Physics 218. Advanced Quantum Field Theory. Professor Dine

Winter, 2009. Homework Set 2. SOLUTIONS.

Generally people did very well with these problems; you are all now functioning at a high level with spinor cross sections and QED. These calculations are discussed at length in the handout and in chapter 5 of the text by Peskin and Schroeder, so instead, I will focus just on the last problem, and do it "from scratch". So let's isolate the most singular term in the scattering cross section.

Recall that there are two diagrams contributing to the Compton amplitude. Calling the initial and final fermion momenta p and p', and the initial and final photon momenta k and k', Define the scattering angles as in lecture, i.e. θ is the angle between the incoming fermion momentum and the outgoing final photon momentum, so $\theta = \pi$ corresponds to the incoming momentum parallel to the outgoing photon momentum.

Then

$$p \cdot k = 2E^2 = p' \cdot k';\ p \cdot k' = E^2(1 + \cos\theta);\ p' \cdot k = E^2(1 + \cos\theta). \tag{1}$$

The first of the Feynman diagrams, where the electron first absorbs the incoming photon and then emits the outgoing photon, has a denominator behaving as $p \cdot k$, and so is non-singular; the second is singular, and we will study its absolute square. This is

$$|M|^2 = e^4 \bar{u}(p') \not\epsilon(k)(\not p - \not k') \not\epsilon(k') u(p) \bar{u}(p) \not\epsilon(k')^* (\not p - \not k') \not\epsilon(k)^* u(p') \tag{2}$$

Averaging over initial spins and polarizations and summing over final spins and polarizations gives a simple expression:

$$\frac{e^4}{4} \mathrm{Tr}(\not p' \gamma^\mu (\not p - \not k') \gamma^\nu \not p \gamma_\nu (\not p - \not k') \gamma_\mu \tag{3}$$

At this point, we can make the simplification $\gamma^\mu \not p \gamma_\mu = -2 \not p$, etc., and use $p - k' = p' - k$ to rewrite this as:

$$e^4 \mathrm{Tr}(\not p' (\not p - \not k') \not p (\not p' - \not k) \tag{4}$$

Now we can use $\not p \not p = p^2 = 0$, etc., to rewrite this as simply:

$$e^4 \mathrm{Tr}(\not p' \not k' \not p \not k) = 4(p' \cdot k' p \cdot k + p' \cdot k p \cdot k' - p \cdot p' k \cdot k') \tag{5}$$

$$= e^4 E^4 (4 + 4\cos(\theta))$$

Now we can compute the differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{16E^4} \int \frac{d^3k}{(2\pi)^2} \delta(|\vec{p}| + ||k| - |\vec{p}'| - ||k'|) |\bar{M}|^2 \tag{6}$$

so, noting that the integral over the delta function gives an additional factor of 1/2:

$$\frac{d\sigma}{d\theta} = \frac{e^4}{32\pi E^2 (1 + \cos\theta)} \tag{7}$$

exhibiting the expected singular behavior at $\theta = \pi$, corresponding to a logarithmic sensitive of the total cross section to s/m^2.