Instructor:	Howard Haber
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#### **RECOMMENDED TEXTBOOKS:**

Theory of Groups and Symmetries, by Alexey P. Isaev and Valery A. Rubakov
Group Theory in Physics: A Practioner's Guide, by Rutwig Campoamor-Stursberg and Michel Rausch de Traubenberg
Group Theory in Physics, by Wu-Ki Tung
Groups, Representations and Physics (2nd edition), by H.F. Jones
Group Theory: A Physicist's Survey, by Pierre Ramond
Group Theory for Physicists, by Zhong-Qi Ma
Lie Groups, Lie Algebras, and Some of Their Applications, by Robert Gilmore
Lie Algebras in Particle Physics (2nd edition), by Howard Georgi
Group Theory in a Nutshell for Physicists, by Anthony Zee

#### Additional outside reading:

Group Theory in Physics: An Introduction, by J.F. Cornwell
Group Theory in Physics, Volume 1, by J.F. Cornwell
Group Theory in Physics, Volume 2, by J.F. Cornwell
Problems and Solutions in Group Theory for Physicists, by Zhong-Qi Ma and Xiao-Yan Gu
Lie Groups and Lie Algebras for Physicists, by Ashok Das and Susumu Okubo
Symmetries, Lie Algebras and Representations, by Jürgen Fuchs and Christoph Schweigert
Symmetry Groups and Their Applications, by Willard Miller Jr.
Group Theory: Birdtracks, Lie's, and Exceptional Groups, by Predrag Cvitanović
(available for free from: http://www.nbi.dk/GroupTheory/version9.0/index.html)
Semi-Simple Lie Algebras and Their Representations, by Robert N. Cahn
(available for free from: http://phyweb.lbl.gov/~rncahn/www/liealgebras/book.html)
Group Theory for Unified Model Building, Physics Reports 79 (1981) 1-128, by Richard Slansky
Mathematics of Classical and Quantum Physics, Chapter 10: Introduction to Group Theory,
by Frederick W. Byron Jr. and Robert W. Fuller

## **Course Outline**

- 1. Introduction to Abstract Group Theory
- 2. Fundamentals of Finite Groups
- 3. Group Representation Theory
- 4. The Symmetric Group and Young Tableaux
- 5. Introduction to Topological Groups and Lie Groups
- 6. SU(2) and SO(3)
- 7. Global and Local Properties of Lie Groups
- 8. Lie Algebras
- 9. Representations of SU(2) and SU(3)
- 10. Complex Semisimple Lie Algebras and their Representations

#### **Course Requirements**

The basic course requirements consist of four problem sets, which will be handed out during the quarter, and a term project. (There will be no exams.) Due to the limited time in a quarter, it will be impossible to do more than sketch some of the most basic applications of group theory to modern physics. To encourage students to delve deeper, all students will be required to complete a term project based on their reading of a particular topic in group theory and its applications to physics. The project may be presented orally or in written form at the end of the term. Oral presentations are encouraged since they will benefit all members of the class. Please follow the following schedule:

Initial choice of topic for term
Short written proposal for term
Oral Presentation of term projectJune 7
Written version of term projectJune 14

All projects should include a one page bibliography (containing references pertinent to the project). Copies of this bibliography should be made available to all students in the class. For those projects presented orally, a xerox of transparencies and a brief set of notes will be acceptable in lieu of a full written version.

I will be available during my office hours for suggestions and consultation on your choice for the term project. If you need some suggestions, you might consider choosing from the following list of possible topics for term projects.

# Suggestions of topics for the term project

- 1. The Crystallographic Point Groups and Space Groups
- 2. Lattices, Bloch's Theorem and Band Theory
- 3. Group Theoretical Treatment of Vibrational Problems
- 4. Group Theory and Molecular Spectra
- 5. Group Theory and the Periodic Table
- 6. Group Theory in Classical Mechanics
- 7. Group Theoretic Methods in Quantum Mechanics
- 8. Group Theory in Condensed Matter Physics
- 9. Group Theory in General Relativity
- 10. Group Theory and the Shell Model in Nuclear Physics
- 11. Group Theory and the Quark Model in Particle Physics
- 12. Non-Abelian Discrete Symmetries in Particle Physics
- 13. Group Structure of Spontaneously Broken Gauge Theories
- 14. Group Theory and Grand Unification
- 15. Group Theory and Monopoles
- 16. The Lorentz and Poincaré Groups in Relativistic Field Theory
- 17. Applications of Clifford Algebras (and spinors) in physics
- 18. Boson and Fermion Realizations of Lie algebras
- 19. Spectrum generating groups (a.k.a. noninvariance groups)
- 20. Coherent states as a problem in group theory

For inspiration, in addition to the recommended outside reading and the many references on group theory for physicists provided in the bibliography that follows, have a look at:

Quantum Theory, Groups and Representations: An Introduction, by Peter Woit (final draft version available from: https://www.math.columbia.edu/~woit/QM/qmbook.pdf) Classical and Quantum Mechanics via Lie algebras, by Arnold Neumaier and Dennis Westra (draft version available from: https://arxiv.org/pdf/0810.1019v2.pdf)

# Bibliography

Other selected references in group theory for physicists:

