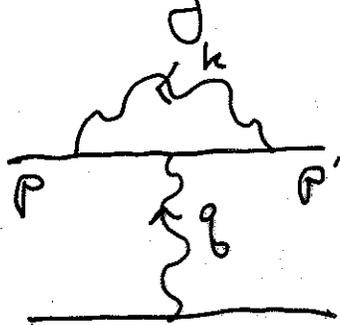


# Infrared Divergences in QED (and eventually QCD)

$$k^0 = -i\epsilon$$

Vertex again:



Focus on vertex. Look at  $k \rightarrow 0$  region of integration

$$\Gamma^\mu \approx - \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma^3 (\not{p}' + m) \gamma^\mu (\not{p} + m) \gamma_3}{(k^2) (2p \cdot k) (2p' \cdot k)}$$

$$= -4 \int \frac{d^4 k}{(2\pi)^4} \frac{p^3 p'_3 \gamma^\mu}{(k^0) (2p \cdot k) (2p' \cdot k)}$$

(Dirac equation in last step)

Let's do  $k^0$  integral. Denominators

$$(k^0 - \vec{k}^2 + i\epsilon) (2p^0 k^0 - 2\vec{p} \cdot \vec{k} + i\epsilon) (2p'^0 k^0 - 2\vec{p}' \cdot \vec{k} + i\epsilon)$$

(2)

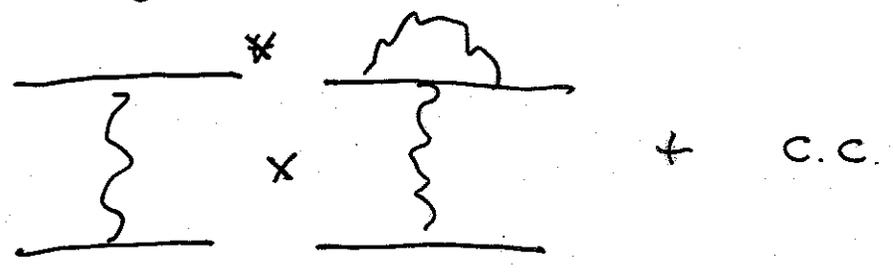
Take  $p^0, p'^0 > 0$ . Poles in 2<sup>nd</sup> & 3<sup>rd</sup> factors in lower half plane. Close ~~bet~~ above!

$$k^0 = -|\vec{k}| + i\epsilon$$

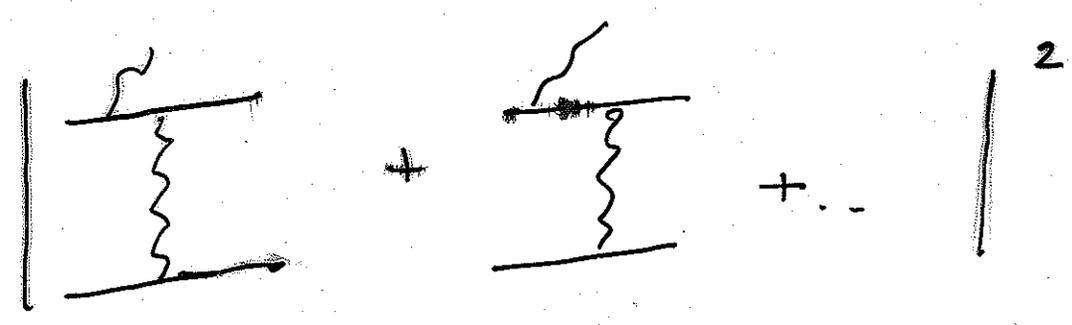
$$T^{\mu\nu} \approx -4i \int \frac{d^3k}{(2\pi)^3} \frac{\gamma^\mu (p \cdot p')}{(2|k|)(2p \cdot k)(2p' \cdot k)}$$

Diverges as  $k \rightarrow 0$ . No excuses. What cancels?

