Little Higgs Models: A Group Theoretic Primer

Eric Carlson

August 25, 2013

1 Introduction

One of the outstanding problems in particle physics is the solution to the Standard Model hierarchy problem. In essence, this problem seeks to understand why the scale of electroweak symmetry breaking (EWSB), dominantly the Higgs mass, does not appear at the grand unification scale, or the Planck scale of quantum gravity which lay 16-18 orders of magnitude higher. The standard model scalar Higgs generically receives quadratically divergent radiative corrections to its mass squared $m_h^2 \propto \Lambda^2$, where Λ is the cutoff scale of new physics. In order to avoid a large fine-tuning of the coupling constants to suppress such terms, one must introduce new dynamics often entailing new symmetries and symmetry breaking patterns. This "naturalness" requirement on the couplings provides strong theoretical motivation for physics beyond the Standard Model entering at the electroweak scale.

Precision electroweak constraints and the LHC era have ushered in a new set of challenges for complete models of EWSB. The leading candidate, Supersymmetry, extends the Poincare group to include a graded Lie algebra connecting bosonic and fermionic degrees of freedom. If realized in nature, the existence of supersymmetry transformations imply that each particle in the Standard Model is linked to new "superpartners", a byproduct of which provides a cancellation of the divergent mass corrections to the Higgs. Minimal implementations of this theory are beginning to face their own issues of tuning as the LHC continues to produce null results on the superpartners that are important in radiative Higgs mass corrections. In particular, limits on the scalar partner of the top quark, or "stop" quark have been pushed higher than the than the values necessary to obtain the correct (measured) relationship between the lightest Higgs and the Z^0 mass. This is the manifestation of the so-called "little hierarchy problem" in supersymmetry.

Awaiting data from the high luminosity LHC, theorists are left to explore other solutions to the hierarchy problem. One such class of theories are composite Higgs models, in which the Higgs boson arises as a composition of fields generated by new dynamics at a high mass scale $\Lambda/4\pi \gtrsim 10$ TeV (constrained by precision electroweak measurements). The Little Higgs subclass of composite models, postulates that these new fields, which have properties analogous to that of the strong nuclear force, carry a relatively large symmetry group, typically at least $SU(3) \times SU(3)$, which is then broken to a smaller stability subgroup. The broken symmetry generators then produce Nambu-Goldstone Bosons (NGBs). If the remaining symmetry is only approximate, then the bosons are called Pseudo-Nambu-Goldstone Bosons (PNGBs) and they can acquire masses much smaller than the cutoff scale of the new strong dynamics. Such a phenomena has already been realized in nature when the $SU(3)_L \times SU(3)_R$ chiral symmetry of the strong force undergoes breaking to the smaller $SU(3)_{\text{diagonal}}$ subgroup, resulting in light, but massive Pion fields composed of up and down type quarks. In nearly complete analogy, the basic mechanism of Little Higgs can be explored. Realistic models, however, must evade a stringent and somewhat nuanced set of constraints, most of which originate from precision electroweak measurements.

This review will provide the basic elements of little Higgs models in the following order. In Section 2 we will discuss symmetry breaking and Nambu Goldstone bosons as well as their transformation properties. In Section 3 we will build a simple model of little Higgs using the breaking pattern $SU(3)/SU(2)$, showing how the Higgs doublet arises as a PNGB and introducing some of the phenomenological constraints. In Section 4, we will introduce the concept of collective symmetry breaking to show how one can evade electroweak constraints and stabilize the scale of EWSB. Finally, in Section 5 we will briefly sketch the steps needed to construct realistic models and conclude.

