GROUP STRUCTURE OF SPONTANEOUSLY BROKEN GAUGE THEORIES

Joseph Connell

June 7, 2017

Phys 251, Group Theory, Dr. Howard Haber, UCSC

Outline

- Global Symmetries
  - ϕ^4 Theory
  - ϕ_i Linear Sigma Model
- Goldstone's Theorem
- Local Symmetries (Gauge Theories)
  - Abelian Higgs Mechanism
  - Non-abelian Gauge Theories
    - Electroweak Theory
    - QCD
    - Grand Unification

Symmetries of $\mathscr{L}$

Spacetime:

- Translations
- Rotations
- Boosts

All QFT's have these $\rightarrow$ Poinaré symmetry

Internal:

- Non-spacetime
- Quantum Numbers
- Noether Charge

Symmetries of $\mathscr{L}$

Symmetry breaking can be:

Explicit: $\mathscr{L}$ does not respect the symmetry, but the perturbation is small $\Rightarrow$ approximate symmetry.

Spontaneous: $\mathscr{L}$ does respect the symmetry, but the ground state breaks it $\Rightarrow$ vacuum expectation value (vev) = v

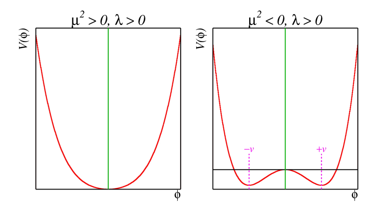
ϕ^4 Theory

In a Lagrangian with one scalar field $\phi(t, x, y, z)$

$$\mathscr{L} = T(\partial_\mu \phi) - V(\phi, \tau)$$

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4$$

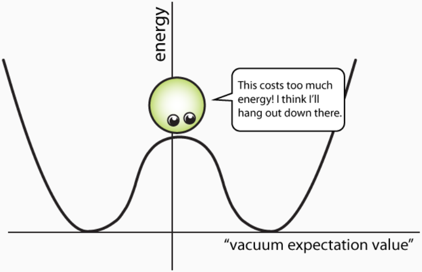
m^2 and λ are τ dependent. For $\tau < \tau_c$, $m^2 \to -m^2$



$\mathscr{L}$ is invariant under $\phi \to -\phi \Rightarrow \mathbb{Z}_2$ symmetry $\{I, -I\}$

ϕ^4 Theory

SPONTANEOUS SYMMETRY BREAKING



However, the system falls into one of its new ground states at $V_{min} = V(\phi_o)$, where $\phi_o = \pm\sqrt{\frac{6m^2}{\lambda}} = \pm v$

ϕ^4 Theory

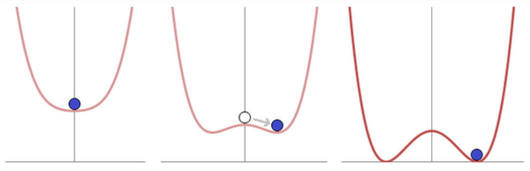
Shifting the origin to ϕ_o, we define a new field

$$\phi(x) = v + \sigma(x)$$

Expanding $\mathscr{L}$ about the classical minimum V_{min}:

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}2m^2\sigma^2 - \sqrt{\frac{\lambda}{6}}m\sigma^3 - \frac{\lambda}{4!}\sigma^4$$

- The $\mathbb{Z}_2$ **symmetry is spontaneously broken** by the σ^3term
- This $\mathscr{L}$ now describes $\sigma(x)$, a **scalar field with mass** = $\sqrt{2}m$



Linear Sigma Model

Generalizing to N fields $\phi_i, i = 1, ..., N$, the symmetry becomes continuous.

