{L} \sin \frac{n_z \pi z}{L}$$

$$\Rightarrow \psi_{111}(x,y,z) = \left(\frac{2}{L}\right)^{3/2} \cos\frac{\pi x}{L} \sin\frac{\pi y}{L} \sin\frac{\pi z}{L}$$

(b) The energy should not change due to shifting the coordinates, so a box corresponding to $0 < x < L$ has the same energy as before.

$\boxed{7.8}$ (a) $\psi_{n_x n_y}(x,y) = \psi_{n_x}(x)\psi_{n_y}(y)$
$= \frac{2}{L}\sin\frac{n_x\pi x}{L}\sin\frac{n_y\pi y}{L}$

(b) $E_{n_x n_y} = E_{n_x} + E_{n_y} = (n_x^2+n_y^2)\hbar^2\pi^2/2mL^2$

(c) 1^{ST} degeneracies occur for the 1^{ST} excited states
$E_{12} = E_{21} = (1^2+2^2)\hbar^2\pi^2/2mL^2$ so $\begin{Bmatrix} n_x \\ n_y \end{Bmatrix} = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$ OR $\begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$

$\boxed{7.9}$ (a) $n=3$ ℓ goes from 0 to $n-1$: $\therefore$ $\ell = 0,1,2$
(b) m goes from $-\ell$ to ℓ: $\ell=0 \Rightarrow m=0$
$\ell=1 \Rightarrow m=\pm1,0$
$\ell=2 \Rightarrow m=\pm2,\pm1,0$
(c) $\ell=2, m=\pm2,\pm1,0 \Rightarrow 5\times2 = 10$ states
$\ell=1$ $m=\pm1,0 \Rightarrow 3\times2 = 6$ states
$\ell=0, m=0 \Rightarrow 2$ states
$\therefore$ total # of states $= \boxed{18}$

$\boxed{7.17}$ (a) $n=6$ $\ell=3$ ($s\to\ell=0$ $p\to\ell=1$ $d\to\ell=2$ $f\to\ell=3$)
(b) $E_n = \frac{Z^2E_0}{n^2}$ where $Z=1$, $n=6$, $E_0 = -13.6\,eV$
$\Rightarrow E_6 = \boxed{-2.2\,eV}$
(c) $|\vec{L}| = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{12}\,\hbar = \boxed{2\sqrt{3}\,\hbar}$
(d) $L_z = m\hbar$ $m=\pm3,\pm2,\pm1,0 \Rightarrow L_z = \boxed{\pm3\hbar,\pm2\hbar,\pm\hbar,0}$

$\boxed{7\text{-}19}$ (a) $\psi_{100} = C_{100}e^{-r/a_0} = \frac{1}{\sqrt{\pi}}(1/a_0)^{3/2}e^{-r/a_0}\Big|_{r=a_0} = \frac{1}{\sqrt{\pi a_0^3}\,e}$

(b) $\psi_{100}^2 = \frac{1}{\pi a_0^3}e^{-2r/a_0}\Big|_{r=a_0} = \frac{1}{\pi a_0^3 e^2}$

(c) $P = (4\pi r^2)\psi^2(r)\Big|_{r=a_0} = \frac{4\pi a_0^2}{\pi a_0^3 e^2} = \frac{4}{a_0 e^2}$

REMARK IF THE $C_{n\ell m}$ NOTATION IS CONFUSING,
use $\psi_{n\ell m} = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$ AND read off
R AND Y FROM THE TABLES.

$\boxed{7\text{-}25}$ LET'S DO THIS PROBLEM BY CONSTRUCTING $\Psi_{n\ell m}$ FROM THE TABLE OF $R_{n\ell}$'s AND $Y_{\ell m}$'s

(a) $\Psi_{200}(r,\theta,\phi) = R_{20}(r)Y_{00}(\theta,\phi) = \left\{\frac{1}{\sqrt{2a_0^3}}\left(1-\frac{r}{2a_0}\right)e^{-\frac{r}{2a_0}}\right\}\cdot\frac{1}{\sqrt{4\pi}}$

AT $r=a_0$, THIS BECOMES

$$= \frac{1}{\sqrt{8\pi a_0^3}}\left(1-\tfrac{1}{2}\right)e^{-1/2} = \frac{e^{-1/2}}{4\sqrt{2\pi a_0^3}}$$

(b) $\psi^2 = \dfrac{e^{-1}}{16\cdot 2\pi a_0^3} = \dfrac{1}{32\pi a_0^3 e} = |\psi|^2$ since Ψ IS REAL

(c) $P = 4\pi r^2|\psi|^2\Big|_{r=a_0} = \dfrac{(4\pi a_0^2)}{32\pi a_0^3 e} = \dfrac{1/8}{a_0 e} = \dfrac{.4059}{a_0}$

$\boxed{7\text{-}26}$ Where is $P_{n=2,\ell=1}(r)$ a maximum? USE RESULTS FROM Problem 7-24:

$$P_{21m}(r) = A\cos^2\theta\, r^4 e^{-r/a_0}$$

$\dfrac{dP}{dr} = 0$ when $r = r_{max}$ (ie, the most probable distance)

so $\dfrac{dP}{dr}\Big|_{r=r_{max}} = 0 = A\cos^2\theta\left\{4r_{max}^3 e^{-\frac{r_{max}}{a_0}} + r_{max}^4\left(\frac{-1}{a_0}\right)e^{-r_{max}/a_0}\right\}$

Solve for r_{max}: $4r_{max}^3 - r_{max}^4 = 0$

IF $r_{max} \neq 0$, then $4 - r_{max}/a_0 = 0 \Rightarrow r_{max} = 4a_0$

QED.