CERN 77-17
Theory Division
1 September 1977

ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE
CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

THE NATURE OF INTRINSIC MAGNETIC DIPOLE MOMENTS

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ERRATUM TO CERN 77-17

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On p. 12, the second footnote should read

**) Dress, Miller and Ramsey[22] give the ratio of electric dipole moment to the proton's charge as less than 10^{-23} cm. Subsequent private communications from Ramsey give less than $\underline{3 \times 10^{-24}\ \text{cm}}$ (as of the end of 1976).

On p. 16, the second and third lines should read

... For the neutron, Ramsey and collaborators are already at the level of a few times $\underline{10^{-24}\ \text{cm}}$, as was noted in Section 4. ...

Geneva - 25 April 1978

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THE NATURE OF INTRINSIC MAGNETIC DIPOLE MOMENTS

J.D. Jackson*)

A lecture given in the
CERN 1977 Summer Student Lecture Programme under the title:
"What can the famous 21 cm astrophysical spectral line of atomic hydrogen
tell us about the nature of magnetic dipoles?"

G E N E V A

1977

*) On leave from the University of California, Berkeley, USA, 1976-1977

CERN - Service d'Information scientifique - RD/269 - 2600 - septembre 1977

ABSTRACT

Although isolated magnetic poles have so far not been unambiguously observed, the notion recurs that they might exist in magnetically neutral groups, bound together to form magnetic dipoles. Perhaps the intrinsic magnetic moments of fundamental particles are just such dipoles. Using only basic ideas of electricity and magnetism and elementary quantum mechanics, a unified pedagogical discussion is given of the hyperfine structure of atomic s-states and the scattering of slow neutrons by magnetic media. All known intrinsic magnetic moments (of electron, muon, proton, neutron, nuclei) are shown to be caused, to a very high precision, by circulating electric currents and not by magnetic charges.

CONTENTS

1. INTRODUCTION

Before having anything more than an intuitive idea of magnetism, most of us are aware of magnetic "poles", the earth's north magnetic pole or the poles of a child's bar magnet (usually painted red, with unpainted ends), for example. Only much later, and often after demonstrations of the lines of magnetic force emanating from magnetic poles, are we led gradually to the idea that isolated magnetic poles (in the sense of isolated electric charges) do not exist. What about magnets then? We are shown that a small loop of current (Amperian loop) produces a magnetic field that has a dipole character at distances from the loop large compared with its dimensions. Atoms have charges orbiting inside them. These charges produce effective loops of current and hence magnetic dipole moments. In bulk, with an applied field, they account for the magnetic properties of most matter. For iron and other ferromagnetic materials we are told that there is actually another source of atomic magnetism -- the *intrinsic* magnetic moment of an electron. For atoms with unpaired electrons, this intrinsic moment always contributes as well as the Amperian currents. Later still, we learn that the proton and neutron also have intrinsic magnetic moments, as do all nuclei with non-zero spin and also the muon. Usually the books are a little vague about the nature of these intrinsic magnetic moments, letting the word "intrinsic" imply that it is beyond the realms of present knowledge or none of your business, or both.

It is common knowledge, then, that magnetic phenomena are caused, not by magnetic poles or charges, but by circulating electric currents and intrinsic magnetic (dipole) moments of fundamental particles. The absence of magnetic charges is codified in the Maxwell equations in the statement, $\vec{\nabla}\cdot\vec{B} = 0$ (in contrast with $\vec{\nabla}\cdot\vec{D} = 4\pi\rho_e$ for electric charges).

Certainly to the present day, no one has observed in a definitive way an isolated magnetic charge*). But what about groups of magnetic charges, north and south poles, bound together? Could the intrinsic magnetic moments of fundamental particles be caused by such bound poles? Why not? The Maxwell equations would look more symmetric with magnetic charge and current densities as well as the conventional terms. In the present era of confined quarks it seems acceptable, to say the least, to postulate magnetic charges that are never seen singly but only in groups with vanishing total magnetic charge. Why not?

Well, one reason is that the 21 cm hyperfine line of atomic hydrogen, famous in astrophysics, would occur at 42 cm instead! Another is that a neutron scattering in iron would respond to $\vec{H}$ instead of $\vec{B}$, as is observed. In short, experiment tells us that *all* magnetic moments are caused by circulating currents. Not only are there no isolated magnetic charges, but there are none bound in pairs (or more) either**).

The purpose of this paper is to describe in essentially classical terms the positive evidence that intrinsic magnetic dipole moments are caused by circulating currents. The reader who objects that this is all well known is in a small minority among physicists, at least to my knowledge. A member of this minority, Casimir, had occasion in 1962 to complain of the prevalent ignorance and confusion on this point[5]. Magnetic poles have a long history

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*) The famous magnetic monopole event of Price et al.[1] has alternative explanations. See Friedlander[2] and Hagstrum[3] and references cited therein.

**) The question of possible particles possessing both electric and magnetic charges is touched on in Section 4 and the Appendix. Schwinger's dyons are an example[4].

in electromagnetism (in the definition of magnetostatic units and as theoretical constructs). Because of Dirac's early brilliant argument on the quantization of electric charge[6)*)], many wish their discovery. They may indeed be found, but they can play no significant role in the magnetic moments of ordinary particles, as we now demonstrate.

2. HYPERFINE STRUCTURE

2.1 For atomic states with non-vanishing orbital angular momentum

The magnetic field of a magnetic dipole $\vec{m}$, located at the origin of coordinates, is

$$\vec{B}_1(\vec{r}) = \frac{3\hat{r}(\hat{r}\cdot\vec{m}) - \vec{m}}{r^3} , \tag{1}$$

where r is the distance from the origin and $\hat{r}$ is a unit vector in the radial direction. If the magnetic moment $\vec{m}$ is of finite extent, Eq. (1) only holds at distances large compared to the extent of the currents or distribution of magnetic charges producing $\vec{m}$. For an electric dipole $\vec{d}$ (made from a distribution of charges, we know), the electric field has exactly the same form as Eq. (1), with $\vec{d}$ replacing $\vec{m}$, again apart from near the distribution producing the dipole. Evidently a dipole field appears the same outside the region of the source, whether caused by a distribution of magnetic charge, or by circulating electric currents.

