Physics 110B. Electricity and Magnetism. Professor Dine

Spring 2006: FINAL EXAM. Take Home. Due Monday, June 10. 4:00 PM (Physics Office)

Problem 1. For an electromagnetic wave of the form:

$$\vec{E} = \vec{E}_o e^{ikz - i\omega t}, \tag{1}$$

use Maxwell's equations to: a. show that the polarization of the wave is perpendicular to its direction of motion.
b. find the relation between ω and k, and from this the velocity of the wave (phase velocity).
c. Find the relation between $\vec{E}$ and $\vec{B}$.

Problem 2. A particle moves in a circle in the $x - y$ plane.
a. Write a formula for the trajectory of the particle, assuming that the radius of the circle is ρ and the frequency of rotation is ω.
b. Compute the dipole moment of the particle, relative to the origin.
c. Compute the $\vec{E}$ and $\vec{B}$ fields far away from the formula for dipole radiation.
d. Compute the $\vec{E}$ and $\vec{B}$ fields from the formulas we derived for the $\vec{E}$ and $\vec{B}$ fields of a charged particle. Remember, radiation requires acceleration.
Problem 3. Work out the following in terms of ordinary three vectors and scalars: a. $x^\mu k_\mu$. If $k = (\omega/c, \vec{k})$, show that $e^{ix^\mu k_\mu}$ is the usual expression for a plane wave.
b. Show that $p^\mu p_\mu$ is $-m^2c^2$, for a particle of mass m.

Problem 4. A relativistic wave equation: The infamous Higgs boson obeys a wave equation similar to that for light. It is simpler, since there is just one field, ϕ. The equation is:

$$[-\vec{\nabla}^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \omega_o^2]\phi = 0. \tag{2}$$

Here ω_o is a constant.
a. Show that $e^{i(\vec{k}\cdot\vec{x} - \omega t)}$ solves the wave equation. What is the relation between ω and k?
b. What is the group velocity of the waves?
c. Now let's do some quantum mechanics and relativity together. If $p = \hbar k$, and $E = \hbar\omega$, what is $p^\mu p_\mu$? Interpret this in terms of the mass of the Higgs boson.
d. Given your result in part b, does the particle ever move faster than the speed of light?

Problem 5. An electromagnetic wave is described, in the gauge $\vec{\nabla} \cdot \vec{A} = 0$ by a vector potential of the form:

$$\vec{A}(\vec{x}, t) = \vec{A}_o e^{i(\vec{k}\cdot\vec{x} - \omega t)}. \tag{3}$$

The scalar potential vanishes, $V = 0$.
a. What does the gauge condition imply about $\vec{A}_o$?
b. Calculate the $\vec{E}$ and $\vec{B}$ fields. Check that they are transverse. Show that they obey the usual relations,

$$\vec{B} = \frac{1}{c}\hat{k} \times \vec{E}. \tag{4}$$

c. Express the energy density and the Poynting vector in terms of $\vec{A}$.

Problem 6. Derive a formula for ϵ for a conductor, assuming that the electrons in the conductor are free (there is no harmonic restoring force). Show that the real and imaginary parts of ϵ are equal in magnitude. Write a formula for the attenuation of a wave in the conductor with distance.