

Presentation: The Effect of extra dimensions on gauge coupling in MSSM

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This presentation is based on the paper written by Keith R. Dienes, Emilian Dudas and Tony Gherghetta on effect of extra dimensions on gauge coupling in minimal supersymmetric standard model.

SUSY

Before we get into the effect of extra dimensions on supersymmetry, we need to first understand what is supersymmetry. A classical definition of symmetry is a transformation that leaves the physical system unchanged or in field theory a transformation that leaves the field lagrangian density not change. As for supersymmetry, it is a symmetry that relates that fermion and boson (assumption that every fermion has a superpartner boson that with same charge) or a transformation turns a bosonic state into a fermionic state, and vice versa. This feature allowed us to make approximate cancellation in huge boson loop contribution to the vacuum energy with fermion loop, so we don't have quadric divergent in the mass of boson in the loop diagram say for Higgs. For example, Two-loop corrections to the Higgs squared mass parameter involving a heavy fermion F that couples only indirectly to the Standard Model Higgs through gauge interactions is given:



With

$$\Delta m_H^2 = C_H T_F \left(\frac{g^2}{16\pi^2} \right)^2 \left[a\Lambda_{UV}^2 + 24m_F^2 \ln(\Lambda_{UV}/m_F) + \dots \right]$$

And these contributions to Δm_H^2 are sensitive both to the largest masses and to the physical ultraviolet cutoff in the theory gives an extremely large contributions to Δm_H^2 . However, with supersymmetry, the contribution of very heavy fermion are canceled by the two-loop effects of some very heavy bosons.

SUSY addressed many issues in modern physics theoretically, yield a profound result in theoretical physics. However, the typical superparticles have the mass term that larger than 100 GeV, although we haven't find exact evidence of superparticles but with 13 TeV LHC in current ATLAS experiment search for superparticles(eg, stau or tau slepton that may play a role in neutralino coannihilation). Despite insufficient experimental evidence, the frame work of

supersymmetry expand the clearly incomplete theory that did not address many problems, namely standard model.

In this presentation, I will talk about low energy supersymmetric extension of the standard model, namely MSSM(minimal supersymmetric standard model).

The MSSM is based one four basic assumption:

- 1, The MSSM is based on the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ (minimal gauge group)
- 2, There are only three generations of spin-1/2 quarks and leptons.(Minimal particle content)
- 3, R-parity conservation where R is defined by

$$R = (-1)^{2s+3B+L}$$

Where L and B are the lepton and baryon numbers and s is spin quantum number. The -1 for supersymmetric particles and 1 for ordinary particles

- 4, Soft supersymmetry breaking which does not allow reappearance of the quadratic divergences. It is defined as followed.

$$V_{\text{soft}} = -\mathcal{L}_{\text{sfermions}} - \mathcal{L}_{\text{Higgs}} - \mathcal{L}_{\text{tril.}}$$

Where each of terms is the following

$$-\mathcal{L}_{\text{sfermions}} = \sum_{i=\text{gen}} m_{\tilde{Q},i}^2 \tilde{Q}_i^\dagger \tilde{Q}_i + m_{\tilde{L},i}^2 \tilde{L}_i^\dagger \tilde{L}_i + m_{\tilde{u},i}^2 |\tilde{u}_{Ri}|^2 + m_{\tilde{d},i}^2 |\tilde{d}_{Ri}|^2 + m_{\tilde{l},i}^2 |\tilde{l}_{Ri}|^2$$

$$-\mathcal{L}_{\text{Higgs}} = m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + B\mu(H_u \cdot H_d + \text{h.c.})$$

and

$$-\mathcal{L}_{\text{tril.}} = \sum_{i,j=\text{gen}} \left[A_{ij}^u Y_{ij}^u \tilde{u}_{Ri} H_u \cdot \tilde{Q}_j + A_{ij}^d Y_{ij}^d \tilde{d}_{Ri} H_d \cdot \tilde{Q}_j + A_{ij}^l Y_{ij}^l \tilde{l}_{Ri} H_u \cdot \tilde{L}_j + \text{h.c.} \right]$$

also

Extra dimensions:

This presentation is intended to introduce the effects of extra spacetime dimensions at intermediate mass scales within the Minimal Supersymmetric Standard Model according to reference[1,2]. In the conventional grand unification scenarios, we have too high of energy scale that so high $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV, we can't do anything experimental at low energy. Thus, the need of bring down unification scale appeared.

Particles states with mass m_0 in MSSM generally have infinite tower of Kaluza-Klein states with masses written as

$$m_n^2 \equiv m_0^2 + \sum_{i=1}^{\delta} \frac{n_i^2}{R^2}$$

Where R is fixed radius from $\delta \equiv D - 4$ extra spacetime dimensions and $n_i \in \mathbb{Z}$ are the corresponding Kaluza-Klein excitation numbers. Also each state exactly mirrors the zero-mode MSSM ground state.