2 Review of Nambu Goldstone Bosons

Nambu-Goldstone Bosons arise whenever a continuous global symmetry is broken. In the case of an exact symmetry, the bosons are precisely massless and couple only through derivatives. As the simplest possible example, consider a complex scalar field with $\mathcal{L} = |\partial_\mu \phi|^2 + V(\phi\phi^*)$. This is invariant under a global $U(1) : \phi \to e^{i\alpha}\phi$. If the potential V gains a minimum away from the origin, ϕ acquires a vacuum expectation value, spontaneously breaking the $U(1)$. Thus we obtain the well-known "Mexican Hat" potential. The fields two degrees of freedom can be decomposed into $\phi(x) = \frac{1}{\sqrt{2}}(f + r(x))\, e^{i\theta(x)/f}$ where f is the vacuum expectation value, $r(x)$ parametrizes radial oscillations about the minimum, and $\theta(x)$ represents rotations in the complex plane. The former requires energy to oscillate up the walls of the potential (i.e. are massive), while the complex rotations are equipotential. These latter excitations are precisely our massless Nambu Goldstone Bosons. In more formal language, we say that the radial field is invariant under the $U(1)$ while the $U(1)$ is non-linearly realized in the transformation $\theta \to \theta + \alpha$. This shift invariance of the field requires that we have only derivative couplings of the NGB fields, implying that they must remain massless.

For the non-abelian case, we simply assign one NGB to each of the broken symmetry generators, which are easily counted once the final stability subgroup is specified. The transformations of the NGBs may seem more complicated, but they are easily parametrized in the more general form of the $U(1)$ case. As an example, let us look at the breaking patter $SU(N) \to SU(N-1)$. This carries $(N^2 - 1) - ((N-1)^2 - 1) = 2N - 1$ NGBs and the Lagrangian containing ϕ is invariant under

$$\phi = \exp\left\{ \frac{i}{f} \left(\begin{array}{c|c} & \vec{\pi} \\ \hline \vec{\pi}^\dagger & \pi_0/\sqrt{2} \end{array} \right) \right\} \begin{pmatrix} \vec{0}_{N-1} \\ f \end{pmatrix} \equiv e^{i\pi/f} \phi_0 \tag{1}$$

where π_0 is real and $\vec{\pi} \equiv (\pi_1, \ldots, \pi_{N-1})$ is made up of $N-1$ complex fields yielding the correct number of NGB's just as in the U(1) case. Let us now complete the analogy and examine how these NGB fields transform under the broken and unbroken symmetries U and U_{N-1} respectively.

Recalling that $U_{N-1}\phi_0 = \phi_0$, under the unbroken $SU(N-1)$ we have

$$\begin{aligned} \phi \to U_{N-1}\phi &= U_{N-1} e^{i\pi} \phi_0 &(2)\\ &= U_{N-1} e^{i\pi} U_{N-1}^\dagger U_{N-1} \phi_0 &(3)\\ &= e^{iU_{N-1}\pi U_{N-1}^\dagger} \phi_0 &(4)\\ & &(5) \end{aligned}$$

Thus, the π fields transform linearly as $\pi \to U_{N-1}\pi U_{N-1}^\dagger$. Under the broken transformations with parameter $\vec{\alpha}$ we have

$$\phi \to U e^{i\pi}\phi_0 = \exp\left\{ i \begin{pmatrix} 0 & \vec{\alpha} \\ \vec{\alpha}^\dagger & 0 \end{pmatrix} \right\} \exp\left\{ i \begin{pmatrix} 0 & \vec{\pi} \\ \vec{\pi}^\dagger & 0 \end{pmatrix} \right\} \phi_0 \tag{6}$$

$$= \exp\left\{ i \begin{pmatrix} 0 & \vec{\pi'} \\ \vec{\pi'}^\dagger & 0 \end{pmatrix} \right\} \phi_0 \tag{7}$$

$$\tag{8}$$

These π' are, in general, a complicated function of $\vec{\pi}$ and $\vec{\alpha}$ and thus the broken transformations are non-linearly realized on the π fields: $\vec{\pi} \to \vec{\pi'} = \vec{\pi} + \vec{\alpha} + g_{\text{Non. Lin.}}(\vec{\alpha}, \vec{\pi})$.

To summarize this section, we have shown that when a symmetry group G is broken to the coset space G/H, it generates a set of excitations known as Nambu-Goldstone Bosons. The number of NGBs is equal to the number of broken generators and the Lagrangian whidch includes these NGBs can only contain derivative interactions! Introducing a mass to the NGBs, for example, will explicitly break the shift symmetry as these fields do not transform linearly under the broken symmetries. In Little Higgs models, the Higgs is realized as a psuedo-NGB and we will have to address this problem of masslessness in order to obtain a non-zero Higgs mass.

