The Gell-Mann matrices are the traceless hermitian generators of the Lie algebra $\mathfrak{su}(3)$, analogous to the Pauli matrices of $\mathfrak{su}(2)$.

The eight Gell-Mann matrices are defined by:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

The Gell-Mann matrices satisfy commutation relation,

$$[\lambda_a, \lambda_b] = 2i f_{abc} \lambda_c \,, \qquad \text{where } a, b, c = 1, 2, 3, \ldots, 8 \,,$$

where there is an implicit sum over c, and the structure constants f_{abc} are totally antisymmetric under the interchange of any pair of indices. The explicit form of the non-zero $\mathfrak{su}(3)$ structure constants are listed in Table 1.

Table 1: Non-zero structure constants[1] f_{abc} of $\mathfrak{su}(3)$.

abc	f_{abc}	abc	f_{abc}
123	1	345	$\frac{1}{2}$
147	$\frac{1}{2}$	367	$-\frac{1}{2}$
156	$-\frac{1}{2}$	458	$\frac{1}{2}\sqrt{3}$
246	$\frac{1}{2}$	678	$\frac{1}{2}\sqrt{3}$
257	$\frac{1}{2}$		

[1] The f_{abc} are antisymmetric under the permutation of any pair of indices.

The following properties of the Gell-Mann matrices are also useful:

$$\mathrm{Tr}(\lambda_a \lambda_b) = 2\delta_{ab} \,, \qquad \{\lambda_a, \lambda_b\} = 2d_{abc}\lambda_c + \tfrac{4}{3}\delta_{ab} \,,$$

where $\{A, B\} \equiv AB + BA$ is the anticommutator of A and B. It follows that

$$f_{abc} = -\tfrac{1}{4} i \,\mathrm{Tr}\big(\lambda_a[\lambda_b, \lambda_c]\big) , \qquad\qquad d_{abc} = \tfrac{1}{4}\mathrm{Tr}\big(\lambda_a\{\lambda_b, \lambda_c\}\big) .$$

The d_{abc} are totally symmetric under the interchange of any pair of indices. The explicit form of the non-zero d_{abc} are listed in Table 2.

Table 2: Non-zero independent elements of the tensor[2] d_{abc} of $\mathfrak{su}(3)$.

abc	d_{abc}	abc	d_{abc}
118	$\frac{1}{\sqrt{3}}$	355	$\frac{1}{2}$
146	$\frac{1}{2}$	366	$-\frac{1}{2}$
157	$\frac{1}{2}$	377	$-\frac{1}{2}$
228	$\frac{1}{\sqrt{3}}$	448	$-\frac{1}{2\sqrt{3}}$
247	$-\frac{1}{2}$	558	$-\frac{1}{2\sqrt{3}}$
256	$\frac{1}{2}$	668	$-\frac{1}{2\sqrt{3}}$
338	$\frac{1}{\sqrt{3}}$	778	$-\frac{1}{2\sqrt{3}}$
344	$\frac{1}{2}$	888	$-\frac{1}{\sqrt{3}}$

[2]The d_{abc} are symmetric under the permutation of any pair of indices.

The d_{abc} can be employed to construct a cubic Casimir operator for $\mathfrak{su}(3)$,

$$C_3 \equiv \tfrac{1}{8} d_{abc}\lambda_a\lambda_b\lambda_c ,$$

where all repeated indices are summed over. The overall factor of $\frac{1}{8}$ is conventional. It is straightforward to prove that,

$$[\lambda_a, C_3] = 0 , \qquad \text{for } a = 1, 2, 3, \ldots, 8 .$$

Since C_3 commutes with all the $\mathfrak{su}(3)$ generators of the defining representation, it follows that C_3 is a multiple of the identity. One can define C_3 for any d-dimensional irreducible representation of $\mathfrak{su}(3)$. Denoting the traceless hermitian generators by R_a,[1]

$$C_3(R) \equiv d_{abc} R_a R_b R_c = c_{3R}\mathbf{I} ,$$

where $\mathbf{I}$ is the $d \times d$ identity matrix. For an irreducible representation of $\mathfrak{su}(3)$ denoted by (n, m), corresponding to a Young diagram with $n + m$ boxes in the first row and n boxes in the second row, the eigenvalue of the cubic Casimir operator is given by:

$$c_3 = \tfrac{1}{2}(m - n)\left[\tfrac{2}{9}(m + n)^2 + \tfrac{1}{9}mn + m + n + 1\right] .$$

[1]The traceless hermitian generators R_a satisfy $[R_a, R_b] = i f_{abc} R_c$. In the defining representation of $\mathfrak{su}(3)$, $R_a = \frac{1}{2}\lambda_a$ and in the adjoint representation of $\mathfrak{su}(3)$, $(R_a)_{bc} = -i f_{abc}$.