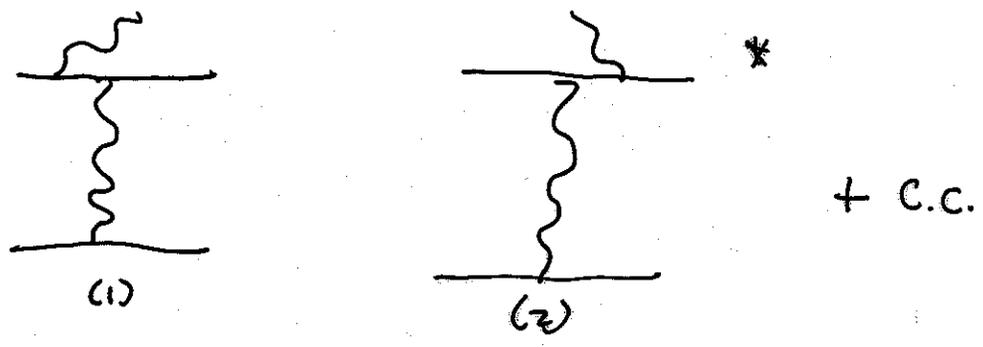
Study cross section



Compare:



Look esp. at interference:



$$M^{(1)} = \frac{\bar{u}(p') \gamma^\mu (\not{p} - \not{k} + m) \not{\epsilon}(k) u(p)}{-2p \cdot k} \times [\text{stuff}]$$

$$\approx \frac{2p \cdot \epsilon}{-2p \cdot k} \bar{u}(p') \gamma^\mu u(p) \times [\text{stuff}]$$

$$M^{(2)} \approx \frac{p' \cdot \epsilon}{2p' \cdot k} \bar{u}(p') \gamma^\mu u(p) \times [\text{stuff}]$$

Squaring (taking interference term) summing over photon polarizations:

$$\frac{d\sigma}{dR} \approx \left( \frac{d\sigma}{dR_0} \right) e^2 \int \frac{d^3k}{(2\pi)^3} \frac{2p \cdot p'}{(p \cdot k)(p' \cdot k)} \frac{1}{(2|k|)}$$

⇒ The divergence cancels divergence in cross section. High energy limit: diverges for  $k \approx p, k \approx p'$

$$\left( \frac{d\sigma}{dR} \right)_0 \approx \frac{\alpha}{\pi} \ln\left(\frac{E}{E_r}\right) \ln\left(\frac{E}{m}\right)$$

"Sudakov double logarithm"

In general, sensitive to expt'l situation

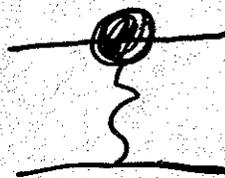
What about bd states? No emission

Log of binding energy.

"Beth log" in Lamb shift.

Effective coupling at vertex: (non-rel. limit)

$$\bar{u}(p') \gamma_\mu \left[ 1 + \frac{\alpha}{3\pi} \frac{q^2}{m^2} \ln\left(\frac{m}{k_{\min}}\right) + \dots \right] u(p)$$

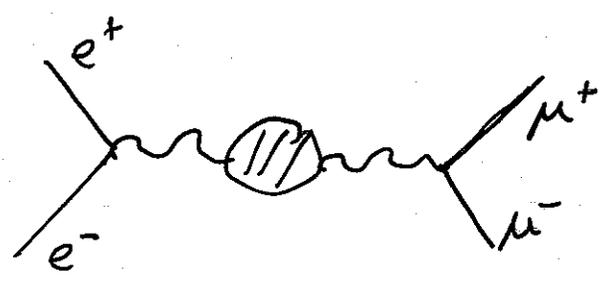

$$\equiv -\frac{Z\alpha}{r} + \frac{4\alpha}{3} \frac{Z\alpha}{m^2} \delta(r) \ln\left(\frac{m}{E_b}\right)$$