3 The Toy Model

We begin with the simplest implementation of the Little Higgs Model. Fortunately, this utilizes familiar symmetry groups and a simple breaking pattern, $SU(3)/SU(2)$[1]. Consider a theory with a global $SU(3)$ symmetry group which is spontaneously broken to $SU(2)$ by a vacuum condensate i.e. the symmetry is spoiled as the vacuum chooses a preferential direction in the internal symmetry space. Note that this symmetry still exists in the Lagrangian, but not in the field configuration[2]. The original $SU(3)$ contained $3^2 - 1 = 8$ independent generators, while $SU(2)$ has only 3. These 5 broken generators produce Nambu-Goldstone Boson (NGB) fields which we denote by $\pi^a(x)$ for $a = 1..5$. Such a vacuum condensate at scale f is described by a

[1]The generalization to $SU(N) \to SU(N-1)$ or $SU(N) \to SO(N)$ is somewhat more complicated, and depends on the exact form of the symmetry breaking tensor multiplet. These more complex cases are not necessary to demonstrate the basic concepts behind Little Higgs models. They are, however, required when building realistic models and these details can be found in the references.

[2]Analogously, rotational invariance of a 2-d Ising model is still present in the Lagrangian even after the system undergoes a phase transition and aligns the spins preferentially along one direction. This is precisely what is implied by spontaneous breaking of a global symmetry.

field ϕ_0 transforming in the fundamental representation of $SU(3)$ where

$$\phi_0 = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \tag{9}$$

We can describe the low-energy dynamics of this system using a "non-linear sigma model". What can happen will happen, and the Lagrangian of this sigma field ϕ must in general include all possible Lorentz-invariant, local operators that can be constructed from $\phi(x)$ and its derivatives[3]. We parametrize $\phi(x)$ in terms of our broken symmetry generators X^a, the NGB fields π^a, and the original vacuum condensate ϕ_0.

$$\phi(x) = \frac{1}{f} \exp\left(\frac{2i\pi^a(x)X^a}{f}\right)\phi_0 \tag{10}$$

Note that the index a is implicitly summed over the 5 broken degrees of freedom. As we are interested in the low energy behavior, we first consider only the lowest order operator we can write down that is not a constant. This is given by $\mathcal{L}_{kinetic} = f^2|\partial_\mu\phi|^2$. Recalling the notation of Section 2, let us rewrite the NGB fields as

$$\pi = \left(\begin{array}{cc|c} 0 & & \multirow{2}{*}{$h(x)$} \\ & 0 & \\ \hline \multicolumn{2}{c|}{h(x)^\dagger} & 0 \end{array}\right) + \left(\begin{array}{cc|c} -\eta(x)/2 & 0 & \multirow{2}{*}{0} \\ 0 & -\eta(x)/2 & \\ \hline \multicolumn{2}{c|}{0} & \eta(x) \end{array}\right) \tag{11}$$

Here we ignore the second term proportional to η and identify $h(x)$ as the Standard Model Higgs doublet. In order to do this, however, we must also introduce gauge interactions to the unbroken $SU(2)$.[4] This is achieved by replacing the derivatives in $\mathcal{L}_{kinetic} = f^2|\partial_\mu\phi|^2$ with their gauge covariant counterparts $D_\mu = \partial_\mu - igW^a_\mu(x)Q^a$. Here W^a_μ are the $SU(2)$ gauge fields and

$$Q^a \equiv \begin{pmatrix} \sigma^a/2 & 0 \\ 0 & 0 \end{pmatrix} \tag{12}$$

Where σ^a are the three Pauli matrices. Now, one can easily expand the Lagrangian and identify the Standard Model Higgs/gauge-boson interactions.

[3]Often denoted Σ, but here denoted ϕ to keep the notation consistent.

[4]Strictly speaking, we must also include the SM $U(1)$ gauge group, but this is an unnecessary complication at this stage.

$$\mathcal{L}_{kinetic} = f^2|D_\mu\phi|^2 \quad = \quad |D_\mu h|^2 + \frac{|h^\dagger D_\mu h|^2}{f^2} + \cdots \tag{13}$$

Several issues still exist at this point: At tree level the Higgs is still a massless NGB. Furthermore, we have explicitly broken the global $SU(3)$ by introducing gauge interactions to the $SU(2)$ sector and the one-loop radiative corrections from the gauge bosons in Eq. 13 produce a Higgs mass term $\mu^2 h^\dagger h$ and quartic couplings $\lambda(h^\dagger h)^2$ as in the Standard Model. Choosing $\mu^2 < 0$ and $\lambda > 0$ induces electroweak symmetry breaking.

