$\mathscr{G}$	Conditions on $\mathbf{A} \in \mathscr{G}$	$\mathscr{L}$	Conditions on $\mathbf{a} \in \mathscr{L}$	n
$GL(N, C)$	—	$gl(N, C)$	—	$2N^2$
$GL(N, R)$	$\mathbf{A}$ real	$gl(N, R)$	$\mathbf{a}$ real	N^2
$SL(N, C)$	$\det \mathbf{A} = 1$	$sl(N, C)$	$\operatorname{tr} \mathbf{a} = 0$	$2N^2-2$
$SL(N, R)$	$\mathbf{A}$ real, $\det \mathbf{A} = 1$	$sl(N, R)$	$\mathbf{a}$ real, $\operatorname{tr} \mathbf{a} = 0$	N^2-1
$U(N)$	$\mathbf{A}^\dagger = \mathbf{A}^{-1}$	$u(N)$	$\mathbf{a}^\dagger = -\mathbf{a}$	N^2
$SU(N)$	$\mathbf{A}^\dagger = \mathbf{A}^{-1}$, $\det \mathbf{A} = 1$	$su(N)$	$\mathbf{a}^\dagger = -\mathbf{a}$, $\operatorname{tr} \mathbf{a} = 0$	N^2-1
$U(p, q)$	$\mathbf{A}^\dagger \mathbf{g} = \mathbf{g}\mathbf{A}^{-1}$	$u(p, q)$	$\mathbf{a}^\dagger \mathbf{g} = -\mathbf{g}\mathbf{a}$	N^2
$SU(p, q)$	$\mathbf{A}^\dagger \mathbf{g} = \mathbf{g}\mathbf{A}^{-1}$, $\det \mathbf{A} = 1$	$su(p, q)$	$\mathbf{a}^\dagger \mathbf{g} = -\mathbf{g}\mathbf{a}$, $\operatorname{tr} \mathbf{a} = 0$	N^2-1
$O(N, C)$	$\tilde{\mathbf{A}} = \mathbf{A}^{-1}$	$so(N, C)$	$\tilde{\mathbf{a}} = -\mathbf{a}$	N^2-N
$SO(N, C)$	$\tilde{\mathbf{A}} = \mathbf{A}^{-1}$, $\det \mathbf{A} = 1$			
$O(N)$	$\tilde{\mathbf{A}} = \mathbf{A}^{-1}$, $\mathbf{A}$ real	$so(N)$	$\tilde{\mathbf{a}} = -\mathbf{a}$, $\mathbf{a}$ real	$\frac{1}{2}(N^2-N)$
$SO(N)$	$\tilde{\mathbf{A}} = \mathbf{A}^{-1}$, $\mathbf{A}$ real, $\det \mathbf{A} = 1$			
$O(p, q)$	$\tilde{\mathbf{A}}\mathbf{g} = \mathbf{g}\mathbf{A}^{-1}$, $\mathbf{A}$ real	$so(p, q)$	$\tilde{\mathbf{a}}\mathbf{g} = -\mathbf{g}\mathbf{a}$, $\mathbf{a}$ real	$\frac{1}{2}(N^2-N)$
$SO(p, q)$	$\tilde{\mathbf{A}}\mathbf{g} = \mathbf{g}\mathbf{A}^{-1}$, $\mathbf{A}$ real, $\det \mathbf{A} = 1$			
$SO^*(N)$	$\tilde{\mathbf{A}} = \mathbf{A}^{-1}$, $\mathbf{A}^\dagger \mathbf{J}\mathbf{A} = \mathbf{J}$	$so^*(N)$	$\tilde{\mathbf{a}} = -\mathbf{a}$, $\mathbf{a}^\dagger \mathbf{J} = -\mathbf{J}\mathbf{a}$	$\frac{1}{2}(N^2-N)$
$Sp(\frac{1}{2}N, C)$	$\tilde{\mathbf{A}}\mathbf{J}\mathbf{A} = \mathbf{J}$	$sp(\frac{1}{2}N, C)$	$\tilde{\mathbf{a}}\mathbf{J} = -\mathbf{J}\mathbf{a}$	N^2+N
$Sp(\frac{1}{2}N, R)$	$\tilde{\mathbf{A}}\mathbf{J}\mathbf{A} = \mathbf{J}$, $\mathbf{A}$ real	$sp(\frac{1}{2}N, R)$	$\tilde{\mathbf{a}}\mathbf{J} = -\mathbf{J}\mathbf{a}$, $\mathbf{a}$ real	$\frac{1}{2}(N^2+N)$
$Sp(\frac{1}{2}N)$	$\tilde{\mathbf{A}}\mathbf{J}\mathbf{A} = \mathbf{J}$, $\mathbf{A}^\dagger = \mathbf{A}^{-1}$	$sp(\frac{1}{2}N)$	$\tilde{\mathbf{a}}\mathbf{J} = -\mathbf{J}\mathbf{a}$, $\mathbf{a}^\dagger = -\mathbf{a}$	$\frac{1}{2}(N^2+N)$
$Sp(r, s)$	$\tilde{\mathbf{A}}\mathbf{J}\mathbf{A} = \mathbf{J}$, $\mathbf{A}^\dagger \mathbf{G}\mathbf{A} = \mathbf{G}$	$sp(r, s)$	$\tilde{\mathbf{a}}\mathbf{J} = -\mathbf{J}\mathbf{a}$, $\mathbf{a}^\dagger \mathbf{G} = -\mathbf{G}\mathbf{a}$	$\frac{1}{2}(N^2+N)$
$SU^*(N)$	$\mathbf{J}\mathbf{A}^* = \mathbf{A}\mathbf{J}$, $\det \mathbf{A} = 1$	$su^*(N)$	$\mathbf{J}\mathbf{a}^* = \mathbf{a}\mathbf{J}$, $\operatorname{tr} \mathbf{a} = 0$	N^2-1

note: A is an invertable matrix, whereas no such condition is imposed on a.

Table 10.1 The real Lie algebras $\mathscr{L}$ of some important linear Lie groups $\mathscr{G}$. $\mathbf{A}$ and $\mathbf{a}$ are $N \times N$ matrices, which are complex unless otherwise stated; $\mathbf{g}$ is an $N \times N$ diagonal matrix with p diagonal elements $+1$ and $q(=N-p)$ diagonal elements -1, $p \geq q \geq 1$. In the last five entries N is even, and $\mathbf{J}$ and $\mathbf{G}$ are the $N \times N$ matrices defined by

$$\mathbf{J} = \begin{bmatrix} 0 & \mathbf{1}_{N/2} \\ -\mathbf{1}_{N/2} & 0 \end{bmatrix}, \qquad \mathbf{G} = \begin{bmatrix} -\mathbf{1}_r & 0 & 0 & 0 \\ 0 & \mathbf{1}_s & 0 & 0 \\ 0 & 0 & -\mathbf{1}_r & 0 \\ 0 & 0 & 0 & \mathbf{1}_s \end{bmatrix}$$

where $1 \leq r \leq \frac{1}{2}N$ and $s = \frac{1}{2}N - r$. Note that $\tilde{A}$ denotes the transpose of the matrix A.