$\boxed{8\text{-}18}$ (a) USE EQN 8-68: $e^{-\alpha} \overset{set}{=} 1 = \frac{N}{V}\frac{h^3}{2(2\pi m_e kT)^{3/2}}$

$\Rightarrow \frac{N}{V} = \frac{2(2\pi m_e kT)^{3/2}}{h^3}$ $kT \approx 2.6\cdot 10^{-2}\text{eV}$

$hc \approx 1240\cdot 10^{-9}\text{eV}\cdot\text{m}$

$m_e c^2 = .51\cdot 10^6\text{eV}$

$\frac{N}{V} = \boxed{2.5\cdot 10^{25}}$

(b) FOR $e^{-\alpha} = 10^{-6}$, MULTIPLY BY 10^{-6} TO GET $\boxed{2.5\cdot 10^{19}}$

$\boxed{8\text{-}33}$ FOR 10 PROTONS, THE HIGHEST STATE FILLED IS E_5. ESTIMATING THE ENERGY USING THE INFINITE POTENTIAL WELL GIVES $E_n = \frac{\pi^2\hbar^2}{2m_pL^2}\cdot n^2$

WHERE $L = 2(3.1\cdot 10^{-15}\text{m})$.

$E_1 = \frac{\pi^2(\hbar c)^2}{2(m_pc^2)(2r)^2} = \frac{\pi^2(197.3\cdot 10^{-9}\text{m}\cdot\text{eV})^2}{8(3.1\cdot 10^{-15}\text{m})^2(938\text{MeV})} = 5.32\text{ MeV}$

PROTONS: $E_F = E_5 = (25)E_1 = \boxed{133\text{ MeV}}$

$\langle E\rangle = \frac{1}{10}\left[2\cdot E_1 + 2\cdot 4E_1 + 2\cdot 9E_1 + 2\cdot 16E_1 + 2\cdot 25E_1\right] = \boxed{58.5\text{ MeV}}$

NEUTRONS: $E_1 = \frac{\pi^2(\hbar c)^2}{2(m_nc^2)(2r)^2} = \frac{\pi^2(197.3\cdot 10^{-9}\text{m}\cdot\text{eV})^2}{8(3.1\cdot 10^{-15}\text{m})(939\text{ MeV})} = 5.31\text{ MeV}$

$E_F = E_6 = 36E_1 = \boxed{191\text{ MeV}}$

$\langle E\rangle = \frac{1}{12}\cdot 2\left[E_1 + 4E_1 + 9E_1 + 16E_1 + 25E_1 + 36E_1\right] = \boxed{80.5\text{ MeV}}$

$\boxed{8\text{-}34}$ BOSONS WILL ALL BE IN E_1 STATE $\Rightarrow 10\times E_1 = \frac{10\hbar^2\pi^2}{2mL^2} = \frac{5\hbar^2\pi^2}{mL^2}$

$\boxed{8\text{-}36}$ (a) FIND FRACTION AT $T = T_c/2$

$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c}\right)^{3/2} = 1 - (1/2)^{3/2} = \boxed{.65}$

(b) AT $T = T_c/4$ $\frac{N_0}{N} = 1 - (1/4)^{3/2} = \boxed{.875}$

COMMENT ON PROBLEM 8-33: THIS CRUDE MODEL NEGLECTS INTERACTIONS BETWEEN PARTICLES AND USES AN ABRUPT RATHER THAN SLOPING WELL, WHICH WOULD CHANGE THE ENERGY. SEE CHAPTER 11.

$\boxed{8-41}$ (a) $P(u)du = Ce^{-Au^2/kT}du$

NORMALISE : $\int_{-\infty}^{\infty} P(u)du = C\int_{-\infty}^{\infty} e^{-Au/kT}du = 1$

$\Rightarrow C\sqrt{\pi kT/A} = 1 \Rightarrow C = \sqrt{A/\pi kT}$

(b) $\langle E\rangle = \langle Au^2\rangle = C\int_{-\infty}^{\infty} Au^2 e^{-Au^2/kT}du$ LET $b \equiv A/kT$

$\int_{-\infty}^{\infty} u^2 e^{-bu^2}du = -\frac{d}{db}\int_{-\infty}^{\infty} e^{-bu^2}du = -\frac{d}{db}\sqrt{\frac{\pi}{b}} = \frac{1}{2}\frac{\sqrt{\pi}}{b^{3/2}}$

$\therefore \langle E\rangle = CA\left[\frac{1}{2}\sqrt{\pi}(kT/A)^{3/2}\right] = \frac{1}{2}\sqrt{\frac{A\pi}{\pi kT}}\frac{kT}{\sqrt{A}}\sqrt{kT} = \frac{kT}{2}$ ✓

