Physics 110B. Electricity and Magnetism. Professor Dine

Spring 2005: MIDTERM EXAM

Problem 1. For a spherical wave (a wave moving outward from the origin) the electric field behaves as $\vec{E} = \frac{A}{r}\hat{\phi}\cos(kr-\omega t)$. (A reminder of the form of the unit vectors in spherical coordinates appears in the picture below). (You can suppose that the wave is moving in empty space).

a. Use Maxwell's equations to determine $\vec{B}$. In spherical coordinates, you need:

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}.$$

b. Calculate the Poynting vector, $\vec{S} = \frac{1}{\mu_o}\vec{E}\times\vec{B}$. You'll need to work out $\hat{\theta}\times\hat{\phi}$. You can do that by drawing a picture.
c. Determine the energy per unit time flowing through a large sphere of radius R.

Problem 2. Consider a wave incident on a boundary between two dielectric media, as indicated in the figure below. The incident, reflected, and transmitted waves are indicated.

a. Derive the boundary conditions for the component of the electric field normal to the plane of incidence and parallel to the plane of incidence from Maxwell's equations (you don't need to solve them).
b. Show that in order that an equation of the form

$$Ae^{iax} + Be^{ibx} = Ce^{icx}$$

hold for all x, $a = b = c$ and $A + B = C$.
c. Use your result in part b, and the assumption that the fields in each region behave as $e^{i\vec{k}\cdot\vec{x}}$ to show that the angle of incidence is equal to the angle of reflection, and to prove Snell's law.

Problem 3. Consider the formula we derived for ϵ,

$$\epsilon = 1 + \frac{Nq^2}{m\epsilon_o}\sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}.$$

We can model a conductor as a collection of unbound charges. So just take all the f_i's to vanish except f_1, and take $\omega_1 = 0$.
a. Write a formula for k in the limit that ω is very small (keep only the dominant term!). Find the real and imaginary parts of k. Remember that $k^2 = \frac{\omega}{c}\epsilon$. You may also want to remember that

$$\sqrt{-i} = \frac{1-i}{\sqrt{2}}$$

b. What is the form of the wave (i.e. e^{ikz}). In particular, what is the absorption coefficient (recall that the absorption coefficient is defined in terms of the intensity; $I \propto e^{-\kappa z}$).

c. For large ω explain why the metal becomes transparent, and determine the speed of light in the material (use the formula above for ϵ.

Some Useful Formulas

1. Snell's law:

$$\frac{\sin(\theta_T)}{\sin(\theta_I)} = \frac{n_1}{n_2}$$

2.

$$n = \sqrt{\epsilon\mu} = \frac{1}{v}c$$

3. Maxwell's Equations in a medium:

$$\vec{\nabla} \cdot \vec{D} = \rho_f \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

where ρ_f and $\vec{J}_f$ are the free charge and current densities.