The magnetic interaction between the magnetic moment of an electron $\vec{\mu}_e$ and a nuclear magnetic moment $\vec{\mu}_N$ receives a contribution,

$$H^{(1)}_{hfs} = -\vec{\mu}_e\cdot\vec{B}_1(\vec{r}) , \tag{2}$$

where $\vec{B}_1$ is Eq. (1) with $\vec{m} = \vec{\mu}_N$. This interaction,

$$H^{(1)}_{hfs} = \frac{1}{r^3}\left[\vec{\mu}_e\cdot\vec{\mu}_N - 3(\hat{r}\cdot\vec{\mu}_e)(\hat{r}\cdot\vec{\mu}_N)\right] , \tag{3}$$

is the familiar interaction energy between two magnetic dipoles with separation, $\vec{r} = r\hat{r}$. It is supplemented by the interaction of the nuclear moment with the magnetic field of the orbiting electron, $\vec{B}_{orbital}(0) = e\vec{L}/mcr^3$,

$$H^{(2)}_{hfs} = -\frac{e}{mc}\frac{1}{r^3}\vec{L}\cdot\vec{\mu}_N . \tag{4}$$

The sum of Eqs. (3) and (4) accounts for the magnetic part of the hyperfine structure of atomic states with electronic orbital angular momentum different from zero.

2.2 For atomic s-states

For atomic s-states further illuminating consideration is necessary. An electron in a s-state spends part of its time at the origin where the nucleus is. Its magnetic moment thus interacts with the nuclear magnetic field in the region where the dipole approximation, Eq. (1), fails. Furthermore, this localized interaction is the only part of the interaction that gives a non-vanishing contribution to the energy: Eq. (3), applying only for $r > R$, say, averages to zero over angles for $\ell = 0$ states, as does Eq. (4). One is led, therefore, to consider the localized interaction,

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*) For an elementary introduction to Dirac monopoles, see Sections 6.12 and 6.13 of Jackson[7)].

$$H_{hfs}^{(0)} = -\vec{\mu}_e \cdot \vec{B}_N(\vec{r}) \ , \qquad r < R \ , \tag{5}$$

where R is some radius near, but outside, the nuclear surface. In first-order perturbation theory, the energy shift cause by this localized interaction is

$$\Delta E_{hfs}^{(\ell=0)} = - \int_{r<R} \psi^\dagger(\vec{r}) \ \vec{\mu}_e \cdot \vec{B}_N(\vec{r}) \ \psi(\vec{r}) \ d^3r \ .$$

Because the nuclear size is much smaller than atomic dimensions, the electronic $\ell = 0$ wave function varies negligibly for $r < R$ and can be approximated by its value at the origin. The energy shift is therefore accurately given by

$$\Delta E_{hfs}^{(\ell=0)} = -|\psi(0)|^2 \ \vec{\mu}_e \cdot \int_{r<R} \vec{B}_N(\vec{r}) \ d^3r \ . \tag{6}$$

[In Eq. (6) it is understood that an expectation value will be eventually taken for the electronic and nuclear spin states.]

The energy shift (6) is proportional to the integral of the nuclear magnetic field over a spherical volume containing all of the sources of that field. It will now be shown that the integral has different values, depending on whether the sources are circulating electric currents or magnetic charges*). First consider the usual assumptions, $\vec{\nabla}\cdot\vec{B} = 0$, $\vec{B} = \vec{\nabla} \times \vec{A}$, with the vector potential $\vec{A}$ given by**)

$$\vec{A}(\vec{r}) = \frac{1}{c} \int \frac{\vec{J}(\vec{r}') \ d^3r'}{|\vec{r} - \vec{r}'|} \ . \tag{7}$$

The required integral is

$$\int_{r<R} \vec{B}(\vec{r}) \ d^3r = \int_{r<R} \vec{\nabla} \times A \ d^3r \ .$$

The volume integral of the curl of $\vec{A}$ can be written as a surface integral. Thus one has

$$\int_{r<R} \vec{B}(\vec{r}) \ d^3r = R^2 \int_{r=R} \hat{r} \times \vec{A} \ d\Omega \ .$$

Substitution of Eq. (7) and interchange of orders of integration permit the integral to be written

$$\int_{r<R} \vec{B}(\vec{r}) \ d^3r = - \frac{R^2}{c} \int d^3r' \ \vec{J}(\vec{r}') \times \int_{r=R} d\Omega \ \frac{\hat{r}}{|\vec{r} - \vec{r}'|} \ . \tag{8}$$

The integral over solid angle is evidently a vector in the direction $\hat{r}'$ (because that is the only direction that survives the integration over the directions of $\hat{r}$). It is straightforward***) to show that

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*) We follow here the derivation of Ref. 7, Sections 4.1 and 5.6. See also p. 222 ff of Good and Nelson[8]).

**) We keep the appearance of classical physics and have a static expression for $\vec{A}$ in terms of a time-independent current density $\vec{J}(\vec{r})$. This has a proper quantum-mechanical justification, the current density being the expectation value of the quantal current operator in the nuclear ground state.

***) See, for example, Ref. 7, p. 140-1.

$$\int_{r=R} d\Omega \frac{\hat{r}}{|\vec{r} - \vec{r}'|} = \frac{4\pi}{3} \hat{r}' \frac{r_<}{r_>^2} , \tag{9}$$

where $r_<$ ($r_>$) is the smaller (larger) of r' and R. By assumption, $r' < R$ where $\vec{J}(\vec{r}') \neq 0$. Thus Eq. (8) becomes

$$\int_{r<R} \vec{B}(\vec{r})\, d^3r = \frac{4\pi}{3c} \int \vec{r}' \times \vec{J}(\vec{r}')\, d^3r' .$$

Since the magnetic dipole moment of a distribution of current is given by

$$\vec{m} = \frac{1}{2c} \int \vec{r}' \times \vec{J}\, d^3r' , \tag{10}$$

the integral of $\vec{B}$ over a spherical volume containing all of the sources of current is

$$\int_{r<R} \vec{B}(\vec{r})\, d^3r = \frac{8\pi}{3} \vec{m} , \tag{11}$$

where $\vec{m}$ is the total magnetic moment of the sources. Note that the radius R does not enter the result. Thus the division between "near" and "far from" the nucleus can be chosen so that the interaction (3) obviously vanishes for s-states, while the approximations leading to Eq. (6) are still quite valid, at least for electronic atoms.

