Physics 110. Electricity and Magnetism. Professor Dine

Spring, 2008. Handout: Some Integrals Useful in Homework

$$I = \int_{-\infty}^{\infty} \frac{dz}{(c+z^2)^{3/2}}.$$

This can be rewritten as:

$$I = -2\frac{d}{dc}\int_{-\Lambda}^{\Lambda} \frac{dz}{(c+z^2)^{1/2}}$$

where it is necessary to take the limit $\Lambda \to \infty$. Split up the integral into regions $|z| < a\sqrt{z}c$ and $|z| > a\sqrt{c}$. Then

$$I = -4\frac{d}{dc}\left(\int_0^{\sqrt{ac}} \frac{dz}{(c+z^2)^{1/2}} + \int_{\sqrt{ac}}^{\Lambda} \frac{dz}{(c+z^2)^{1/2}}\right)$$

If $a \gg 1$, then

$$I \approx -4\frac{d}{dc}\left(\int_0^{\sqrt{ac}} \frac{dz}{(c+z^2)^{1/2}} + \int_{\sqrt{ac}}^{\Lambda} \frac{dz}{z^{1/2}}\right)$$

The first integral in parenthesis is hard, but, by the change of variables $z = \sqrt{c}u$, it is readily seen to be independent of c, so the derivative of this term is zero. The second is $\ln(\Lambda/\sqrt{ac})$. So performing the derivative,

$$I = 2/c$$

(Professor Haber points out that this can be done by substituting $z = \sqrt{c}\sinh(u)$, and the identity $\sinh^2 + 1 = \cosh^2$).

Also useful:

$$\int_0^{2\pi} \frac{\cos\phi d\phi}{A + B\cos\phi} = \frac{2\pi}{B}\left(1 - \frac{A}{\sqrt{A^2 - B^2}}\right).$$

$$\int_0^{2\pi} \frac{1}{A + B\cos\phi} = \frac{2\pi}{\sqrt{A^2 - B^2}}.$$