Physics 110B. Electricity and Magnetism. Professor Dine

Spring 2008: QUIZ 1

Problem 1. Using Gauss's theorem, explain why the equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$

implies that charge is conserved (reminder: need to integrate over a volume, V).

Solution: Integrating over a volume V, with surface S, and using Gauss's theorem:

$$\frac{d}{dt} \int_V d^3x \rho = -\int_S d^2a \hat{n} \cdot \vec{J}$$

The right hand side is the total flow of charge out of the volume (in Gauss's theorem, $\hat{n}$ is the outward pointing normal) so this equation says that the change of charge in the volume per unit time is equal to the total charge flowing out of the volume per unit time.

Problem 2. Verify, using the index approach, that for a constant magnetic field, $\vec{B}$, a suitable vector potential is

$$\vec{A} = -\frac{1}{2} \vec{x} \times \vec{B}$$

Solution: We should have

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

so, in components (i.e. with indices):

$$\begin{aligned} \vec{B}_i &= \epsilon_{ijk} \partial_j \frac{-1}{2} \epsilon_{klm} \partial_j \partial_k B_m \\ &= -\frac{1}{2} \left[\partial_j x_i B_j - \partial_j x_j B_i \right] \\ &= -\frac{1}{2} (1-3) B_i \\ &= B_i \end{aligned}$$

Problem 3. Verify that $A\cos(kx - \omega t + \delta)$ satisfies the wave equation in one dimension,

$$\frac{\partial^2 f}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

for a particular relation between k and ω (and determine that relation).

Solution:

$$\frac{\partial^2}{\partial t^2}\cos(kx - \omega t) = -\omega^2 \cos(kx - \omega t)$$

$$\frac{\partial^2}{\partial x^2}\cos(kx - \omega t) = -k^2 \cos(kx - \omega t)$$

so the equation is satisfied if $\omega = vk$.

Formulas You Might Need

1. $$(\vec{A} \times \vec{B})_i = \epsilon_{ijk} A_j B_k \tag{1}$$

2. $$\epsilon_{ijk}\epsilon_{k\ell m} = (\delta_{i\ell}\delta_{jm} - \delta_{im}\delta_{j\ell}) \tag{2}$$

3. $$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \vec{\nabla} \times \vec{A} \tag{3}$$