To summarize, we have taken a global $SU(3)$ symmetry and broken it at a scale f by introducing gauge interactions on the $SU(2)$ subgroup. In the process we have built a model of electroweak symmetry breaking consistent with the Standard Model Higgs doublet which provides the interactions and longitudinal degrees of freedom to the massive gauge bosons[5]. The remaining quadratic divergences to the Higgs mass squared are induced by the loop diagrams and ultimately depend on the on the UV completion of the low-energy theory. In our effective field theory approach, these diagrams will in general be cut-off at some scale $\Lambda \sim 4\pi f$. The leading order contributions to the Higgs potential are then gauge-boson loops which produce

$$\mu^2 = c\frac{g^2}{16\pi^2}\Lambda^2 \sim cg^2f^2, \qquad \lambda = c'\frac{g^2}{f^2 16\pi^2}\Lambda^2 \sim c'g^2$$

where c and c' are of order one in a natural theory. The Higgs mass has recently been measured at the LHC to be $m_h = \sqrt{2}|\mu| = \sqrt{c}gf = 125$ GeV and thus we must significantly tune c and c' in order to maintain consistency with a much higher cut-off scale $f = \Lambda/4\pi$.

Precision electroweak constraints have placed stringent bounds on new generic, strongly coupled theories. In particular, dimension-6 Higgs/Gauge-Boson operators require $\Lambda \gtrsim 9$ TeV unless they are suppressed by other means. Thus we have traded a Planck scale hierarchy problem for a little hierarchy problem at scale Λ.

Suppose instead of the above, we had gauged the entire $SU(3)$. In this case, each of the 8 NGBs would be eaten by the 8 gauge bosons. The quadratically divergent term now looks appears as

[5] Note that we are still neglecting the η component of the NGBs.

$$\frac{g^2}{16\pi^2}\Lambda^2\phi^\dagger \left(\begin{array}{cc|c} 1 & & 0 \\ & 1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)\phi \tag{14}$$

Clearly, this has no Higgs doublet as all of the non-zero terms are diagonal. All of the degrees of freedom were taken by the longitudinal modes of the gauge bosons. On the other hand, the quadratic divergence is in the vacuum energy, and not the Higgs mass term. Perhaps we can combine the behavior of these two gauge structures, and obtain a Higgs doublet while also stabilizing its mass. The elegance of Little Higgs models is in the solution to this issue.

4 Collective Symmetry Breaking

Our goal is to solve the phenomenological problems presented above. If we can break the global symmetry group G to H at a scale f, then $f \gtrsim 1$ TeV, so that $4\pi f \sim 10$ TeV, would evade the current precision electroweak constraints. Thus, we would like to modify our symmetry structure to achieve this while keeping in mind that we must preserve the gauge interactions achieved above.

Let us start with a product group of two identical global symmetry groups $G = G_1 \times G_2 = SU(3) \times SU(3)$ where we gauge a full $SU(3)$ of G_1 and G_2 using the same set of gauge covariant derivatives. The fields ϕ_1 and ϕ_2 are then characterized by two non-linear sigma models with aligned vacuum expectation values:

$$\phi_1 = e^{i\pi_1 f}\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}, \qquad \phi_2 = e^{i\pi_2 f}\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \tag{15}$$

The kinetic term now appears as the simple sum of the two kinetic terms $\mathcal{L}_{kinetic} = |D_\mu\phi_1|^2 + |D_\mu\phi_2|^2$. Each term individually contributes self energy diagrams which are quadratically divergent shown in the top row of Figure 4, but these diagrams containing only one species of ϕ do not contribute to the Higgs mass (i.e. they simply duplicate the last example of the previous section). However, there is now an additional set of diagrams shown in the bottom row of Figure 4 which mix ϕ_1 and ϕ_2 through a loop. This diagram is no longer invariant under the two $SU(3)$'s of G_1 and G_2, but is instead invariant under a diagonal $SU(3)$ subgroup that is also gauged. Thus, there are two linear combinations π_1 and π_2. One produces 5 exact NGB's which are eaten by the gauge bosons, and the orthogonal linear combination produces 5 PNGB's identified with the $SU(2)$ Higgs doublet h and singlet η.

