$$\int \frac{d^n q}{(2\pi)^n}\,\frac{1}{(q^2+2q{\cdot}p-m^2+i\varepsilon)^r} = \quad i(-1)^r(p^2+m^2)^{2-\epsilon-r}(4\pi)^{\epsilon-2}\,\frac{\Gamma(\epsilon+r-2)}{\Gamma(r)}$$

$$\int \frac{d^n q}{(2\pi)^n}\,\frac{q^\mu}{(q^2+2q{\cdot}p-m^2+i\varepsilon)^r} = -i(-1)^r(p^2+m^2)^{2-\epsilon-r}(4\pi)^{\epsilon-2}\,\frac{\Gamma(\epsilon+r-2)}{\Gamma(r)}\,p^\mu$$

$$\begin{aligned}\int \frac{d^n q}{(2\pi)^n}\,\frac{q^\mu q^\nu}{(q^2+2q{\cdot}p-m^2+i\varepsilon)^r} &= \quad i(-1)^r(p^2+m^2)^{2-\epsilon-r}(4\pi)^{\epsilon-2}\,\frac{\Gamma(\epsilon+r-3)}{\Gamma(r)}\\ &\qquad \times\left[(\epsilon+r-3)p^\mu p^\nu-\tfrac{1}{2}g^{\mu\nu}(p^2+m^2)\right]\end{aligned}$$

$$\begin{aligned}\int \frac{d^n q}{(2\pi)^n}\,\frac{q^\mu q^\nu q^\alpha}{(q^2+2q{\cdot}p-m^2+i\varepsilon)^r} &= -i(-1)^r(p^2+m^2)^{2-\epsilon-r}(4\pi)^{\epsilon-2}\,\frac{\Gamma(\epsilon+r-3)}{\Gamma(r)}\\ &\qquad \times\left[(\epsilon+r-3)p^\mu p^\nu p^\alpha-\tfrac{1}{2}(g^{\mu\nu}p^\alpha+g^{\mu\alpha}p^\nu+g^{\nu\alpha}p^\mu)(p^2+m^2)\right]\end{aligned}$$

$$\begin{aligned}\int \frac{d^n q}{(2\pi)^n}\,\frac{q^\mu q^\nu q^\alpha q^\beta}{(q^2+2q{\cdot}p-m^2+i\varepsilon)^r} &= \quad i(-1)^r(p^2+m^2)^{2-\epsilon-r}(4\pi)^{\epsilon-2}\,\frac{\Gamma(\epsilon+r-4)}{\Gamma(r)}\\ &\qquad \times\Big\{(\epsilon+r-3)(\epsilon+r-4)p^\mu p^\nu p^\alpha p^\beta-\tfrac{1}{2}(\epsilon+r-4)(g^{\mu\nu}p^\alpha p^\beta\\ &\qquad +g^{\mu\alpha}p^\nu p^\beta+g^{\mu\beta}p^\mu p^\alpha+g^{\nu\alpha}p^\mu p^\beta+g^{\nu\beta}p^\mu p^\alpha+g^{\alpha\beta}p^\mu p^\nu)(p^2+m^2)\\ &\qquad +\tfrac{1}{4}(g^{\mu\nu}g^{\alpha\beta}+g^{\mu\alpha}g^{\nu\beta}+g^{\mu\beta}g^{\nu\alpha})(p^2+m^2)^2\Big\}\end{aligned}$$

where ε (not to be confused with ϵ) is a positive infinitesimal constant and ϵ is related to the number of spacetime dimensions via

$$g^{\mu\nu}g_{\mu\nu}=n\equiv 4-2\epsilon\,.$$

In addition, we shall *define*

$$\int \frac{d^n q}{(2\pi)^n}\,\frac{1}{(q^2)^r}\equiv 0\,,$$

which corresponds to setting $p^2=m^2=0$ in the first integral above under the assumption that $\epsilon<2-r$. However, in the dimensional regularization procedure, we shall adopt the above definition for *all* r.