As the massive Kaluza-Klein states fall into representations of $N = 2$ supersymmetry we know that gauge group particle content of the MSSM is augmented by $N=2$ vector supermultiplet and $N = 2$ hypermultiplet for the two Higgs doublets at mass level N .

$$V = \begin{pmatrix} A_{\mu}^{(n)} & \phi^{(n)} \\ \lambda^{(n)} & \psi^{(n)} \end{pmatrix}, \quad H = \begin{pmatrix} H_1^{(n)} & H_2^{(n)} \\ \psi_{H_1}^{(n)} & \psi_{H_2}^{(n)} \end{pmatrix}$$

Since when $\delta \geq 2$, we can be complexify the compact coordinates in string theory so we can work with $\delta=1,2$.

We have

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z}$$

As one loop corrections to the gauge coupling g_i , where $\alpha_i \equiv g_i^2/4\pi$ and b_i defined as

$$(b_1, b_2, b_3) = (33/5, 1, -3)$$

For $M_Z \equiv 91.17$ GeV as low energy reference scale, we have gauge coupling

$$\begin{aligned} \alpha_Y^{-1}(M_Z)|_{\overline{\text{MS}}} &\equiv 98.29 \pm 0.13 \\ \alpha_2^{-1}(M_Z)|_{\overline{\text{MS}}} &\equiv 29.61 \pm 0.13 \\ \alpha_3^{-1}(M_Z)|_{\overline{\text{MS}}} &\equiv 8.5 \pm 0.5 \end{aligned}$$

These coupling leads to unification relation

$$\alpha_1(M_{\text{GUT}}) = \alpha_2(M_{\text{GUT}}) = \alpha_3(M_{\text{GUT}}) \approx \frac{1}{24}$$

at the unification scale $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV

We have seen that one-loop MSSM gauge couplings run only logarithmically with energy scale μ , the fact that we need many orders higher energy before unify with arbray coupling at the weak scale. We could essentially running the gauge coupling faster, say exponentially so that we could unify at higher rate without getting into the extreme large energy scale. And to accomplish this, we could add extra dimensions to the theory.

The existence of extra dimensions is the basic assumption of string theory and they remain compactified at very small scale (usually Planck scale). However, it is not impossible that extra dimension have effect on scale which is larger than the Planck scale as some studies have shown that the effects of extra dimensions below the Planck scale have played a role in understanding the strong-coupling behaviour of string theory. However, much of these shield gauge coupling from the effect of gauge coupling. In the study done by P. Hořava and E. Witten [6] gauge coupling were shielded from the effect of extra dimension but gravitational coupling are not but as paper [1] which this presentation is based on states that existence of extra spacetime dimensions at an intermediate mass scale influence the gauge couplings that one should not neglect. Also, string theory is not really needed here which we can study the effect within MSSM.

However, with extra dimension, we loss the renormalizability of theory. However there exists a renormalizable theory that are essentially equivalent to the theory I will talk about in the following.

In the case of extra dimension, we need to add massive Kaluza-Klein states in the loops. We could do the calculation only involve Kaluza-Klein states of a single Dirac fermion charged under a U(1) gauge group then generalized to MSSM with simpler calculation. For the vacuum polarisation contribution of a single Dirac fermion with Kaluza-Klein excitations we have

$$\Pi_{\mu\nu}(k^2) = - \sum_{n_i=-\infty}^{\infty} g^2 \int_0^\infty \frac{d^4 q}{(2\pi)^4} \text{Tr} \left(\gamma_\mu \frac{1}{\not{q} - m_n} \gamma_\nu \frac{1}{\not{k} + \not{q} - m_n} \right)$$

We will take $m_0=0$ for simplicity since we have in previous that R^{-1} is much larger than current observable scale we could then neglect m_0 according to previous given equation of mass of particles states in MSSM. Then, by Jacobi ϑ_3 function

$$\vartheta_3 \left(\frac{it}{\pi R^2} \right) \approx R \sqrt{\frac{\pi}{t}}$$

Use relationship $\Pi_{\mu\nu}(k^2) = (k_\mu k_\nu - g_{\mu\nu} k^2) \Pi(k^2)$ we can write $\Pi(0)$ as

$$\Pi(0) = \frac{g^2}{12\pi^2} \int_0^\infty \frac{dt}{t} \left\{ \vartheta_3 \left(\frac{it}{\pi R^2} \right) \right\}^\delta$$

introduce infrared and ultraviolet regulators

$$\int_0^\infty dt \longrightarrow \int_{r\Lambda^{-2}}^{r\mu_0^{-2}} dt$$

Take $R \rightarrow 0$ we have Jacobi $\vartheta_3 \rightarrow 1$ so we arrive with

$$\Pi(0) = -\frac{g^2}{6\pi^2} \ln \frac{\Lambda}{\mu_0} = \frac{g^2 b}{8\pi^2} \ln \frac{\Lambda}{\mu_0}$$