Let us understand this more concretely by examining a term in the Lagrangian which breaks the individual symmetries. This involves two gauge bosons A_μ, and two scalar fields ϕ_i.

$$\mathcal{L} \supset |gA_\mu\phi_1|^2 + |gA_\mu\phi_2|^2 \tag{16}$$

Suppose now that we set the coupling g to zero for the ϕ_2 term. This would allow for independent $SU(3)$ transformations on ϕ_1 and ϕ_2. These symmetries would then be exact, and the Higgs cannot obtain a mass, as it must be an exact NGB. Likewise for the reciprocal case on the coefficient to ϕ_1. What this implies is that the entire symmetry group G must have non-zero gauge couplings in order for the Higgs to be realized as a massive Pseudo-Nambu-Goldstone-Boson. This phenomena is known as collective symmetry breaking because the gauge couplings collectively break the individual G_1 and G_2 $SU(3)$'s.

The point of all this was that the only diagram contributing to the Higgs mass is logarithmically, rather than quadratically divergent in the cutoff. The one-loop diagram shown in the bottom row of Figure 4 contributes to a log-divergent mass parameter, μ, such that

$$\mu^2 \sim \frac{g^2}{16\pi^2} f^2 \log\left(\frac{\Lambda^2}{f^2}\right) = \frac{g^2}{8\pi^2} f^2 \log\left(4\pi\right) \tag{17}$$

Recall that we needed $f \sim 1$ TeV to evade constraints on higher dimensional operators suppressed by the new strong force scale Λ. If we also want the Higgs quartic coupling λ to be of order 1, this produces the correct EWSB scale. There are now three scales in our theory: (i) the Higgs (PNGB) scale at $\mu \sim 100$ GeV, (ii) the symmetry breaking scale $f \sim 1$ TeV, and (iii) the scale of the new strong dynamics which must occur at $\Lambda \gtrsim 10$ TeV. We now have a prototype for how the Little Higgs mechanism can solve the hierarchy problem, and subsequently, the little hierarchy problem.

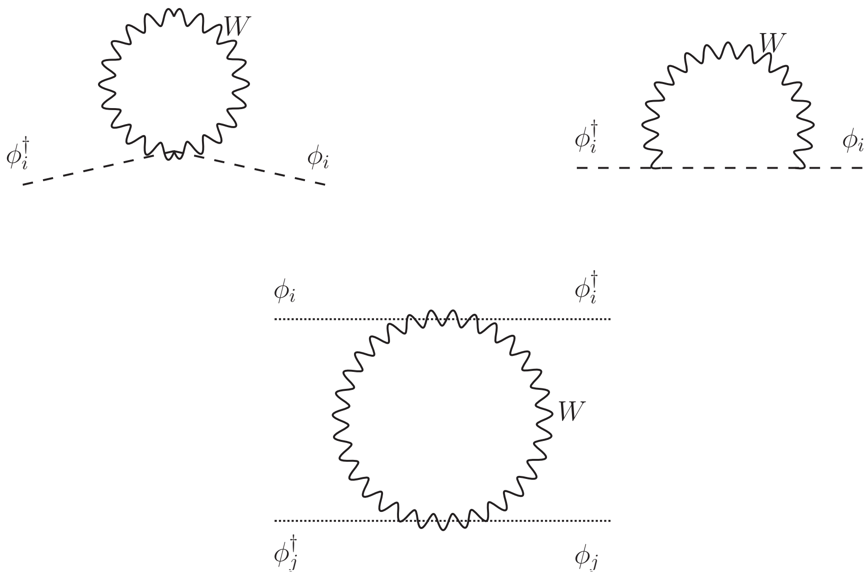

Figure 1: (Top) Quadratically divergent loop diagrams contributing to the vacuum energy. These diagrams involve only a single field ϕ_i. (Bottom) Logarithmically divergent diagrams which contribute to the Higgs mass. Such diagrams break $SU(3)_1 \times SU(3)_2 \to SU(3)_{diag}$ and allow for a stable electroweak symmetry breaking scale.

