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Physics 110 B

Homework Set 4

Solutions

Professor Dine

(2)

$$10.2 \quad \vec{E} = -\frac{\mu_0 k}{2} (ct - |x|) \hat{z} \quad |x| < ct$$

$$\vec{B} = \frac{\mu_0 k}{2c} (ct - |x|) \hat{y}$$

(a)  $t_1 = \frac{d}{c}$  : fields vanish in the region

$$t_1 = \frac{d+h}{c}$$

$$U = \frac{1}{2} \left[ \epsilon_0 \vec{E}^2 + \frac{1}{\mu_0} \vec{B}^2 \right]$$

$$= \frac{1}{2\mu_0} \left[ \mu_0 \epsilon_0 E^2 + \vec{B}^2 \right]$$

$$= \frac{\mu_0 k^2}{4} [ct - |x|]^2$$

$$\text{So } E = \frac{e \omega \mu_0 k^2}{4} \int_d^{d+h} dx [ct - x]^2$$

$$= -\frac{e \omega \mu_0 k^2}{4} \left[ \left( ct - \frac{d+h}{3} \right)^3 - \left( \frac{ct-d}{3} \right)^3 \right]$$

$$= \frac{\epsilon_0 \mu_0^2 \alpha^2 e \omega h^3}{12}$$

$$\vec{S}(x) = \frac{1}{\mu_0} (\vec{B} \times \vec{E}) = \frac{1}{\mu_0 c} E^2 [-\hat{z} \times [\pm \hat{y}]]$$

$$= \pm \frac{\mu_0 \alpha^2}{4c} (ct - |x|)^2 \hat{x}$$

(3)

For  $|x| < ct$ , energy flows in bottom

So

$$\frac{dW}{dt} = \phi = \int \vec{S}(d) \cdot d\vec{a} = \frac{\mu_0 \alpha^2 \ell w}{4c} (ct - d)^2$$

$$W = \int_{t_1}^{t_2} P dt = \frac{\mu_0 \alpha^2 \ell w}{4c} \int_{d/c}^{(d + \ell) / c} dt (ct - d)^3$$

$$= \frac{\mu_0 \alpha^2 \ell w \ell}{12c^2}$$

(4)

10.3.

$$V(\vec{r}, t) = 0 \quad \vec{A}(\vec{r}, t) = -\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = 0$$

This is the field of a point charge.

(5)

10.7

Suppose

$$\vec{\nabla} \cdot \vec{A} \neq -\mu_0 \epsilon_0 \frac{\partial V}{\partial t}$$

Call

$$\vec{\nabla} \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t} = f(t)$$

Look for a function  $\lambda$ , s.t.

$$\vec{\nabla} \cdot \vec{A}' + \mu_0 \epsilon_0 \frac{\partial V'}{\partial t} = 0$$

$$\vec{\nabla} \cdot (\vec{A} + \vec{\nabla} \lambda) + \mu_0 \epsilon_0 \left( \frac{\partial V}{\partial t} - \frac{\partial^2 \lambda}{\partial t^2} \right) = 0$$

or

$$\left[ \vec{\nabla}^2 - \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \right] \lambda = -f(t)$$

But we know how to solve for  $\lambda$ , using  
our Green's function:

$$\lambda = \int dt' G(\vec{x} - \vec{x}', t - t') f(t')$$

$$\vec{E}, \vec{B} = a\omega \left[ -\sin(\omega t_r) \cos(\omega t) \right. \\ \left. + \sin(\omega t_r) \cos(\omega t_r) = 0 \right]$$

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$$S_0 \left( 1 - \frac{\hat{n} \cdot \vec{\sigma}}{c} \right) = 1$$

$$V(z, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{z^2 + a^2}}; \quad A(z, t) = \frac{q\omega a}{4\pi\epsilon_0 c^2 \sqrt{z^2 + a^2}} \\ \times \left[ -\sin(\omega t_r) \hat{x} + \cos(\omega t_r) \hat{y} \right]$$

10.14 We need to develop the square root in 10.42

$$(c^2 t - \vec{r} \cdot \vec{v})^2 + (c^2 v^2)(r^2 - c^2 t^2)$$

$$= c^4 t^2 + (\vec{r} \cdot \vec{v})^2 - 2c^2 t \vec{r} \cdot \vec{v} + c^2 r^2 - v^2 r^2 \\ + v^2 c^2 t^2$$

$$\text{Compare } R^2 v^2 \sin^2 \theta = (\vec{r} \cdot \vec{v})^2 - R^2 v^2$$

$$= [(\vec{r} \cdot \vec{v} t) \cdot \vec{v}]^2 - (\vec{r} \cdot \vec{v} t)^2 v^2$$

$$= (\vec{r} \cdot \vec{v} - v^2 t)^2 - r^2 v^2 - v^4 t^2 + 2\vec{r} \cdot \vec{v} t v^2$$

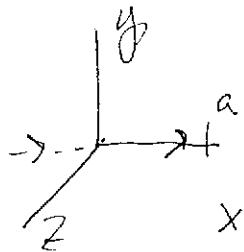
$$= (\vec{r} \cdot \vec{v})^2 - r^2 v^2$$

So

$$R^2 - R^2 \frac{v^2}{c^2} \sin^2 \theta = r^2 + v^2 t^2 - 2\vec{r} \cdot \vec{v} t - \frac{(\vec{r} \cdot \vec{v})^2}{c^2} + \frac{r^2 v^2}{c^2}$$

which is  $\frac{1}{c^2}$  times what we found above.

10.25



$$\vec{X} = \vec{V} t = \hat{X} v t$$

$$t' = 0 = t_r$$

$$\vec{S} = \epsilon_0 [\vec{E}^2 \vec{v} - \vec{v} \cdot \vec{E} \vec{E}]$$

$$d\vec{a} = 2\pi r dr \hat{X} \quad \text{so}$$

$$P = \epsilon_0 \int (E^2 v - E_x^2 v) 2\pi r dr$$

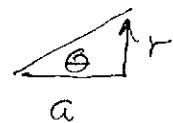
$$= \epsilon_0 \int (E^2 v \sin^2 \theta) 2\pi r dr$$

$$E = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2} \frac{\hat{R}}{\left[\left(1-\frac{r}{R}\right)^2 \sin^2 \theta\right]^{3/2}}$$

$$P = 2\pi\epsilon_0 v \left(\frac{q}{4\pi\epsilon_0}\right)^2 \frac{1}{r^4} \int_0^\alpha \frac{r \sin^2 \theta}{R^4 \left[\left(1-\frac{r}{R}\right)^2 \sin^2 \theta\right]^{3/2}} d\theta$$

$$r = \frac{a}{\cos \theta} \sin \theta$$

$$= a \tan \theta$$



$$P = \frac{v}{2\gamma^4} \frac{q^4}{4\pi\epsilon_0} \frac{1}{a^2} \int_0^{\pi/2} \frac{\sin^3 \theta \cos \theta}{[1 - (v/c)^2 \sin^2 \theta]^3} d\theta$$

This is an elementary integral. Let

$$u = \sin^2 \theta; du = 2 \sin \theta \cos \theta d\theta$$

gives

$$P = \frac{v q^4}{32\pi\epsilon_0 a^2}$$