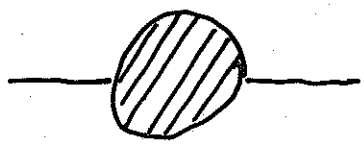
Systematics of Renormalization

– An Introduction

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}(i\not{\partial} - m_0)\psi - e_0 \bar{\psi}\gamma_\mu \psi$$

$$\equiv -\frac{i Z_3 g_{\mu\nu}}{q^2}$$



$$= \frac{i Z_2}{\not{p} - m}$$

Define $A^\mu = Z_3^{1/2} A_\mu^r$

$$\psi = Z_2^{1/2} \psi_r$$

$$e_0 Z_2 Z_3^{1/2} = e Z_1$$

$$\mathcal{L} = -\frac{1}{4} Z_3 (F_r^{\mu\nu})^2 + Z_2 \bar{\psi}_r (i\not{\partial} - m_0)\psi_r - e_0 Z_2 Z_3^{1/2} A_\mu^r \times \bar{\psi}_r \gamma^\mu \psi_r$$

Now idea is to perturb in e, with nice propagators.

So define:

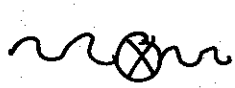
$$\delta_3 = Z_3 - 1; \quad \delta_2 = Z_2 - 1 \quad \delta_m = Z_2 m_0 - m$$

$$\delta_1 = Z_1 - 1 = e_0/e\,(Z_2 Z_3)^{1/2} - 1$$

By design, propagators for <u>renormalized</u> fields are simple:

$$\frac{-i g_{\mu\nu}}{q^2}, \qquad \frac{i}{\not{p} - m}$$

Treat δ's as perturbations; extra vertices



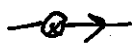
$$= -i\left(g^{\mu\nu} q^2 - q^\mu q^\nu\right)\delta_3$$

$$i\left(\not{p}\,\delta_2 - \delta_m\right)$$

$$-ie\gamma^\mu \delta_1$$

Counterterms chosen so that

$\Pi(q^2=0) =$  $+$  $= 0$

$\Sigma(\not{p}=m) =$  $+$  $= 0$

$$\frac{d}{d\not{p}}\,\Sigma(\not{p})\Big|_{\not{p}=m} = 0$$

$-ie\Gamma^\mu(q=0) = -ie\gamma^\mu$