Now suppose that the source of the nuclear magnetic moment were magnetic charges, rather than electric currents. Let the magnetic charge density be $\rho_M(\vec{r})$ and the resulting magnetic field $B'(\vec{r})$. Then in complete analogy with electrostatics one has $\vec{B}' = -\vec{\nabla}\, \Phi_M'$ and

$$\Phi_M'(\vec{r}) = \int \frac{\rho_M(\vec{r}')}{|\vec{r} - \vec{r}'|}\, d^3r' . \tag{12}$$

Thus

$$\int_{r<R} \vec{B}'(\vec{r})\, d^3r = -\int_{r<R} \vec{\nabla}\, \Phi_M'\, d^3r = -R^2 \int_{r=R} \hat{r}\, \Phi_M'\, d\Omega .$$

Substitution of Eq. (12) and interchange of orders of integration, as before, give

$$\int_{r<R} \vec{B}'(\vec{r})\, d^3r = -R^2 \int d^3r'\, \rho_M(\vec{r}') \int_{r=R} d\Omega \frac{\hat{r}}{|\vec{r} - \vec{r}'|} .$$

The angular integral at $r = R$ is the same as before, namely Eq. (9). Again, $r' = r_<$, $R = r_>$ and the integral of $\vec{B}'$ is

$$\int_{r<R} \vec{B}'(\vec{r})\, d^3r = -\frac{4\pi}{3} \int \vec{r}'\, \rho_M(\vec{r}')\, d^3r' .$$

The definition of a magnetic moment $\vec{m}'$ caused by a distribution of magnetic charge is

$$\vec{m}' = \int \vec{r}'\, \rho_M(\vec{r}')\, d^3r' . \tag{13}$$

Thus one finds, in contrast to Eq. (11),

$$\int_{r<R} \vec{B}'(\vec{r})\, d^3r = -\frac{4\pi}{3} \vec{m}' \tag{14}$$

if the sources are magnetic charges.

The hyperfine energy shift, Eq. (6), for s-states in one-electron atoms can evidently be written

$$\Delta E_{hfs}^{(\ell=0)} = -\frac{8\pi}{3}\lambda\left\langle\vec{\mu}_e\cdot\vec{\mu}_N\right\rangle|\psi(0)|^2 , \tag{15}$$

where $\lambda = 1$ for circulating currents and $\lambda = -\frac{1}{2}$ for magnetic charges as the source of the nuclear magnetic moment*). Equation (15) with $\lambda = 1$ was originally derived by Fermi from the Dirac equation with the nuclear moment contributing an external vector potential[9].

2.3 Comparison with experiment

The hyperfine interaction in s-states is particularly simple for nuclei with spin $\frac{1}{2}$. The energy splitting (15) can be written

$$\Delta E = \frac{8\pi}{3}\lambda|\mu_e|\ \mu_N\left\langle\vec{\sigma}_e\cdot\vec{\sigma}_N\right\rangle|\psi(0)|^2 ,$$

where $|\mu_e|$ is the absolute value of the electronic magnetic moment (the Bohr magneton), μ_N is the nuclear magnetic moment, and the $\vec{\sigma}$'s are Pauli spin operators. The spins combine to give singlet (F = 0) and triplet (F = 1) states. Since $\vec{\sigma}_e\cdot\vec{\sigma}_N = +1$ for triplet and -3 for singlet, the energy splitting between the singlet and triplet states is

$$\Delta E = \frac{32\pi}{3}|\lambda|\ |\mu_e|\ |\mu_N|\ |\psi(0)|^2 , \tag{16}$$

with the singlet state lower in energy if $\lambda\mu_N > 0$. With $|\psi(0)|^2 = Z^3/\pi a_0^3 n^3$, $|\mu_e| = e\hbar/2m_e c$, and the nuclear moment expressed in units of the nuclear magneton ($\mu_n = e\hbar/2m_p c$), the hyperfine splitting is**)

$$\Delta E = \frac{8|\lambda|}{3}\frac{|\mu_N|}{\mu_n}\left(\frac{m_e}{m_p}\right)\left(\frac{Z}{n}\right)^3\alpha^4 m_e c^2 . \tag{17}$$

For atomic hydrogen, $\mu_p = 2.7928\ \mu_n$ (known to high precision from other experiments) and the ground state splitting corresponds to a frequency ($\Delta E/h$) of $\nu = 1421|\lambda|$ MHz, or a wavelength of $21.09/|\lambda|$ cm. Experimentally the frequency difference is known[10] with fantastic accuracy ($\pm 1.2/10^{12}$). The first nine significant figures are $\nu_{obs} = 1420.40575$ MHz, establishing beyond a shadow of a doubt that $\lambda = 1$, not $-\frac{1}{2}$. The singlet state lies lower than the triplet, as required for $\lambda = 1$.

For muonium, an atom comprised of an electron and a positive muon, Eq. (17) applies with $|\mu_N|/\mu_n = m_p/m_\mu$. Numerically, the ground-state muonium splitting is $\nu = 4519|\lambda|$ MHz, compared with the experimental $\nu_{obs} = 4463.303 \pm 0.001$ MHz [11]. Actually, with the reduced mass correction $(1 + m_e/m_\mu)^{-3}$ and the first-order anomalous magnetic moment factor $(1 + \alpha/2\pi)^2$, the theoretical result becomes $\nu = 4464.19|\lambda|$ MHz, the discrepancy with experiment (for $\lambda = 1$) now only 2 parts in 10^4, attributable to known higher-order corrections.

For positronium, the "nucleus" is a positron and the reduced mass correction is a factor of $\frac{1}{8}$. The calculated splitting corresponds to a frequency difference of $\nu = 116.8|\lambda|$ GHz. The observed splitting[12] is almost a factor of two larger, $\nu_{obs} = 203.3856 \pm 0.0014$ GHz. Something is evidently enhancing the splitting of the triplet and singlet states! That

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*) In principle, we should keep separate track of the orbital part of the moment (caused by circulating protons and surely having $\lambda = 1$) and the intrinsic part, but we ignore this point here. Our comparison with experiment mainly concerns protons, electrons, and muons, as the "nuclei" of atoms.

**) For the purpose at hand, we neglect reduced mass, relativistic, and radiative corrections.

something is the virtual annihilation of the electron-positron pair. This causes an additional upward shift of the triplet state, making the positronium separation $^7/_4$ times the value of Eq. (17) for $\lambda = 1$, namely $\nu = 204.4$ GHz. The remaining small difference from experiment is presumed to be attributable to relativistic and radiative corrections.

For the hydrogen atom, the muonium atom, and the positronium atom, the ground-state "hyperfine" splitting is in good agreement with theoretical calculation *provided* the nuclear magnetic moment is attributed to circulating electric currents. There is gross disagreement between theory and experiment if the magnetic moment is attributed to magnetic charges. Since the "nuclei" in question are the proton, the muon, and the electron, it is therefore established that the intrinsic magnetic moments of these fundamental particles are caused by circulating electric currents*).