$\boxed{8\cdot 44}$ $N = e^{-\alpha}\frac{4\pi(2m_e)^{3/2}V}{h^3}\int_0^{\infty} E^{1/2}e^{-E/kT}dE$

$\int_0^{\infty} E^{1/2}e^{-\beta E}dE = \Gamma(3/2)/\beta^{3/2}$ $\quad \Gamma(\frac{3}{2}) = \frac{1}{2}\Gamma(\frac{1}{2}) = \frac{1}{2}\sqrt{\pi}$

$e^{-\alpha} = \frac{2Nh^3(1/kT)^{3/2}}{4\pi(2m_e)^{3/2}V\sqrt{\pi}} = \frac{Nh^3}{2(2\pi m_e kT)^{3/2}V}$ AS REQUIRED.

$\boxed{8-46}$ SEE NEXT PAGE

$\boxed{\text{DINE SPECIAL}}$ $n(E)dE = g(E)f(E)dE$

WE KNOW THAT THE DEGENERACY IS $2l+1$, BUT WE NEED IT AS A FUNCTION OF ENERGY : SET $g(E)dE = g(l)dl = (2l+1)dl$

$\Rightarrow g(E) = g(l)\frac{dl}{dE}$ $\quad \frac{dE}{\cancel{dl}} = \frac{l(l+1)\hbar^2}{2I} \approx \frac{l^2\hbar^2}{2I}$

$\frac{dl}{dE} = \frac{d}{dE}\left(\frac{\sqrt{2IE}}{\hbar}\right) = \frac{\sqrt{2I}}{\hbar}\frac{1}{2E^{1/2}} = \sqrt{\frac{I}{2E\hbar^2}}$

$\Rightarrow n(E)dE = \sqrt{\frac{I}{2E\hbar^2}}\frac{1}{e^{\alpha}e^{-\beta E}-1}dE$

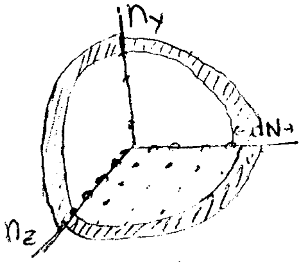
IT IS NOT REQUIRED TO INTEGRATE THIS.

$\boxed{8-46}$ (a) $E = pc, \quad p = \hbar k = \hbar\frac{\pi}{L}\sqrt{n_x^2+n_y^2+n_z^2} = \frac{\hbar\pi N}{L}$

$$\Rightarrow \boxed{E = \frac{\hbar c\pi N}{L}}$$

(b) RECALL SPHERICAL VOLUME ELEMENT IS $dV = 4\pi r^2 dr$

In n-SPACE, WE USE $4\pi N^2 dN$



WANT POSITIVE OCTANT: MULTIPLY BY 1/8

2 POLARIZATIONS: MULTIPLY BY 2

$$\Rightarrow \frac{4\pi(2)}{8}N^2dN = \pi N dN$$

(c) $E = \frac{N\hbar c\pi}{L} \Rightarrow N = \frac{LE}{\hbar c\pi}$ $\quad \frac{dN}{dE} = \frac{L}{\hbar c\pi}$ SOLVE FOR dN

$$g(E)dE = \pi N^2 dN = \pi\left(\frac{LE}{\hbar c\pi}\right)^2\left(\frac{L\,dE}{\hbar c\pi}\right)$$

$$\Rightarrow g(E)dE = \frac{L^3 E\,dE}{\pi^2(\hbar c)^3} \qquad \hbar c = 2\pi hc \Rightarrow 8\pi(L/hc)^3E^2dE$$

$$n(E)dE = g(E)f_{BE}(E)dE = \frac{8\pi(L/hc)^3E^2dE}{e^{+\beta E}-1}$$ SINCE $e^{-\alpha}=1$

$$d\lambda = dE\cdot\left|\frac{d\lambda}{dE}\right|$$

IN DISCUSSION WHEN I DISCUSSED THIS PROBLEM I LEFT OUT THE ABSOLUTE VALUE SIGN. SO THE NEGATIVE SIGN ACTUALLY GOES AWAY.

$$\frac{d\lambda}{dE} = -\frac{hc}{E^2} \Rightarrow d\lambda = \frac{hc}{E^2}dE = \frac{dE}{hc} \Rightarrow dE = (hc\ \ ^2)d\lambda$$

$$u(E)dE = \frac{E}{L^3}n(E)dE = \frac{8\pi(L/hc)^3(hc/\lambda)^3(hc/\lambda^2)d\lambda}{L^3(e^{\beta hc/\lambda}-1)}$$

$$= \frac{8\pi hc/\lambda^5\,d\lambda}{e^{\beta hc/\lambda}-1} \Rightarrow u(\lambda) = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT}-1}$$