Physics 222. Quantum Field Theory 3. Professor Dine

Spring, 2011. Homework Set 2. **Solutions**.

1. Do the exercises in the note on two-component spinors.

   Solution: This is reasonably straightforward. A couple of points are important and worth noting. There is a close parallel with the theory of a complex scalar. Here one writes:

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega(k)}} \left(a(k) e^{-ik\cdot x} + b^\dagger(k) e^{ik\cdot x} \right). \tag{1}$$

   a is an annihilation operator for particles, $b^\dagger$ a creation operator for antiparticles.

   Similarly, for a complex, two-component spinor, one has:

$$\chi(x)_\alpha = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega(k)}} \left(a(k) u_\alpha e^{-ik\cdot x} + b^\dagger(k) v_\alpha e^{ik\cdot x} \right) \tag{2}$$

$$\chi^\dagger(x)_{\dot\alpha} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega(k)}} \left(a^\dagger(k) u^*_{\dot\alpha} e^{ik\cdot x} + b(k) v^*_{dot\alpha} e^{-ik\cdot x} \right) \tag{3}$$

   u and v are positive and negative solutions of the *Weyl* equation; they have opposite helicity. So $a^\dagger$ and $b^\dagger$ create particles of the *opposite* helicity, e.g. left-handed, right-handed neutrinos. **As some of you noted, this can all be inferred by starting with four component spinors in the Weyl basis.**

   Also worth noting is the structure of the kinetic terms in the lagrangian. Working these out starting from four component spinors yields

$$\mathcal{L} = i\chi\sigma^\mu\partial_\mu\chi^* + i\phi^*\bar\sigma^\mu\partial_\mu\phi. \tag{4}$$

   We have argued that the χ, ϕ fields are equivalent, in the sense of behavior under Lorentz transformations (they may be associated with fields with different charges, etc.). We can see this explicitly. Define $\tilde\phi = \sigma^2\phi$. Then take reverse the order of the ϕ, ϕ^* terms; using the fact that $\sigma^2\sigma^{iT}\sigma = \sigma^i$, and similarly $\sigma^2\sigma^0\sigma^2 = \sigma^0$, and noting that there are two sign reversals, one following from the anticommutation relations of the ϕ's, and one from integrating by parts, we see that the second term is just:

$$i\tilde\phi\sigma^\mu\partial_\mu\tilde\phi^*. \tag{5}$$

   One further remark is worthwhile, especially for those of you who know something about supersymmetry, and for our upcoming discussions of supersymmetry. In addition to spinors expanded in terms of a and $b^\dagger$, we can define spinors with a only (and their conjugates with $a^\dagger$). These are called "Majorana spinors", examples of which will be the gauginos of supersymmetry.

2. Do the exercises in the note on the Coulomb gauge.

   Solution: This is reasonably straightforward and everyone who attempted did ok with this (some people had trouble with the signs of the A^i propagator; each component here is like a scalar field, i.e. they create positive energy states, so signs should be the same). This is a good exercise to do; it is a pedestrian but insightful way to understand why we can use the nice, covariant form for the propagator in electrodynamics.

3. Exploit transversality to work out the gauge boson propagator for a non-abelian gauge theory in Feynman gauge (much of this we did in class).

   Solution: This is essentially done in your textbook and caused no difficulty.

4. PS 16.2.

   This caused no difficulty. The vertex for one gauge boson is just that of scalar electrodynamics, which you have encountered before, except for the matrix T^a. For two gauge bosons interacting with the scalar, one gets a T^a for each boson,

   Solution: This caused no difficulty. The vertex for one gauge boson is just that of scalar electrodynamics, which you have encountered before, except for the matrix T^a. For two gauge bosons interacting with the scalar, one gets a T^a for each boson,