One example of a nucleus may be considered, namely $^3_2$He. It consists of a pair of protons and a neutron, all assumed in s-states. The protons are in a singlet spin state; the angular momentum and magnetic moment of $^3_2$He are presumably caused solely by the neutron. The measured magnetic moment ($\mu_N = -2.1275\ \mu_n$) is, however, different from the neutron's moment ($-1.91315\ \mu_n$) by about 11%, showing that non-central forces and exchange currents modify the simple expectation somewhat. For the singly-ionized $^3_2$He ground state, the hyperfine interval is observed to be[13] $\nu_{obs} = 8665.649867 \pm 0.000010$ MHz. With the known magnetic moment, Eq. (17) yields an expected value of $\nu = 8660.9|\lambda|$ MHz, while correcting of the electron's anomalous moment and the reduced mass effect gives $8665.7|\lambda|$ MHz. The remaining difference is less than 1 part in 10^5 for $\lambda = 1$ and can be attributed to nuclear structure effects and relativistic and radiative corrections. At the level of choosing between $\lambda = 1$ and $\lambda = -^1/_2$, $^3_2He^+$ shows that a bound neutron's magnetism, as well as the 11% contribution from orbital motion and exchange currents, is caused by circulating currents.

There is one logical weak point in the discussion so far. If *magnetic charges* exist inside particles like the proton, they can *give rise* to *electric dipole moments*. Such dipole moments can cause shifts in energy levels of atoms and molecules (also parity-violating effects) that need to be considered along with the magnetic effects. The agreement with $\lambda = 1$ might be accidental; $\lambda = -^1/_2$ magnetic effects combined with the *electric* dipole splittings could *a priori* give the same results as $\lambda = 1$ alone! These questions are examined in the Appendix. The conclusion is that it is impossible -- the electric dipole moments are known experimentally (from experiments other than hyperfine interactions) to be sufficiently small as not to be significant factors in the hyperfine splittings.

2.4 Casimir's derivation

In passing we remark that, while we have examined the origin of nuclear magnetism and treated the electronic magnetic moment as given, Casimir[5] does the opposite. He writes the interaction as between nuclear moment $\vec{\mu}_N$ and *electronic* magnetic field,

*) The reader may have a conceptual difficulty for the electron and the muon, if not for the proton, in imagining circulating currents inside a point particle. I have no classical panacea for this. I can only observe that the problem is there for magnetic charges and circulating currents alike. In the one case, as the area of the loop goes to zero, the current must go to infinity. In the other, as the distance between poles goes to zero, the magnetic charge must go to infinity.

$$H^{(0)}_{hfs} = -\vec{\mu}_N \cdot \vec{B}_e(0) \ . \tag{18}$$

Here $\vec{B}_e(0)$ is the magnetic field of a s-state electron, evaluated at the origin. This field is given by

$$\vec{B}_e(0) = \frac{1}{c} \int \frac{\vec{r} \times \vec{J}_e(\vec{r})}{r^3} \, d^3r \ ,$$

where $\vec{J}_e(\vec{r}) = c \, \vec{\nabla} \times (\psi^\dagger \, \vec{\mu}_e \, \psi)$ is the magnetization current. It is straightforward to show that if ψ is a spherically symmetric wave function vanishing at infinity the field at the origin is

$$\vec{B}_e(0) = \frac{8\pi}{3} \, (\psi^\dagger \, \vec{\mu}_e \, \psi)_{r=0} \ . \tag{19}$$

When combined with Eq. (18), Eq. (19) yields the previous expression, Eq. (15), with $\lambda = 1$.

Our procedure has the slight pedagogical virtue of treating the nature of nuclear magnetism separately from the quantum mechanics of the electron, whereas Casimir's discussion necessarily has the nature of the magnetism and the quantum mechanics intertwined. The result is, of course, the same (for $\lambda = 1$).

3. MAGNETIC SCATTERING OF NEUTRONS

The nature of the magnetic dipole moment of the free neutron can be uncovered by study of the magnetic scattering of neutrons in ferromagnetic materials. Such scatterings, basically of the neutron by the electrons through interaction between their magnetic moments, were considered theoretically by Bloch[14)] and Schwinger[15)] in 1936/37. Bloch's original treatment is equivalent to assuming the neutron's moment is caused by magnetic charges; Schwinger assumed currents as the source of magnetization. Shortly after, Bloch remarked on the two possibilities[16)], noting that one corresponded to a small excluded volume at the neutron shaped like a disc and the other to a needle. The idea of disc- and needle-shaped cavities in defining $\vec{B}$ and $\vec{H}$ in material media is familiar from elementary electricity and magnetism. It leads to the equivalent question: Does a neutron interact magnetically with $\vec{B}$ or $\vec{H}$ in a ferromagnetic medium?

3.1 Scattering by the electronic magnetization

In order to see how the different origins of magnetic moment influence the scattering, we first consider the classical interaction between the neutron magnetic moment and that of the electron from Section 2. The s-state interaction (15) is equivalent to a contact coupling,

$$H^{(0)}_{int} = -\frac{8\pi}{3} \lambda \, \vec{\mu}_e \cdot \vec{\mu}_N \, \delta(\vec{r}_N - \vec{r}_e) \ . \tag{20}$$

When combined with the dipole-dipole interaction, Eq. (3), it yields the total interaction between the magnetic moments of the neutron and electron,

$$H_{int} = \frac{1}{r^3} \left[\vec{\mu}_e \cdot \vec{\mu}_N - 3(\vec{\mu}_e \cdot \hat{r})(\vec{\mu}_N \cdot \hat{r})\right] - \frac{8\pi}{3} \lambda \, \vec{\mu}_e \cdot \vec{\mu}_N \, \delta(\vec{r}) \ , \tag{21}$$

where $\vec{r}$ is the relative coordinate and $\lambda = 1$ for magnetism caused by circulating currents and $\lambda = -\frac{1}{2}$ for magnetism caused by magnetic charges.

The magnetic scattering of a neutron by a sample*) of ferromagnetic material can be calculated in the first Born approximation. The magnetic scattering amplitude is

$$f(\vec{p}',\vec{p}) = -\frac{m_N}{2\pi\hbar^2}\int d^3r_N \int d^3r_e \, |\psi(\vec{r}_e)|^2 \, e^{i\vec{q}\cdot\vec{r}_N} H_{int} \, .$$

Here $\psi(\vec{r}_e)$ is the electronic spatial wave function and $\vec{q} = \vec{p} - \vec{p}'$ is the neutron's momentum transfer. By introducing the magnetization (magnetic moment density), $\vec{M}(\vec{r}) = \vec{\mu}_e \, |\psi(\vec{r})|^2$ and its Fourier transform,

$$\vec{m}(\vec{q}) = \int \vec{M}(\vec{r}) \, e^{i\vec{q}\cdot\vec{r}} \, d^3r \, , \qquad (22)$$

the scattering amplitude can be written

$$f(\vec{q}) = \frac{m_N}{2\pi\hbar^2}\int d^3r \, e^{i\vec{q}\cdot\vec{r}} \left[\frac{3\vec{m}(\vec{q})\cdot\hat{r} \; \vec{\mu}_N\cdot\hat{r}}{r^3} - \frac{\vec{m}(\vec{q})\cdot\vec{\mu}_N}{r^3} + \frac{8\pi}{3}\lambda \, \vec{m}(\vec{q})\cdot\vec{\mu}_N \, \delta(\vec{r})\right] .$$