Which $b = -4/3$ as the β -function coefficient

We have an $N = 2$ vector multiplet and an $N = 2$ hypermultiplet for each Kaluza-Klein massive level. We have that from β -function and ϑ_3 function

$$(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (3/5, -3, -6)$$

We then can have gauge couplings corrections of the form

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(\mu_0) - \frac{b_i - \tilde{b}_i}{2\pi} \ln \frac{\Lambda}{\mu_0} - \frac{\tilde{b}_i}{4\pi} \int_{r\Lambda^{-2}}^{r\mu_0^{-2}} \frac{dt}{t} \left\{ \vartheta_3 \left(\frac{it}{\pi R^2} \right) \right\}^\delta$$

The result then can be reduced to the form

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(\mu_0) - \frac{b_i - \tilde{b}_i}{2\pi} \ln \frac{\Lambda}{\mu_0} - \frac{\tilde{b}_i X_\delta}{2\pi\delta} \left[\left(\frac{\Lambda}{\mu_0} \right)^\delta - 1 \right]$$

in the limit $\Lambda R \gg 1$.

One would have obtained this result also by performing all loop integrals in a $(\delta+4)$ -dimensional spacetime. However, this feature also means that the theory is a non renormalizable theory.

At scale below μ_0 , we can replace the first one with the usual logarithmic running but for any scale above μ_0 we can use this expression. The physics scale is depend on Λ and is parameterized by the coefficient X_δ .

With appropriate choice of X_δ We could treat Λ as the mass scale for new physics this effective non renormalizable theory. Which is

$$X_\delta = \frac{\pi^{\delta/2}}{\Gamma(1 + \delta/2)} = \frac{2\pi^{\delta/2}}{\delta\Gamma(\delta/2)}$$

We could get final expression

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\Lambda}{M_Z} + \frac{\tilde{b}_i}{2\pi} \ln \frac{\Lambda}{\mu_0} - \frac{\tilde{b}_i X_\delta}{2\pi\delta} \left[\left(\frac{\Lambda}{\mu_0} \right)^\delta - 1 \right]$$

By use logarithmic running for scale below μ_0 and the approximation we have for scale above μ_0 .

We have then unification scale that valid for all $\Lambda \geq \mu_0$.

What is remarkable is that there always exists a value of Λ for which the gauge couplings unify and are independent of r , δ , μ_0 , the number η of chiral MSSM generations.

We see here we are not actually calculating the running of the gauge coupling. We calculate the quantum correction which depends explicitly on the cutoff parameter Λ we introduced. So we are in fact getting correction through the calculation of $\alpha_i(\Lambda)$. However, the Λ dependence is essentially the same as scale-dependence calculation we could get from a theory that is renormalizable. Still, we could only say this theory is an effective theory due to its nature of non-renormalizability and valid up to some mass scale M . We have to face the fact that Λ is not an intrinsic physical parameter and it is ultimately on the form of the regulator in which it seems to appear. We could associate the cutoff parameter Λ with the physical mass scale M but with poor choice of regulator, it may not reveal any useful information about M . Since we can shorten the tower at a suitable energy level without seriously altering the results of a given calculation in this theory which underlies the potential of a new complete renormalizable field theory that will give the same result in certain situations. Thus we can associate Λ with mass M without much of a problem. Even the photon decay issue could be avoided through new symmetry related to extra dimension t by making a choice for the modings of the GUT fields with respect to the Z_2 orbifold that produces $N = 1$ MSSM states in the $\eta = 0$ minimal scenario where proton decay will be odd functions of these extra spacetime coordinates, thus their wave functions vanish at the orbifold fixed points. It implies that perturbative proton-decay diagrams vanish and it is independent of the number of extra dimensions or the energy scale. In case where $\eta=1,2$, where not all of the chiral MSSM generations are restricted to orbifold fixed points. It may not present a problem as long as $\mu_0 > \sim 10^{12}$ GeV.

As we have seen that extra dimension can impact the low-energy parameters also it could be interpreted as the classical scaling of dimensionful gauge coupling. However, other questions arise from here also. First supersymmetry may not be a necessary component of this theory which non-SUSY SM can lead to low energy gauge coupling unification. This scenario can also be embedded in string theory which can be realized in certain string and D-brane settings. And it is also possible to consider the effects of extra spacetime dimensions on the running of the soft SUSY-breaking masses.

Take away

However, in the MSSM the energy range between μ_0 and Λ (unification scale) is relatively small due to the steep behaviour in the evolution of the couplings. For example, for an extra dimension the ratio Λ/μ_0 that have upper limit of the order of 30, which will largely reduce for higher to be less than 6. The power law running may not provide the correct situation to a high precision in high dimension as stated at reference [7].

There are many issues still in the model proposed by DDF, as illustrated in the reference paper[7-10] and addressed many of them. But this model still opened a new approach to grand unification and large-radius string compactifications.

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