Physics 218. Advanced Quantum Field Theory. Professor Dine

Winter, 2009. Homework Set 3. Due Wed, Feb. 18.

Problem numbers refer to your textbook.

1. Calculate the vacuum polarization in QED, using dimensional regularization and the Feynman parameter trick (don't do the Feynman parameter integral at this stage). Verify that the result is transverse.

   Solution: This appears in many texts, e.g. in Peskin and Schroeder. It is a quite straightforward exercise, and everyone did well with it.

2. Consider the limit $q^2 \to 0$. Write an expression for the renormalized coupling in terms of the bare coupling.

   Solution: This problem was meant to be simple, and just to get you thinking about the meaning of coupling constant renormalization. Using, in the small q^2 limit,

$$\Pi(q^2) = \Pi(0) = -\frac{4/3e^2}{16\pi^2}(\frac{2}{\epsilon} + \ln(\mu^2/m_e^2)) \tag{1}$$

   Note that in dimensional regularization, we have defined

$$e_0^2 = e^2\mu^\epsilon. \tag{2}$$

   We have, for example thinking about Coulomb scattering

$$\frac{e^2}{q^2} = \frac{e_0^2}{q^2}\frac{1}{1 + \frac{4/3e_0^2}{16\pi^2}(\frac{2}{\epsilon} + \ln(\mu^2/m_e^2))} \tag{3}$$

   from which we can read off the renormalized charge (we can, if we like, expand in powers of e^2; in the correction, we can use either e_0^2 or e^2, since these are the same to this order):

$$e^2 = \frac{e_0^2}{1 + \frac{4/3e_0^2}{16\pi^2}(\frac{2}{\epsilon} + \ln(\mu^2/m_e^2))}$$

3. Consider now the following slightly unrealistic problem. Imagine a world in which the muon is stable, and both the muon and proton are 1000 times more massive than their observed value. In this world, there is a stable $\mu - p$ atom. Discuss the modifications of the Coulomb force law you expect from the vacuum polarization due to the electron and muon. Note that in an atom, the typical momentum transfers are of order αm (where m is the lighter charged particle).

   Solution: Now we have two contributions to the vacuum polarization tensor.

$$\Pi(q^2) = -\frac{4/3e^2}{16\pi^2}(\frac{2}{\epsilon} + \ln(\mu^2/q^2)) + -\frac{4/3e^2}{16\pi^2}(\frac{2}{\epsilon} + \ln(\mu^2/m_\mu^2)) \tag{4}$$

where we have worked in the limit $m_e^2 \ll q^2 \ll m_\mu^2$. Note as a result the second term has significant dependence on $\vec{q}$, or in coordinate space, on $\vec{r}$.

For "muonium", a system with a muon bound to a proton, these corrections have actually been measured and calculated (one can not make the extreme approximation we have made above and the corrections must be evaluated numerically). The splittings of the first few levels are of order 10^{-2}-10^{-4}. (See, for example, Lifshitz and Pitaevski, *Relativistic Quantum Theory*, pp. 468-470.