The remaining integration yields the result,

$$f(\vec{q}) = -\frac{2m_N}{\hbar^2}\left[\vec{\mu}_N\cdot\hat{q} \; \vec{m}(\vec{q})\cdot\hat{q} - \left(\frac{2\lambda + 1}{3}\right) \vec{\mu}_N\cdot\vec{m}(\vec{q})\right] . \qquad (23)$$

This can be written

$$f(\vec{q}) = \frac{2m_N}{\hbar^2} \, |\mu_N| \, m(\vec{q}) \; \vec{\sigma}_N\cdot\vec{a} \, , \qquad (24a)$$

where

$$\vec{a} = \begin{cases} \hat{q}(\hat{m}\cdot\hat{q}) - \hat{m} & (\lambda = 1, \text{"currents"}) \\ \hat{q}(\hat{m}\cdot\hat{q}) & (\lambda = -\tfrac{1}{2}, \text{"charges"}) \, . \end{cases} \qquad (24b)$$

3.2 Magnetic scattering by $\vec{B}$ or $\vec{H}$?

Before comparison with experiment, it is enlightening to show explicitly how the two possibilities correspond to the neutron's magnetic moment coupled to $\vec{B}$ or $\vec{H}$ of the magnetic medium[17]. Since the interaction is either $-\vec{\mu}_N\cdot\vec{B}$ or $-\vec{\mu}_N\cdot\vec{H}$, it is sufficient in first-order perturbation theory to examine the Fourier transform of $\vec{B}$ or $\vec{H}$. If the magnetization $\vec{M}(\vec{r})$ is assumed given, the field $\vec{H}$ can be derived from a magnetic scalar potential Φ_M according to $\vec{H} = -\vec{\nabla}\,\Phi_M$, where

$$\Phi_M(\vec{r}) = -\int \frac{\vec{\nabla}'\cdot\vec{M}(\vec{r}')}{|\vec{r} - \vec{r}'|} \, d^3r' .$$

The Fourier transform of $\vec{H}$ is

$$\vec{h}(\vec{q}) = \int \vec{H}(\vec{r}) \, e^{i\vec{q}\cdot\vec{r}} \, d^3r = \int d^3r \, e^{i\vec{q}\cdot\vec{r}} \, \vec{\nabla} \int d^3r' \, \frac{\vec{\nabla}'\cdot\vec{M}(\vec{r}')}{|\vec{r} - \vec{r}'|} \, .$$

Some integrations by parts, interchange of orders of integration, and shift of origin, yield

$$\vec{h}(\vec{q}) = -\vec{q} \int \vec{q}\cdot\vec{M}(\vec{r}') \, e^{i\vec{q}\cdot\vec{r}'} \, d^3r' \int \frac{e^{i\vec{q}\cdot\vec{R}}}{R} \, d^3R \, .$$

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*) "Sample" can be a single atom or molecule, a lattice cell, or a macroscopic block. When it is relevant, the "sample" will be specified.

The first integral is just $\vec{q}$ dotted into $\vec{m}(\vec{q})$, Eq. (22), and the second integral is $4\pi/q^2$. Therefore the Fourier transform of $\vec{H}$ is

$$\vec{h}(\vec{q}) = -4\pi\ \hat{q}(\vec{m}\cdot\hat{q}) \ . \tag{25}$$

Since $\vec{B} = \vec{H} + 4\pi\vec{M}$, the Fourier transform of $\vec{B}$ is directly

$$\vec{b}(\vec{q}) = \int \vec{B}(\vec{r})\ e^{i\vec{q}\cdot\vec{r}}\ d^3r = -4\pi[\hat{q}(\vec{m}\cdot\hat{q}) - \vec{m}] \ . \tag{26}$$

Comparison of Eqs. (25) and (26) with the two forms in Eq. (24) shows the correspondences:

Source of neutron's magnetism	Magnetic scattering interaction in magnetic medium
Circulating currents	$-\vec{\mu}_N\cdot\vec{B}$
Magnetic charges	$-\vec{\mu}_N\cdot\vec{H}$

The magnetic scattering amplitude (24a) can be written in the alternative form[17),]

$$f(\vec{q}) = -\frac{2m_N}{\hbar^2}\ |\mu_N|\ \vec{\sigma}_N\cdot[\vec{h}(\vec{q}) + 4\pi C\ \vec{m}(\vec{q})] \ , \tag{27}$$

where C = 1 for "circulating currents" and C = 0 for "magnetic charges". For $\vec{q} \neq 0$, Eq. (24) is preferred because it exhibits explicitly through $\vec{a}$ the dependence of the magnetic scattering on the relative orientation of $\vec{m}$ and $\vec{q}$. For forward scattering, however, Eq. (27) is preferable. The direction of $\hat{q}$ depends on how the limit of $\vec{q} \to 0$ is defined. Thus $\vec{a}$ is ambiguous at q = 0. The form (27) provides an unambiguous definition since $\vec{h}(\vec{q})$ and $\vec{m}(\vec{q})$ become proportional to the average macroscopic quantities, $\vec{H}$ and $\vec{M}$, of the ferromagnetic medium as $\vec{q} \to 0$. Ekstein[18)] has shown that a more careful consideration of the microscopic scattering amplitude leads to an improvement of Eq. (24), that is the same for $\vec{q} \neq 0$, but has an unambiguous limit agreeing with Eq. (27) as $\vec{q} \to 0$.

3.3 Comparison with experiment

If the neutron energy is low enough or the angle of scattering small enough, the momentum transfer q is small enough that q^{-1} is large compared to atomic or crystal lattice dimensions. Then $\vec{m}(\vec{q})$ becomes equal to the magnetic moment of the sample and $\hat{m}$ defines the direction of magnetization in the sample. In general, there is nuclear scattering as well as magnetic scattering. Assuming the nuclear scattering amplitude to be neutron-spin-independent, the scattered intensity for low-energy, unpolarized neutrons will be proportional to

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\sum_{if} |\langle f|A_n + A_{mag}\ \vec{\sigma}_N\cdot\vec{a}|i\rangle|^2$$

or

$$\frac{d\sigma}{d\Omega} = |A_n|^2 + |A_{mag}|^2\ |\vec{a}|^2 \ ,$$

where A_n is the nuclear scattering amplitude, the vector $\vec{a}$ is defined in Eq. (24b) and A_{mag} is the magnetic scattering coefficient of $\vec{\sigma}_N\cdot\vec{a}$ in Eq. (24a).