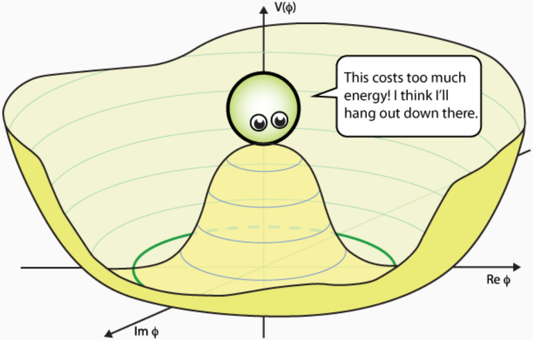
$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}m^2(\phi^i)^2 - \frac{\lambda}{4}[(\phi^i)^2]^2$$

This $\mathscr{L}$ is invariant under the rotation group O(N):

$$\phi^i \rightarrow R^{ij}\phi^j \tag{1}$$

Now V_{min} is satisfied for any ϕ_o^i vector with length $\phi_o^i = \sqrt{\frac{m^2}{\lambda}} = v$ and **arbitrary direction**.

N=2 Visualization



Circular minima: $\phi_1^2 + \phi_2^2 = \frac{m^2}{\lambda^2}$

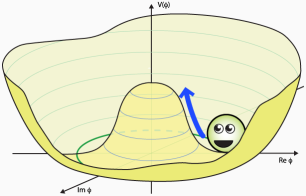
Linear Sigma Model

Pick the N^{th} field to minimize $V(\phi)$:

$$\phi_o^i = (0, 0, ..., 0, v)$$

Define shifted fields:

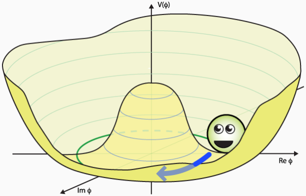
$$\phi^i(x) = (\pi^k(x), v + \sigma(x)) \qquad k = 1, .., N-1$$



The radial curvature of $V(\phi)$ makes $\sigma(x)$ a **massive field** (mass term in $\mathscr{L}$).

Linear Sigma Model

- $\mathscr{L}$ will still have an unbroken O(N-1) symmetry.
- O(N-1) is the little group H that leaves ϕ_o invariant.
- The $\pi^k(x)$ fields rotate in an (N-1)-dimensional trough.



For N=4, the 3 massless ”pion” fields correspond to the real particles:

Pions π^+, π^0, and π^-

Goldstone's Theorem

Spontaneous breaking of continuous global symmetries implies the existence of massless, spinless particles.

Massless Nambu-Goldstone bosons occur for each generator of the broken symmetry.

Ex: N=4, O(N-1) has 3 generators $\Rightarrow$ 3 massless pions: $\pi^{\pm,o}$

Goldstone's Theorem

$\mathscr{L}$ remains invariant under the little group O(N-1)

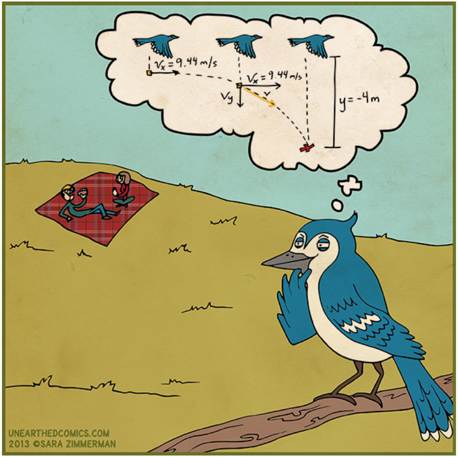
$$\phi_i' = e^{i\alpha^a T^a_{ij}}\phi_j = (I + i\alpha^a T^a_{ij} + ...)\phi_j$$

where the T^a are generators of the Lie group that satisfy the Lie algebra:

$$[T^a, T^b] = if^{abc}T^c$$

A massless Goldstone boson arises for each generator T.

Pause



$V_x = 9.44$ m/s
$V_x = 9.44$ m/s
V_y
V
y = -4m
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Gauge Theories

A ”gauged” transformation has **local spacetime dependence**.

$$\alpha \to \alpha(x)$$

For example, a $\mathscr{L}$ can be invariant under an abelian U(1) gauge transformation G:

$$\phi \to e^{i\alpha(x)}\phi(x) \qquad \text{and} \qquad A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x)$$

where A_μ is a gauge field.

To keep $\mathscr{L}$ invariant under G, we need a covariant derivative D_μ:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ieA_\mu$$

For a complex scalar field coupled to itself and an EM field:

$$\mathscr{L} = |D_\mu \phi|^2 - V(\phi) - \frac{1}{4}(F_{\mu\nu})^2$$

where $F_{\mu\nu}$ is the EM field-strength tensor.