Splits  $S_{1/2}$  &  $P_{1/2}$  (degenerate in Dirac theory)

$$\Delta E_n = \frac{4\alpha}{3} \frac{Z\alpha}{m^2} |\psi_{\text{non}}(0)|^2 \left[ \ln(\alpha^{-1}) + \mathcal{O}(1) \right]$$

$Z^0$  line shape:

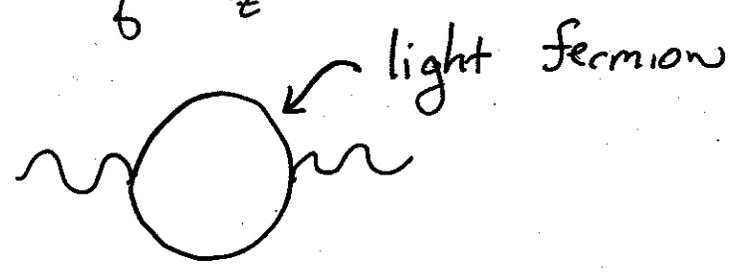
brings our various one-loop analysis together.



Need full standard model to obtain complete answer. Treat  $Z^0$  like massive photon

$g Z^0 \bar{\psi} \gamma_\mu \psi$  couplings

$$\frac{-g_{\mu\nu}}{q^2 - M_Z^2} : \text{bare propagator.}$$



$$\begin{aligned} \Upsilon(q^2) &= \frac{8g^4}{16\pi^2} \int d\alpha \left[ \frac{2}{\epsilon} - \ln(-q^2 \alpha(1-\alpha)) \right] \\ &= \Upsilon_R + i\Upsilon_i \quad \Upsilon_i = \frac{g^2}{4\pi} \end{aligned}$$

Why an imaginary part?

$$Z_0 \rightarrow f + \bar{f}$$

( $q^2$  near  $M_Z^2$ ).

[unitarity; as for optical theorem.

Or: KL rep: integration over spectral function]

$\Gamma_R$ : absorbed into renormalization of charge.

$$\text{So } \mathcal{M} \propto \frac{Z}{q^2 - M_Z^2 + i\Gamma}$$

— Breit-Wigner.

$$\text{[note on KL: } \int_{M_0^2}^{\infty} \frac{dM^2 \rho(M^2)}{q^2 - M^2 + i\epsilon}$$

$$q^2 > M_0^2$$

$$\text{In part: } 2\pi \delta(q^2) ]$$

But line shape not exactly BW.



Double logs, so corrections not so small.

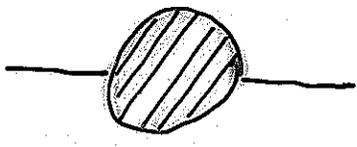
# Systematics of Renormalization

- An Introduction

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} (i\not{\partial} - m_0) \psi - e_0 \bar{\psi} \gamma_\mu \psi A^\mu$$



$$= -\frac{iZ_3 g_{\mu\nu}}{q^2}$$



$$= \frac{iZ_2}{\not{p} - m}$$

Define  $A^\mu = Z_3^{1/2} A_\mu^r$

$$\psi = Z_2^{1/2} \psi^r$$

$$e_0 Z_2 Z_3^{1/2} = e Z_1$$

$$\mathcal{L} = -\frac{1}{4} Z_3 (F_r^{\mu\nu})^2 + Z_2 \bar{\psi}^r (i\not{\partial} - m_0) \psi^r - e_0 Z_2 Z_3^{1/2} A_\mu^r \bar{\psi}^r \gamma^\mu \psi^r$$

Now idea is to perturb in  $e_0$  with nice propagators.

So define:

$$\delta_3 = Z_3 - 1; \quad \delta_2 = Z_2 - 1; \quad \delta_m = Z_2 m_0 - m$$

$$\delta_1 = Z_1 - 1 = e_0/e (Z_2 Z_3)^{1/2} - 1$$