The dependence of the scattering on the relative orientation of the magnetization and the momentum transfer direction is contained in $|\vec{a}|^2$. Explicitly,

$$|\vec{a}|^2 = \begin{cases} 1 - (\hat{m}\cdot\hat{q})^2 = \sin^2\alpha & (\text{"currents or } \vec{B}\text{"}) \\ (\hat{m}\cdot\hat{q})^2 \quad = \cos^2\alpha & (\text{"charges or } \vec{H}\text{"}) \end{cases} \tag{28}$$

where α is the angle between $\hat{m}$ and $\hat{q}$.

Two experiments on the scattering of thermal neutrons by magnetized substances were performed in 1949 and 1950. One[19] involved the Bragg scattering of neutrons from magnetite (Fe_3O_4) powder. The crystal structure is such that the (111) reflection is almost wholly caused by magnetic scattering, with only 2% nuclear contribution. The results of the experiment are shown in Figs. 1a and 1b. The scattered intensities for the sample unmagnetized, then magnetized with $\alpha = 90°$, then with $\alpha = 0°$, are displayed in Fig. 1a. Clearly the intensity pattern is in far better agreement with a $\sin^2\alpha$ dependence than $\cos^2\alpha$! The intensity for the sample unmagnetized is approximately $2/3$ of the intensity when $\alpha = 90°$, as expected for a spherical average of $\sin^2\alpha$. Data at two intermediate angles, together with the $\alpha = 0°$, $90°$ data, are shown in Fig. 1b in the form of a plot of $|\vec{a}|^2$ versus α. The data follow the $\sin^2\alpha$ curve quite adequately. The neutron is thus shown to interact magnetically with $\vec{B}$, not $\vec{H}$, or equivalently to have a magnetic dipole moment that is caused by circulating currents, not magnetic charges.

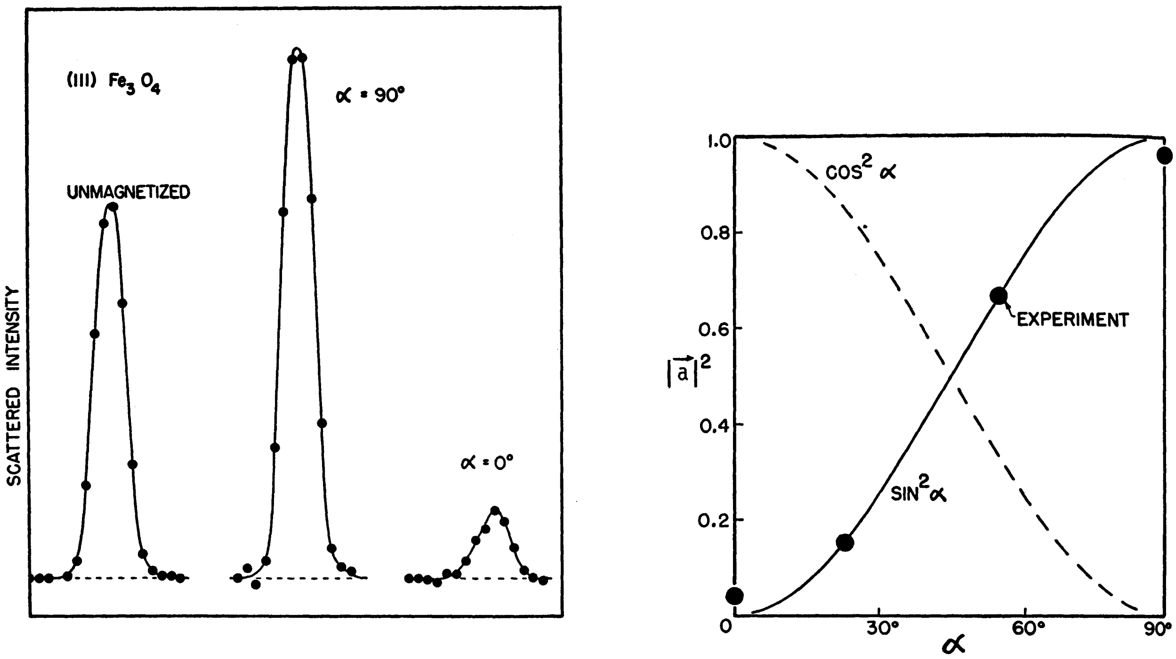

Fig. 1 a) Diffracted neutron intensity (111, from Fe_3O_4 powder) for no magnetization and for a sample magnetized with $\alpha = 90°$ and $\alpha = 0°$.
b) Variation of normalized scattered intensity, that is $|\vec{a}|^2$ of Eq. (28), versus the angle α.
[Figures 1 and 2 of Shull, Wollan and Strauser[19].]

The other experiment[20] consisted of the glancing reflection of slow neutrons by a magnetized iron slab. The simplest way to think about the experiment is in terms of an index of refraction for neutron waves. It is well known that the square of the index of refraction for particles, as well as light, can be expressed in terms of the forward scattering amplitude*),

*) See, for example, Ref. 7, Section 9.14 for a derivation for light waves.

$$n^2 = 1 + \frac{4\pi N}{k^2} f_{scatt}(0°) ,$$

where $f_{scatt}(0°)$ is the average coherent scattering amplitude per atom, N is the number of atoms per unit volume, and k is the wave number of the particle. For thermal neutrons, with coherent s-wave nuclear scattering per atom described by the amplitude A_n and magnetic scattering given by Eq. (27), the square of the index of refraction for spin state j is

$$n_j^2 = 1 + \frac{4\pi N\ An}{k^2} - \frac{|\mu_N|}{E} (\vec{H} + 4\pi C\vec{M})\cdot\langle j|\vec{\sigma}_N|j\rangle .$$

Here $|\mu_N| = 1.91\ e\hbar/2m_p c$ is the magnitude of the neutron's magnetic moment, k and E are the neutron's wave number and kinetic energy, while $\vec{H}$ and $\vec{M}$ are the macroscopic fields of the medium.