Gauge Theories with SSB

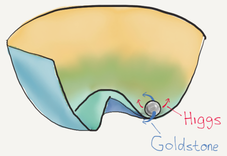
What happens if we combine local gauge invariance and symmetry breaking?

Abelian Higgs Model

Take a **complex** scalar field with the ϕ^4 potential:

The U(1) global symmetry is broken by $\phi_o = v$
Expanding about v:

$$\phi(x) = \phi_o + \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x))$$



The rewritten $\mathscr{L}$ describes a massive ϕ_1 field and ϕ_2 is a massless Goldstone boson.

Abelian Higgs Model

The kinetic term, with a gauged transformation, D_μ, and SSB:

$$|D_\mu \phi|^2 = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 + (\sqrt{2} e \phi_o) A_\mu \partial^\mu \phi_2 + e^2 \phi_o^2 A_\mu A^\mu + \ldots$$

A_μ is a massive gauge field coupled to ϕ_2.

Abelian Higgs Model

Unitary Gauge: Pick $\alpha(x)$ such that ϕ_2 is eliminated from the theory.

$$\mathscr{L}_{unitary} = (\partial_\mu \phi)^2 + e^2 \phi^2 A_\mu A^\mu - V(\phi) - \frac{1}{4}(F_{\mu\nu})^2$$

Higgs Mechanism: If $V(\phi)$ favors $\phi_o = v$, this $\mathscr{L}$ describes a **massive gauge boson** A_μ and a massive field ϕ_1:

$$\phi_1 = h = \text{Higgs Boson}$$

Non-Abelian Gauge Theories

Generalize to local gauge symmetries with non-commuting generators.

Non-abelian examples:

- QCD
- Electoweak
- Standard Model
- Grand Unified Theories

Non-Abelian Gauge Theories

Quantum Chromodynamics:

Gauge Group: $SU(3)$

The strong interactions can be described by an unbroken SU(3) color gauge theory with a scalar field in the adjoint representation.

The adjoint rep. has $n^2 - 1$ generators.

$$\mathrm{SU}(3) \Rightarrow 3^2 - 1 = 8 \text{ massless gluons}$$

Non-Abelian Gauge Theories

Electroweak:

$$SU(2)_L \bigotimes U(1)_Y$$

- $SU(2)_L$: weak isospin, 3 generators
- $U(1)_Y$: weak hypercharge, 1 generator

The symmetry is broken below the unification energy (246 GeV):

$$SU(2) \bigotimes U(1)_Y \Rightarrow U(1)_{EM}$$

- 3 massive bosons: $W^{\pm}, Z^{o} \rightarrow$ weak force
- 1 massless boson: γ photon $\rightarrow$ EM force

Non-Abelian Gauge Theories

<u>Standard Model:</u>

$$SU(3) \bigotimes SU(2)_L \bigotimes U(1)_Y$$

- $SU(3)$: strong force
- $SU(2) \bigotimes U(1)_Y$: electroweak force

If we break the second of these in a more complicated way, we get an extended **Higgs sector**, which is necessary for SUSY, MSSM, and CPV for early universe baryon-antibaryon asymmetry.

Non-Abelian Gauge Theories

Grand Unified Theories:

Georgi-Glashow Model, SU(5):

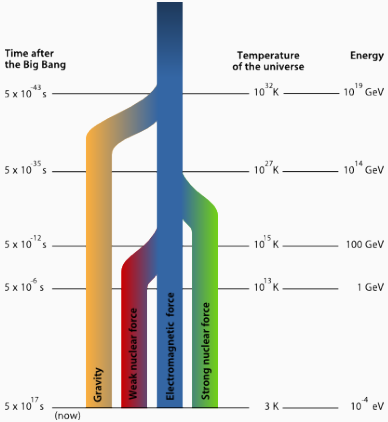
$$SU(5) \supset SU(3) \bigotimes SU(2)_L \bigotimes U(1)_Y$$

*Predicts proton decay (never observed)

Other candidates:

- SO(10)
- SU(8)
- O(16)

Grand Unified Theories:



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QUESTIONS?