Physics 218. Advanced Quantum Field Theory. Professor Dine

Winter, 2009. Homework Set 6. Due Mon., March 16.

Problem numbers refer to your textbook.

1. Derive the Feynman rules for non-Abelian gauge theories (this is just an exercise; obviously you can find this in your text, Peskin and Schroeder, or in a sketchy form, in your class notes. You can use either the path integral form, or (in my view simpler) just use Wick's theorem, which follows from either the path integral or the operator approach).

2. Verify our calculation of the asymptotic freedom of unbroken, non-Abelian gauge theories. You can consider, as we did in class, the force between two heavy quarks, or consider alternative approaches.

3. Before the discovery of "neutral currents" in the early 1970's (processes mediated by gauge bosons) an interesting alternative to the $SU(2) \times U(1)$ model was a theory written down by Georgi and Glashow (and known as the Georgi-Glashow model). In this theory, the gauge group is simply $SU(2)$, and the Higgs fields form a triplet of the gauge group, ϕ_a (real; you might also want to write this as $\vec{\phi}$).

   (a) Write a potential which leads to symmetry breaking (i.e. so that the theory lies in the Higgs phase). Write the form of the solution. Show that the unbroken gauge symmetry is $U(1)$.

   (b) Determine the spectrum of the theory. Thinking of the $U(1)$ as electromagnetism, show that the states have definite charge.

   (c) Write the Feynman rules for this theory in the 't Hooft-Feynman gauge.

   While the Georgi-Glashow theory does not describe nature, it is an interesting model. One of its most interesting features (which we will talk about in Physics 222) is that the *classsical* solutions of this theory have finite energy, static solutions which describe *magnetic monopoles.* It is the simplest field theory with this feature.

4. Write the fermionic couplings, in some detail, for a two generation version of the standard model (i.e. $e, \mu, \nu_\mu, \nu_\tau, u, c, d, s$, couplings in particular to W and Z, and to Higgs). Verify that the KM mixing involves a single angle, known as the Cabibo angle. If it is not familiar to you, check in the particle data group listings the value of this angle; look over the general form of the CKM matrix for three generations, and familiarize yourself with the experimental numbers.