Assuming $\vec{H}$ and $\vec{M}$ to be parallel and choosing spin states with quantization axis in this direction yield the indices of refraction for the two spin states

$$n_\pm^2 = 1 + \frac{4\pi N\ An}{k^2} \mp |\mu_N| \left(\frac{H + 4\pi CM}{E}\right) . \qquad (29)$$

In many substances the nuclear scattering amplitude A_n is negative ("hard sphere" scattering). The index of refraction without magnetization is less than unity. If neutrons are incident at a glancing angle θ on a flat slab of such material, there will be a critical angle θ_c (measured up from the plane) below which there will be total reflection (the total "internal" reflection of optics). For n^2 less than but close to unity, Snell's law gives this angle as

$$\theta_c \approx (1 - n^2)^{\frac{1}{2}} = \left[\frac{4\pi N(-A_n)}{k^2} \pm |\mu_N| \left(\frac{H + 4\pi CM}{E}\right)\right]^{\frac{1}{2}} . \qquad (30)$$

For an unpolarized beam, both spin states are present. If the magnetic contribution is significant, there will be two critical angles, one for each spin state. For a magnetized slab, the maximum magnetic effect will be seen with magnetization parallel to the slab's surface. The magnetic field $\vec{H}$ is relatively small outside the surface and continuity of tangential $\vec{H}$ means that inside it is also small. Thus the magnetization contribution in Eq. (30) is dominant; C = 1 and C = 0 will be clearly distinguished by experiment.

For magnetized iron, the relative magnitudes of the two terms in Eq. (30) are such that the angles differ in size by a factor of two or more. The results of the reflection experiment are shown in Fig. 2. The reflected neutron intensity is plotted as a function of the angle of reflection. Because the neutron beam is not monoenergetic and the index of refraction depends on energy, the sudden onset of reflection at the critical angle is smeared out. The curve labelled "BLOCH" is the result expected if there were no magnetic term in Eq. (30) (C = 0). The curve labelled "DIRAC" is the prediction with the magnetic terms present (C = 1). We see again, in a different experiment, that the neutron's magnetic moment is caused by circulating currents, and not by separated equal and opposite magnetic charges.

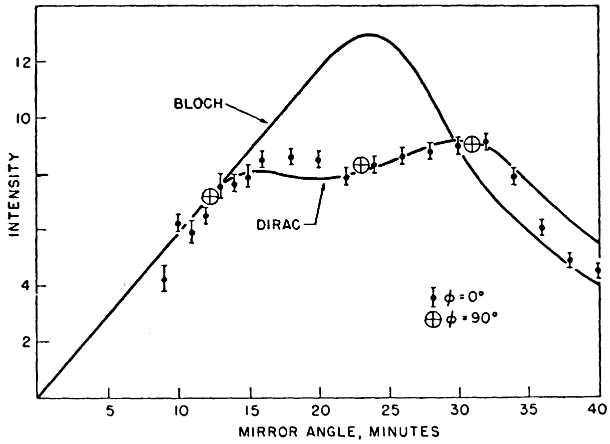

Fig. 2 Intensity of neutrons reflected from a magnetized iron slab (mirror) as a function of the mirror angle (θ) in minutes of arc. The curve marked BLOCH corresponds to C = 0 in Eq. (30) (no spin-dependent effect in this experiment); the curve marked DIRAC corresponds to C = = 1, with two critical angles θ_c for each neutron energy. Most of the data points are for magnetization parallel to the plane of scattering, but the three circled crosses are data with magnetization perpendicular to that plane. The effect is the same. [Figure 4 of Hughes and Burgy[20].]

4. SUMMARY AND CONCLUSIONS

It is generally accepted that macroscopic magnetic phenomena are produced for the most part by circulating electric currents, that is, electric charges in motion. There is, as far as we know today, no magnetic analogue of electric charge, no isolated magnetic pole. There are, however, intrinsic magnetic dipole moments (of the electron, muon, proton, neutron, and in fact of any fundamental particle of non-zero spin). From time to time the question arises as to whether magnetic charges might exist, not isolated, but in bound pairs of an equal and opposite strength or more complex groupings of differing magnitudes and signs, but vanishing total magnetic charge. They might then be responsible for the intrinsic magnetic moments of the fundamental particles. We have demonstrated, by examining the experimental facts on the hyperfine structure of atomic s-states and the magnetic scattering of neutrons, that the answer to the second part of the question is no. For the neutron, proton, electron, and muon at least, experiment establishes that their intrinsic magnetic moments are caused by circulating electric currents.

It is possible that bound groups of magnetic charges exist; indeed there are some models of hadrons in which the constituents possess both electric and magnetic charge[4]. The quantitative agreement of the observed ground-state hyperfine splitting in atomic hydrogen (to very high precision) with the calculations of conventional quantum electrodynamics[21] shows that such constituents ("dyons") contribute negligibly to the proton's magnetic moment through their magnetic charge*). Furthermore, the extremely small upper limit on the electric dipole moment of the neutron**) requires the postulation in such models of a very strong magnetic-charge-exchange interaction between dyons, that effectively averages the magnetic charge density to zero inside a hadron.

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*) When the level of comparison gets below 1 part in 10^5, the somewhat uncertain hadronic dynamics of the proton start to enter significantly. There could thus be a magnetic charge contribution of that order of magnitude. See, however, the Appendix.

**) Dress, Miller and Ramsey[22] give the ratio of electric dipole moment to the proton's charge as less than 10^{-23} cm. Subsequent private communications from Ramsey give less than 4×10^{-25} cm (as of the end of 1976).

Magnetic charges may exist in bound pairs, or even singly, but if they do, they have nothing (or almost nothing) to do with the magnetic properties of media or fundamental particles. All is circulating electric current.

Acknowledgements

It seems appropriate to acknowledge here my indebtedness to and admiration of V.F. Weisskopf, teacher, longtime friend, and shining example of a physicist who seeks always to understand the fundamentals and to convey that understanding to others. He had nothing to do with this paper, but I hope that he and others enjoy reading it.

I wish to express my gratitude to J. Prentki and the Theory Division at CERN for their warm welcome, and to the Regents of the University of California for granting the sabbatical leave and leave of absence that made a year at CERN possible. In matters of physics, I thank J.R. Ellis for a conversation about gauge theories and electric dipole moments, and H.A. Schuessler for the example of the hyperfine structure in ${}^3_2He^+$.

APPENDIX

LIMITS ON ELECTRIC DIPOLE MOMENTS

If magnetic charges are the source of magnetic dipole moments in nuclei and fundamental particles, these systems will in general possess electric dipole moments. It was pointed out originally by Landau[23] that the existence of electric dipole moments requires violation of both parity conservation and time-reversal invariance. Particles having both electric and magnetic charges necessarily violate both these symmetries*), and so provide a mechanism for the existence of electric dipole moments (EDM), although it is generally believed that EDMs are caused only by weak interaction effects.

