Physics 218. Advanced Quantum Field Theory. Professor Dine

Winter, 2009. Homework Set 5. Due Wed, March 4.

Problem numbers refer to your textbook.

1. As a model for the Z_0 boson, consider a massive vector with a coupling to a massless spinor,

$$\mathcal{L}_I = g Z^\mu \bar{\psi} \gamma_\mu (1 - \gamma_5) \psi. \tag{1}$$

   Calculate the lifetime of Z at lowest order in perturbation theory.

2. For the model of the previous problem, suppose that the field, ψ, carries electric charge. Discuss the corrections to the Z lifetime to order e^2. Don't actually compute them, but discuss the types of divergences which occur at order e^2. Interpret the ultraviolet divergences and explain what resolves the various infrared divergences. Is the lifetime finite in the next order?

3. Pions as Goldstone bosons in the strong interactions. As a model for the pions, define a field, M, which is a two by two matrix. Take the symmetry to be $SU(2)_L \times SU(2)_R$, where

$$M \rightarrow U_L M U_R \tag{2}$$

   where U_L and U_R are (distinct) $SU(2)$ matrices. Show that the lagrangian:

$$\mathcal{L} = \text{Tr } M^\dagger M + \frac{\mu^2}{2} \text{Tr } M^\dagger M - \frac{\lambda}{4} \text{Tr } (M^\dagger M)^2 \tag{3}$$

   is invariant under the symmetry.

   Show that at the minimum of the potential, M has the form

$$M = \sigma_0 \tag{4}$$

   (i.e. it is proportional to the unit matrix). Argue that an $SU(2)$ subgroup of the original symmetry group is preserved; this can be identified with ordinary isospin. Writing

$$M = \sigma_0 + \delta\sigma + \vec{\pi}(x) \cdot \vec{\sigma} \tag{5}$$

   show that the $\vec{\pi}$ fields are massless, and that they form a triplet of isospin.