5 Discussion and Conclusions

At this point, the group theory which is generic to little Higgs models has been presented. We have reviewd NGB's, shown how the SM Higgs doublet can arise as a Pseudo-Nambu-Goldstone-Boson when an $SU(3)$ global symmetry is broken to a gauged $SU(2)$ subgroup. However, this proved inadequate when considering current precision electroweak constraints on strongly coupled theories. These constraints required us to reintroduce tuning to suppress new effective operators at the scale of the strong dynamics, albeit much less tuning than in the Standard Model. Finally, we demonstrated how the concept of collective symmetry breaking can be used to ensure quadratically divergent contributions to the Higgs mass squared do not occur at one loop, while logarithmically divergent radiative corrections are able to reproduce the correct electroweak symmetry breaking scale, with a stipulation on the yet unmeasured Higgs quartic coupling.

In an analogous construction, one can use this collective breaking to eliminate the quadratic divergences to the Higgs mass arising from the heavy fermion sector. In particular, the top quark contribution. In this case, it is the Yukawa couplings which explicitly break the global symmetries and they must be also be collectively broken.

The Littlest Higgs model embeds the Standard model into an $SU(5)/SO(5)$ non-linear sigma model with ϕ_0 given by a symmetric $SU(5)$ tensor multiplet. This breaking produces 24-10=14 NGB's. An $[SU(2)\times U(1)]^2$ subgroup of the original $SU(5)$ is gauged and one follows effectively the same procedure as discussed above. The $[SU(2)\times U(1)]^2$ is broken to a diagonal subgroup identified with the SM gauge groups and the couplings can be set to the SM weak and hypercharge gauge couplings. The remaining freedom is parameterized by two mixing angles. At this stage, the collective symmetry breaking again mandates that the Higgs mass arises from diagrams containing *at least 2* gauge couplings, and ensuring that the one-loop quadratic divergences vanish.

In order to stabilize contributions from the fermion sector, i.e. the top quark, one must minimally add a pair of weak-singlet Weyl fermions which couple to the third generation SM quark doublet and singlet through two couplings λ_1 and λ_2. At this point, the top quark acquires mass in the usual way, and collective symmetry breaking ensures that the quadratic divergences to the Higgs mass vanish while there are again log-divergent diagrams involving both λ_1 and λ_2. The relations among these couplings and the top mass is likely to be within the reach of the LHC upgrade and will provide some of the earliest constraints on little Higgs models outside of the current precision electroweak data.

Even excluding scales new strong dynamics, variations of the Little Higgs mechanism can lead to a very rich phenomenology which is largely within the reach of LHC upgrades. Several models impose a discrete Z_2 symmetry known as "T-party" in order to alleviate tension with additional electroweak observables. This stabilizing symmetry naturally produces a thermal WIMP dark matter candidate, the LTP, which is likely probable by next generation direct and indirect detection experiments, as well as at the LHC and ILC. This discriminant dark matter complementarity offers significant unique features features.

While a complete description of EWSB lies tantalizingly just beyond current experimental reach, it is as much exciting as confounding and pessimistic. Little Higgs models are capable of addressing the most fundamental problem facing particle physics beyond the standard model, the hierarchy problem, and yet make up just a fragment of the possible solutions. If the Little Higgs is not realized in nature, it nonetheless provides a beautiful example of group theories applications in particle physics and elegantly relates differing mass scales through a simple gauge and global symmetry structure.

References

1. *Group theory of the spontaneously broken gauge symmetries*; Phys. Rev. D 9, 1723-1739 (1974)

2. *Electroweak symmetry breaking from dimensional deconstruction*; Nima Arkani-Hamed, Andrew G. Cohen, Howard Georgi

3. *Vacuum symmetry and the pseudo-Goldstone phenomenon*; Phys. Rev. D 12, 508-512 (1975)

4. *Little Higgs Review*; Martin Schmaltz, David Tucker-Smith http://arxiv.org/pdf/hep-ph/0105239v4.pdf

5. *Little Higgs Models and Their Phenomenology*; Maxim Perelstein http://arxiv.org/pdf/hep-ph/0512128v1.pdf

6. *Little Higgs Dark Matter*; Andreas Birkedal, Andrew Noble, Maxim Perelstein, Andrew Spray http://arxiv.org/abs/hep-ph/0603077

7. *Notes on the spontaneous breaking of SU(N) and SO(N) via a second-rank tensor multiplet*; Just google it! Howard Haber