Simple semi-classical considerations can indicate plausible magnitudes or upper limits for EDMs. We suppose that the magnetic charges occur as bound pairs of equal and opposite magnitude $\pm e_M$. Since the observed magnetic moment of a particle of mass m is always of order

$$|\vec{\mu}| = O\left(\frac{e\hbar}{mc}\right) ,$$

where e is the charge of the proton, the effective average separation of the magnetic charges must be

$$|\vec{r}| = O\left(\frac{e}{e_M}\frac{\hbar}{mc}\right) .$$

If $e/e_M = O(1)$, the separation is of the order of the particle's Compton wavelength. If $e/e_M = O(1/137)$, as for Dirac monopoles, the mean separation is much smaller.

The size of an EDM can be estimated in analogy with the size of orbital magnetic moments. Just as an orbiting particle with electric charge e and mass M gives rise to an orbital magnetic dipole moment,

$$\vec{\mu}_{orbital} = \frac{e}{2Mc} \vec{L} ,$$

so an orbiting magnetic charge e_M with mass M will give rise to an EDM,

$$\vec{d}_{orbital} = \frac{e_M}{2Mc} \vec{L} .$$

With $|\vec{L}| = O(\hbar)$, $M = O(m)$, one finds $|\vec{d}| = O(|\vec{\mu}|)$, independent of the value of e_M. This should be viewed as an upper limit. The magnetic and electric dipole moments are defined in terms of magnetic charge and current densities as

$$\vec{\mu} = \int \vec{r}\, \rho_M(\vec{r})\, d^3r$$

$$\vec{d} = \frac{1}{2c} \int \vec{r} \times \vec{J}_M(\vec{r})\, d^3r .$$

The differing vectorial character of the two expressions, and also the presence of a spin contribution to $\vec{J}_M$, prevents explicit connections, except in very simple models.

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*) See, for example, Ref. 7, Section 6.12.

The naïve argument leads one to expect EDMs of the order of or less than $(d/e)_{max} \approx$ $\approx 2 \times 10^{-11}$ cm for an electron and 10^{-14} cm for a proton or neutron. Actually, analyses of various aspects of atomic energy levels, especially the Lamb shift ($2s_{1/2}$-$2p_{1/2}$ splitting in hydrogen) set early upper limits on $|d/e|$ for both the electron[24)] and proton[25)]. The present discrepancy between QED theory and experiment for the Lamb shift is[26)] about 0.03 MHz, or 3 parts in 10^5. Attributing all of this to EDM, one finds the limit $|d/e| < 2 \times 10^{-14}$ cm for electron and proton. Such a limit says nothing significant for the proton, but shows that the EDM of an electron is less than 10^{-3} times the largest plausible value if magnetic charges are the source of intrinsic magnetic moments.

Such a small EDM could not interfere with our conclusions about the value of λ in Section 2.3. Consider, for example, the positronium hyperfine splitting. Salpeter[24)] shows that an EDM for the electron (and positron) would contribute an additional energy splitting,

$$\Delta E_{EDM} = \pm\frac{5}{4}\,\xi^2(\Delta E)_0 \ ,$$

where $(\Delta E)_0$ is the conventional magnetic contribution, Eq. (17) for $\lambda = 1$, ξ is the ratio d/μ_e, and the $\pm$ signs correspond to different choices of the behaviour of the EDM under charge conjugation. If indeed $\lambda = -\frac{1}{2}$, the magnetic plus annihilation terms would subtract instead of add, leaving the triplet state only $\frac{1}{7}$ as far above the singlet as is observed. If the difference were attributed to the EDM contribution it would require $\xi^2 = \frac{6}{5}$, but other atomic evidence sets the upper limit, $\xi < 0.001$! (If we accept $\lambda = 1$, the present difference between QED and experiment for positronium of about 1 part in 10^4 itself sets an upper limit of $\xi < 0.02$ or $d/e < 4 \times 10^{-13}$ cm.)

These upper limits from nearly twenty years ago are adequate for our purposes, but for the record we note that the present upper limits are many orders of magnitude smaller. The discovery of CP and T violation in neutral kaon decays rekindled the interest in EDMs, and experiments were mounted to go far beyond the $|d/e| < 10^{-13}$ cm range into the domain expected from weak interactions. On dimensional grounds, this domain is $|d/e| \lesssim O\,(G^2 m_p^3)$, where $G = 10^{-5}\ m_p^{-2}$ is the Fermi beta-decay coupling constant, the parameter characteristic of weak interactions. Thus $|d/e| \lesssim 10^{-24}$ cm might be expected, although several orders of magnitude smaller is quite likely*).

For the electron, atomic beam magnetic resonance experiments with applied electric fields have been performed on highly polarizable heavy atoms, where the EDM effect is enhanced one hundredfold. For caesium, the null result is interpreted as setting the upper limit[27)], $|d/e|_{electron} < 3 \times 10^{-24}$ cm. For the metastable $^3P_2$ state of xenon, the limit is[28)] $|d/e|_{electron} = (0.7 \pm 2.2) \times 10^{-24}$ cm (90% confidence level). For positive and negative muons, the signature of an EDM was sought in the CERN Muon Storage Ring (g-2) experiment as a vertically oscillating component of the muon polarization (having the same frequency as the precession of the horizontal polarization). No positive effect was seen, setting the upper limit[29)] $|d/e|_{muon} < 1.05 \times 10^{-18}$ cm (95% confidence level).

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*) In the older literature, one sees estimates, $|d/e| = O\,(Gm_p) \approx 10^{-19}$ cm, but present gauge theories of weak and electromagnetic interactions give

$$|d/e| = O\left[f\ G(\alpha\ \Delta m_q^2/m_W^2)\ m_p\right] = O\left[f\ G^2(\Delta m_q^2)\ m_p\right] ,$$

where Δm_q^2 is a difference of squares of quark masses and f is a dimensionless factor that may be as small as 10^{-6}.

The upper limits on EDMs for the proton and neutron have achieved remarkably small values, too. For the neutron, Ramsey and collaborators are already at the level of a few times 10^{-25} cm, as was noted in Section 4. For the proton, an electric-deflection molecular beam resonance experiment on TlF gave the limit[30] $|d/e|_{\text{proton}} = (7 \pm 9) \times 10^{-21}$ cm. Such minute EDMs are of no relevance for our considerations on the source of intrinsic magnetic moments. For the dyon model[4] of hadrons, however, they are a severe constraint. It is necessary to have a mechanism for smearing out the different magnetic charges of the dyons so that the effective magnetic charge density is zero at all points to better than perhaps 1 part in 10^8. The magnetic dipole moments of hadrons can then be caused only by circulating electric currents (the moving *electric* charges of the dyons, plus the intrinsic magnetic moments of the dyons, themselves caused by circulating currents!).

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