DUE: TUESDAY MARCH 9, 2010

To receive full credit for the following problems, you must exhibit the intermediate steps that lead you to your final results. The nth problem in Boas from section $a.b$ is designated by $a.b$–n.

1. This problem is inspired by problem 3.11-33 of Boas on p. 159.

(a) Compute the eigenvalues of the symmetric 2×2 matrix

$$A = \begin{pmatrix} a & c \\ c & b \end{pmatrix},$$

where a, b and c are arbitrary *real* numbers.

(b) Show that the eigenvalues of A are real and the eigenvectors are perpendicular.

(c) A 2×2 real orthogonal matrix S with unit determinant must have the following form:

$$S = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

Find an expression for θ in terms a, b and c such that

$$S^{-1}AS = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix},$$

where λ_1 and λ_2 are the eigenvalues obtained in part (a).

HINT: Derive an expression for $\tan 2\theta$. Can you determine which quadrant the angle θ lives in?

2. Boas, p. 159, problem 3.11–30.

3. Boas, p. 160, problem 3.11–42.

4. Boas, p. 161, problem 3.11–54.

5. Boas, p. 161, problem 3.11–58.

6. Consider the matrix

$$M = \begin{pmatrix} 0 & b \\ 0 & a \end{pmatrix},$$

where a and b are arbitrary complex numbers.

(a) Compute the eigenvalues of M.

(b) Find a matrix C such that $C^{-1}MC$ is diagonal.

(c) Compute e^M.

HINT: Denote $D = C^{-1}MC$ where D is the diagonal matrix obtained in part (b). Show that

$$e^M = e^{CDC^{-1}} = Ce^D C^{-1}. \tag{1}$$

Employing the results of parts (a) and (b), first evaluate e^D and then use eq. (1) to compute e^M.

(d) Verify that $\det(e^M) = e^{\mathrm{Tr}\, M}$.

7. Boas, p. 161, problem 3.11–60.

8. Boas, p. 171, problem 3.12–4.

9. Boas, p. 171, problem 3.12–9. Carry this out *only* for Boas problem 3.12–4.

10. Boas, p. 172, problem 3.12–16.

11. Boas, p. 184, problem 3.14–15.

12. Boas, p. 184, problem 3.14–16.