Physics 222. Quantum Field Theory 3. Professor Dine

Spring, 2011. Homework Set 5. Due Tues, June 14.

1. Consider a supersymmetric version of the $SU(5)$ grand unified theory. Take Σ to be a chiral superfield in the adjoint representation, and take the superpotential to be

$$W(\Sigma) = m \mathrm{Tr}\Sigma^2 + \frac{\lambda}{3}\mathrm{Tr}\Sigma^3. \tag{1}$$

Verify that there are (up to gauge transformations) three stationary points:

$$\Sigma = 0;\ \Sigma = \frac{m}{\lambda}\mathrm{diag}(1,1,1,1,-4);\ \Sigma = \frac{m}{\lambda}\mathrm{diag}(2,2,2,-3,-3) \tag{2}$$

What is the gauge symmetry in each of these vacua?

2. Consider a $U(1)$ gauge theory, with a neutral field, X, and two charged fields, $\phi^{\pm}$.

a. Show that the D terms vanish if $\phi^+ = \phi^- = v$ in the vacuum, i.e. that there is a one complex parameter set of vacuum states.

b. For fixed v, compute the spectrum. Basically you should find a massive gauge field, a massive Dirac fermion, arising from the Yukawa couplings between the gaugino and the fermionic components of ϕ^+ and ϕ^- ($g\sqrt{2}\lambda(\phi^{+*}\psi^+ - \phi^{-*}\psi^-)$, and one more massive scalar. This scalar arises from expanding D about the vacuum; you should find

$$D \propto v\Phi$$

where Φ is a (real) scalar field; the square of this is a mass term for Φ.