Physics 217. Quantum Field Theory. Professor Dine

Fall, 2013. Homework Set 3

Solutions

1. Peskin 4.2

Solution: $\mathcal{L}_I = \mu\Phi\phi\phi$. Note that there are two fields, of different masses. Ignoring the interaction, the free field expansions would be:

$$\phi = \int \frac{d^3p}{(2\pi)^3\sqrt{2(p^2+m^2)}}[a_p e^{-ip\cdot x} + \text{c.c.}]. \quad \Phi = \int \frac{d^3p}{(2\pi)^3\sqrt{2(p^2+M^2)}}[A_p e^{-ip\cdot x} + \text{c.c.}] \tag{1}$$

Note the two different masses and the two types of creation and annihilation operators appearing in these expressions. Note that p^0 in the exponents of these two expressions means something different.

We need to study an amplitude of the type:

$$\langle p_1 p_2 | T(\mu\Phi\phi\phi)|k_\Phi\rangle \tag{2}$$

$$= \langle 0|a_{p_1}a_{p_2}T(\mu\Phi\phi\phi)A_k^\dagger|0\rangle\sqrt{2E_{p_1}2E_{p_2}2E_k}.$$

The generalization of our contraction rules are immediate;

$$\phi \hookrightarrow |p\rangle = 1 \quad \Phi \hookrightarrow |k\rangle = 1. \tag{3}$$

So in the amplitude, noting the two identical ϕ particles, we get $= 2\mu$. So the lifetime is:

$$\Gamma = \frac{1}{2}\frac{1}{2M}\int\frac{d^3p_1 d^3p_2}{(2\pi)^2}\delta^{(4)}(p_1+p_2-k)|\mathcal{M}|^2 \tag{4}$$

$$= \frac{1}{2M}\frac{4\mu^2}{2}\frac{1}{(2\pi)^2}\int d^3p_1\delta(2\sqrt{p_1^2+m^2}-M)\frac{1}{4E_{p_1}^2}.$$

Here, we have gone to the cm frame and integrated over $\vec{p}_2$. Remaining integral gives

$$4\pi\frac{p^2}{2\left(\frac{\partial E}{\partial p}\right)} = 2\pi E p. \tag{5}$$

So

$$\Gamma = \frac{\mu^2}{8\pi M}\frac{pE}{e^2} = \frac{\mu^2 p}{4\pi M^2}. \tag{6}$$

$$E = \frac{M}{2}; p = \sqrt{\frac{M^2-4m^2}{4}}.$$

2. Peskin 4.3 (a)

Solution:

$$\Phi^i = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}[a^i(p)e^{-ip\cdot x} + \text{c.c.}] \tag{7}$$

where $p^0 = \sqrt{\vec{p}^2 + m^2}$, i.e. we have N types of particles, all with the same mass.

$$[a_i, a_j^\dagger] = \delta_{ij}\delta(\vec{p} - \vec{p}')$$

Then, for contractions of fields, we have

$$\Phi^i(x) \hookrightarrow \Phi^j(y) = \delta^{ij} D_F(x - y). \tag{8}$$

To obtain the vertex, consider scattering of four particles of momenta p_1, p_2, k_1, k_2, carrying indices i, j, k, ℓ, respectively:

$$|p, i\rangle = \sqrt{2E_p} a^{i\dagger}(p)|0\rangle, \tag{9}$$

etc. So we need to study

$$-i\frac{\lambda}{8}\langle 0|a^k(k_1)a^\ell(k_2)(\Phi^m\Phi^m)(\Phi^n\Phi^n)a^{i\dagger}(p_1)a^{j\dagger}(p_2)|0\rangle. \tag{10}$$

Now the contraction of Φ^m with an external state is just the usual contraction of a scalar, except that it vanishes unless the state carries index m, i.e.

$$\Phi^m \hookrightarrow |k, i\rangle = \delta^{im}. \tag{11}$$

So to determine the vertex, all we need to do is list the possible contractions. These yield:

$$i\frac{\lambda}{8}8(\delta^{mk}\delta^{m\ell}\delta^{ni}\delta^{nj} + \delta^{mk}\delta^{mi}\delta^{n\ell}\delta^{nj} + \delta^{m\ell}\delta^{mi}\delta^{nk}\delta^{nj}) \tag{12}$$

where the factor of 8 arises because several contractions give the same result. So the vertex is, finally, as given in the problem.