We can expand about $\epsilon=0$ by using

$$\Gamma(-N+\epsilon)=\frac{(-1)^N}{N!}\left[\frac{1}{\epsilon}+\psi(N+1)+\mathcal{O}(\epsilon)\right]\,,$$

where N is a non-negative integer, $\psi(x) \equiv \Gamma'(x)/\Gamma(x)$ with $\Gamma'(x) \equiv d\Gamma(x)/dx$,

$$\psi(1) = -\gamma\,, \qquad\qquad \psi(N+1) = -\gamma + \sum_{k=1}^{N} \frac{1}{k}\,,$$

and $\gamma = -\Gamma'(1) = 0.5772\cdots$ is the Euler-Mascheroni constant. If the $\mathcal{O}(\epsilon)$ terms are needed, then one must use $x\Gamma(x) = \Gamma(x+1)$ until $\Gamma(1+\epsilon)$ is reached, and then use

$$\log\Gamma(1+\epsilon) = -\gamma\epsilon + \sum_{k=2}^{\infty} \frac{(-\epsilon)^k}{k}\,\zeta(k)\,,$$

where $\zeta(k)$ is the Riemann zeta function.

When fermions are involved, we need to consider the Dirac matrix algebra in n-dimensions. The Dirac gamma matrices are denoted by γ^0, $\gamma^1, \gamma^2, \ldots, \gamma^{n-1}$. The n-dimensional analog of γ_5 is somewhat problematical, although we shall treat it as anticommuting with all the other gamma matrices. The following n-dimensional gamma matrix relations must be used:

1. $\left\{\gamma^\mu\,,\, \gamma^\nu\right\} = 2g^{\mu\nu}$
2. $\left\{\gamma^\mu\,,\, \gamma_5\right\} = 0$
3. $(\gamma_5)^2 = \mathbb{1}$
4. $\gamma_\mu\gamma^\mu = n$
5. $\gamma_\mu\gamma^\alpha\gamma^\mu = (2-n)\gamma^\alpha$
6. $\gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\mu = 4g^{\alpha\beta} + (n-4)\gamma^\alpha\gamma^\beta$
7. $\gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\rho\gamma^\mu = -2\gamma^\rho\gamma^\beta\gamma^\alpha + (4-n)\gamma^\alpha\gamma^\beta\gamma^\rho$
8. Trace formulae are unchanged. In particular, $\mathrm{Tr}\gamma^\mu\gamma^\nu = 4g^{\mu\nu}$.

The 4 in the last trace formula is purely conventional. However, note that it is crucial to use $g^{\mu\nu}g_{\mu\nu} = n$ in all calculations in n-dimensions before taking the $\epsilon \to 0$ limit.

Finally, we record some of the Feynman parameter formulae:

$$\frac{1}{A^\alpha B^\beta} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 dx\, \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\big[xA + (1-x)B\big]^{\alpha+\beta}}$$

$$\frac{1}{A^\alpha B^\beta C^\delta} = \frac{\Gamma(\alpha+\beta+\delta)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\delta)} \int_0^1 x\,dx \int_0^1 dy\, \frac{x^{\alpha+\beta-2}\,y^{\alpha-1}(1-x)^{\delta-1}(1-y)^{\beta-1}}{\big[xyA + x(1-y)B + (1-x)C\big]^{\alpha+\beta+\delta}}$$

and more generally,

$$\frac{1}{A_1^{\alpha_1}A_2^{\alpha_2}\cdots A_N^{\alpha_N}} = \frac{\Gamma(\alpha_1+\alpha_2+\cdots+\alpha_N)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\cdots\Gamma(\alpha_N)} \int_0^1 dx_1 \int_0^1 dx_2 \cdots \int_0^1 dx_N\, \delta\left(\sum_{j=1}^{N} x_j - 1\right)$$
$$\times \frac{x_1^{\alpha_1-1}\,x_2^{\alpha_2-1}\cdots x_N^{\alpha_N-1}}{(x_1A_1 + x_2A_2 + \cdots + x_NA_N)^{\alpha_1+\alpha_2+\cdots+\alpha_N}}$$