Physics 101B. Modern Physics. Professor Dine

Measurement in Quantum Mechanics

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1 Copenhagen

[**This handout is for fun, and "culture". You won't be tested on this.**]

Shortly after Schrodinger and Heisenberg proposed their formulations of quantum mechanics, an interpretation of the theory developed, centered around Niels Bohr's school in Copenhagen. After all, it was not obvious what the wave function meant, or Heisenberg's more abstract notion of "state."

Crucial to the thinking of the Copenhagen school, was that the only sensible questions were those which have, at least in principle, and operational, experimental meaning. So the notion of an electron's simultaneous momentum and position, for example, was not meaningful.

The basic ingredients in the interpretation were:

1. States: these are described by wave functions, ψ, or more abstractly by "state vectors," $|\psi>$. These more abstract state vectors may describe, not only a particle's position, but its spin, and perhaps other variables.

2. Operators: for every measurable quantity, there is associated an operator. Examples include the coordinate, $\vec{x}$; acting on a wave function, ψ, it multiplies the wave function by $\vec{x}$, i.e.

$$\vec{x}_{op}\psi(\vec{x}) = \vec{x}\psi(\vec{x}) \tag{1}$$

   and similarly the momentum:

$$\vec{p}_{op}\psi(\vec{x}) = -i\hbar\vec{\nabla}\psi(\vec{x}) \tag{2}$$

   and finally the spin; acting on a state of spin 1/2, and spin pointing the in $+z$ direction, it gives

$$S^2|\psi>= \hbar^2\frac{1}{2}(1+\frac{1}{2})|\psi> \qquad S_z|\psi>= \frac{1}{2}\hbar|\psi> \tag{3}$$

3. Any state can be written as a linear combination, with complex coefficients, of states which are eigenfunctions of the various observables. So, for example, in the case of the hydrogen atom, we know that states can be taken to be eigenstates of the energy, the total orbital angular momentum, the z component of the angular momentum, and the z-component of the spin. So any state of the system can be written:

$$|\Psi>= \sum_{n,\ell,m_\ell,m_s} C(n,\ell,m_\ell,m_s)|n,\ell,m_\ell,s,m_s> \tag{4}$$

4. The results of the process of measuring an observable, say L_z is to leave the system in one (and only one) eigenstate of that observable. The probability that the system will be in any particular state is given by the absolute square of the corresponding expansion coefficient, e.g.

$$|C(n,\ell,m_\ell,m_s)|^2 \tag{5}$$

So what was our wierd Stern-Gerlach story?

We started with a magnetic field in the z direction, which *measured* S_z by splitting the two beams. The two possible outcomes were $\pm\frac{\hbar}{2}$. We absorbed the beam with negative S_z. The beam with positive S_z was passed through a Stern-Gerlach apparatus in the x direction. This constitutes a measurement of S_z. The possible results are again $\pm\frac{\hbar}{2}$. We absorbed again the state of negative S_x. This leaves a state with positive S_x. It turns out that:

$$|S_x = \frac{1}{2} >= \frac{1}{\sqrt{2}}(|S_z = \frac{1}{2} > +|S_z = -\frac{1}{2} >). \tag{6}$$

So the expansion coefficients, C, in this problem, are both $\frac{1}{\sqrt{2}}$. So when we finally pass the system through another magnetic field in the z direction, half the beam emerges with spin up, half with spin down.

This problem is discussed in some detail in the Feynman lectures on physics, volume 3. This is not impenetrable (he gave these lectures for freshman at Cal Tech; admittedly, most of them were lost most of the time). Feynman also discusses other examples of this sort of phenomena. The K mesons are perhaps the most interesting. (Perhaps you can get Dave Dorfan to talk to you about this; he has actually done some of these experiments!).

2 Post-Copenhagen?

This measurement story is rather bizarre. There has been a lot of effort through the years to provide alternatives (e.g. some of you have heard about hidden variable theories; to date, these have all failed – they contradict). There have also been attempts which are more successful to avoid saying that the wave function “collapses into a particular state” upon measurement. But while the words are sometimes arguably more palatable, the physics remains the same.