Problem numbers refer to your textbook.

1. As a model for the $Z_0$ boson, consider a massive vector with a coupling to a massless spinor,

$$\mathcal{L}_I = g Z^\mu \bar{\psi} \gamma_\mu (1 - \gamma_5) \psi.$$  

Calculate the lifetime of $Z$ at lowest order in perturbation theory.

2. For the model of the previous problem, suppose that the field, $\psi$, carries electric charge. Discuss the corrections to the $Z$ lifetime to order $e^2$. Don’t actually compute them, but discuss the types of divergences which occur at order $e^2$. Interpret the ultraviolet divergences and explain what resolves the various infrared divergences. Is the lifetime finite in the next order?

3. Pions as Goldstone bosons in the strong interactions. As a model for the pions, define a field, $M$, which is a two by two matrix. Take the symmetry to be $SU(2)_L \times SU(2)_R$, where

$$M \rightarrow U_L M U_R$$  

where $U_L$ and $U_R$ are (distinct) $SU(2)$ matrices. Show that the lagrangian:

$$\mathcal{L} = \text{Tr} \ M^\dagger M + \frac{\mu^2}{2} \text{Tr} \ M^\dagger M - \frac{\lambda}{4} \text{Tr} \ (M^\dagger M)^2$$  

is invariant under the symmetry.

Show that at the minimum of the potential, $M$ has the form

$$M = \sigma_0$$  

(i.e. it is proportional to the unit matrix). Argue that an $SU(2)$ subgroup of the original symmetry group is preserved; this can be identified with ordinary isospin. Writing

$$M = \sigma_0 + \delta \sigma + \vec{\pi}(x) \cdot \vec{\sigma}$$  

show that the $\vec{\pi}$ fields are massless, and that they form a triplet of isospin.