Systematics of Renormalization

- An Introduction

\[ L = - \frac{1}{4} F_{\mu \nu}^2 + \bar{\psi} (i \gamma \cdot \mathbf{D} - m_0) \psi - \bar{\psi} \gamma^\mu \gamma^\nu \psi \]

\[ \text{Define } A^\mu = Z_3 \gamma^\mu A_\mu^r \]

\[ \gamma = Z_2 \frac{1}{2} \gamma^r \]

\[ e_0 Z_2 \gamma^r = e \bar{\psi} \]

\[ L = - \frac{1}{4} Z (F_\mu^\nu)^2 + Z \bar{\psi} r (i \gamma \cdot \mathbf{D} - m_0) \psi - e_0 Z_2 \bar{\psi} \gamma^\mu \gamma^\nu \psi \]

Now idea is to perturb in \( e_3 \) with nice propagators.

So define:

\[ \delta_3 = Z_3 - 1 \quad \delta_2 = Z_2 - 1 \quad \delta_m = Z_2 m_0 - m \quad \delta_1 = Z_1 - 1 = e_0 / (Z_2 Z_3)^{1/2} - 1 \]
By design, propagators for renormalized fields are simple:

\[-i \frac{\square}{q^2}, \frac{i}{q} \phi_{-m} \]

Treat δ's as perturbations; extra vertices

\[ \square \phi \phi = -i (q^\mu q^\nu - g^{\mu\nu}) \delta \]

\[ i (\phi \delta \phi - \delta \phi) \]

\[ -i e Y^A \phi \]

Counterterms chosen so that

\[ \Pi(q^2=0) = m \phi \phi + m \square \phi = 0 \]

\[ \Sigma(\phi=m) = m + m \phi \phi = 0 \]

\[ \frac{d}{d\phi} \Sigma(\phi) \bigg|_{\phi=m} = 0 \]

\[ -ie \Gamma^m (q=0) = -ie Y^A \]

\[ + \]

\[ - \]

\[ + \]

\[ - \]