INSTRUCTIONS: This is a one-hour exam. During the exam, you may refer to the textbook, the class handouts (including solution sets to homework and practice problems) or your own personal notes. Collaboration with your neighbor is strictly forbidden.

In answering the questions, it is not sufficient to simply give a final result. You must provide the intermediate steps needed to arrive at the final solution in order to get full credit. The point value of each problem is indicated in brackets. The total number of points available is 100.

1. [20] Consider the real valued function:

$$g(x) = \left(\frac{3}{x^3} - \frac{1}{x}\right)\sin x - \frac{3}{x^2}\cos x\,.$$

(a) Compute $\lim\limits_{x\to 0} g(x)$.

(b) Find the *behavior* of $g(x)$ as $x \to 0$.

2. [30] Consider the real-valued function:

$$f(x) = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right)\,.$$

(a) Determine the Taylor series expansion of $f(x)$ about the point $x = 0$. Write the series using summation notation (that is, you will need to determine the general term in the series).

(b) Determine all possible values of x for which the series obtained in part (a) converges.

(c) Evaluate explicitly the sum

$$\sum_{n=0}^{\infty} \frac{1}{2^{2n}}\,\frac{1}{2n+1}\,.$$

Use your calculator to compute the sum of the first four terms of the series, and compare this numerical approximation with the exact result.

HINT: You may find the results of part (a) helpful in this regard.

3. [30] Evaluate the following quantities. If complex, express the quantity in $x + iy$ form. If the quantity is multi-valued, you should provide all possible values.

(a) 1^{π}

(b) $\mathrm{Arg}(\sin i)$

(c) $\mathrm{Im}\ \ln(i - 1)$

4. [20] Assume that p is a real parameter such that $-1 < p < 1$.

(a) Compute the following sum:

$$\sum_{n=0}^{\infty} p^n e^{in\theta} .$$

(b) Using the results of part (a), compute the sum

$$\sum_{n=0}^{\infty} p^n \cos(n\theta) .$$

Verify that your result for the sum in part (b) has the correct form in the $\theta \to 0$ limit.