Physics 214. Electricity and Magnetism. Professor Dine

Winter, 2011. Homework Set 2. Due Thurs. Feb. 3

Problem numbers refer to your textbook.

1. Jackson 11.3

   This problem is pretty straightforward

2. Jackson 11.6

   In this problem, one needs a handle on the idea that the acceleration is constant in the rest frame *at any instant*. In terms of the four acceleration,

   $$a^\mu = \frac{da^\mu}{d\tau} \tag{1}$$

   this means that

   $$a^2 = -g^2 \tag{2}$$

   (think of the acceleration as along the x direction). Another useful fact follows from the fact that $u^2 = 1$, from which it follows that

   $$a^\mu u_\mu = 0. \tag{3}$$

   Use these equations to write an equation for $\frac{du^x}{d\tau}$, and solve as a function of τ. Knowing u^x and $u^2 = 1$, you should be able to figure out how much time has elapsed on earth.

3. Jackson 11.17

   In this problem, the idea is to avoid doing complicated Lorentz transformations for the fields (eqns. 11.149), but to remember that $F_{\alpha\beta}$ is a tensor, and to try and write it in terms of quantities X^α, U^α, etc., which transform as tensors. In particular, if we can write an expression which takes the correct form in the rest frame. In the rest frame, the result should always be the Coulomb field for $\vec{E}$, and vanishing $\vec{B}$. So one wants to check whether these expressions reduce to the Coulomb expression in that frame. Some care is required in the different parts, as what is simultaneous in one frame is not simultaneous in another. it is important that, whatever one does, terms involving the time should drop out.

4. Jackson 11.18. In part c., you need consider only the first of the two vector potentials.

In this problem, you need to establish that in the limit of large γ, the expression for $\vec{E}$ involves a δ function. In particular, consider

$$\frac{\gamma}{(\gamma^2 x^2 + a^2)^{3/2}}. \tag{4}$$

Claim: this is proportional to $\delta(x)$ as $\gamma \to \infty$. To check this, you can graph the function, to see that it is highly peaked at $x = 0$. You can study

$$\int dx f(x) \frac{\gamma}{(\gamma^2 x^2 + a^2)^{3/2}} \tag{5}$$

and show that this is proportional to $f(0)$ plus terms of order $1/\gamma$.