1. Consider a supersymmetric version of the $SU(5)$ grand unified theory. Take $\Sigma$ to be a chiral superfield in the adjoint representation, and take the superpotential to be

$$W(\Sigma) = m \text{Tr} \Sigma^2 + \frac{\lambda}{3} \text{Tr} \Sigma^3.$$  

(1)

Verify that there are (up to gauge transformations) three stationary points:

$$\Sigma = 0; \quad \Sigma = \frac{m}{\lambda} \text{diag}(1, 1, 1, 1, -4); \quad \Sigma = \frac{m}{\lambda} \text{diag}(2, 2, 2, -3, -3)$$

(2)

What is the gauge symmetry in each of these vacua?

2. Consider a $U(1)$ gauge theory, with a neutral field, $X$, and two charged fields, $\phi^\pm$.

a. Show that the $D$ terms vanish if $\phi^+ = \phi^- = v$ in the vacuum, i.e. that there is a one complex parameter set of vacuum states.

b. For fixed $v$, compute the spectrum. Basically you should find a massive gauge field, a massive Dirac fermion, arising from the Yukawa couplings between the gaugino and the fermionic components of $\phi^+$ and $\phi^-$ ($g\sqrt{2}\lambda(\phi^+\psi^+ - \phi^-\psi^-)$), and one more massive scalar. This scalar arises from expanding $D$ about the vacuum; you should find

$$D \propto v \Phi$$

where $\Phi$ is a (real) scalar field; the square of this is a mass term for $\Phi$. 