Lectures on Group Theory for Physicists, by A.P. Balachandran and C.G. Trahern Lie Algebras Part 1: Finite and Infinite Dimensional Lie Algebras and Applications in Physics. by G.G.A. Bäuerle and E.A. de Kerf Lie Groups and Lie Algebras: A Physicist's Perspective, by Adam M. Bincer Group Theory and General Relativity, by Moshe Carmeli Symmetries in Quantum Mechanics: From Angular Momentum to Supersymmetry, by M. Chaichian and R. Hagedorn Group Theory in Physics, Volume 3, by J.F. Cornwell Group Theory: Application to the Physics of Condensed Matter, by Mildred S. Dresselhaus, Gene Dresselhaus and Ado Jorio Symmetries and Condensed Matter Physics, by Michael El-Batanouny and Frederick Wooten Group Theoretical Methods and Their Applications, by A. Fässler and E. Stiefel *Lie Groups, Physics and Geometry*, by Robert Gilmore Quantum Mechanics: Symmetries, by Walter Greiner and Berndt Müller Symmetries and Curvature Structure in General Relativity, by G.S. Hall Group Theory and its Application to Physical Problems, by Morton Hammermesh Lie Algebras and Applications (2nd edition), by Francesco Iachello Lectures on Groups and Vector Spaces for Physicists, by Chris J. Isham An Introduction to Tensors and Group Theory for Physicists (2nd edition), bu Nadir Jeevanjee Theory of Groups in Classical and Quantum Physics, by Théo Kahan Primer for Point and Space Groups, by Richard L. Liboff Symmetries in Physics: Group Theory Applied to Physical Problems, by W. Ludwig and C. Falter Lectures on Advanced Mathematical Methods for Physicists, by Sunil Mukhi and N. Mukunda Group Structure of Gauge Theories, by L. O'Raifeartaigh Generalized Coherent States and their Applications, by A. Perelomov Lie Groups and Algebras with Applications to Physics, Geometry and Mechanics, by D.H. Sattinger and O.L. Weaver A Course in the Application of Group Theory to Quantum Mechanics, by I.V. Schensted *Physics from Symmetry*, by Jakob Schwichtenberg Relativity, Groups, Particles, by Roman U. Sexl and Helmuth K. Urbantke Group Theory and Physics, by S. Sternberg Shattered Symmetry, by Pieter Thyssen and Arnout Ceulemans Group Theory and Quantum Mechanics, by Michael Tinkham Group Theory and its Applications to the Quantum Mechanics of Atomic Spectra, by E. Wigner Quantum Theory, Groups and Representations: An Introduction, by Peter Woit Classical Groups for Physicists, by Brian G. Wybourne

# Selected references in topology and differential geometry for physicists:

An Introduction to Spinors and Geometry, by A.M. Benn and R.W. Tucker Differential Geometry and Lie Groups for Physicists, by Marián Fecko The Geometry of Physics: An Introduction (3rd edition), by Theodore Frankel Fundamental Groups and Covering Spaces, by Elon Lages Lima Topology: An Introduction with Application to Topological Groups, by George McCarty A First Course in Topology: Continuity and Dimension, by John McCleary Geometry, Topology and Physics (2nd edition), by M. Nakahara Geometry and Topology, by Miles Reid and Balázs Szendrői Geometrical Methods of Mathematical Physics, by Bernard Schutz

## Other selected mathematical references:

*Basic Lie Theory*, by Hossein Abbaspur and Martin Moskowitz Conformal Groups in Geometry and Spin Structures, by Pierre Anglès An Introduction to Lie Groups and the Geometry of Homogeneous Spaces, by A. Arvanitoyeorgos Matrix Groups: An introduction to Lie group theory, by Andrew Baker Theory of Group Representations and Applications, by A.O. Barut and R. Raczka Representations of Linear Groups, by Rolf Berndt Representations of Compact Lie Groups, by Theodor Bröcker and Tammo tom Dieck Lie Groups (2nd edition), by Daniel Bump Lectures on Lie Groups and Lie Algebras, by R..W. Carter, G. Segal and I.G. Macdonald Lie Groups, by P.M. Cohn Matrix Groups, by Morton L. Curtis The Geometry of Octonians, by Tevian Dray and Corinne A. Manogue Transformation Groups for Beginners, by S.V. Duzhin and B.D. Chebotarevsky *Clifford Algebras: An Introduction*, by D.J.H. Garling Representations of the Rotation and Lorentz Groups and their Applications, by I.M. Gel'fand, R.A. Minlos and Z. Ya. Shapiro Basics of Lie Groups, by Michel Gourdin Lie Algebras: Theory and Algorithms, by W.A. De Graaf Continuous Groups of Transformations, by Luther Eisenhart Introduction to Lie Algebras, by Karin Erdmann and Mark Wildon Lie Groups, Lie Algebras, and Representations (2nd edition), by Brian C. Hall Spinors and Calibrations, by F. Reese Harvey Lie Groups, Lie Algebras, by Melvin Hausner and Jacob Schwartz The Structure and Geometry of Lie Groups, by Joachim Hilgert and Karl-Hermann Neeb Lectures on Representation Theory, by Jing-Song Huang Introduction to Lie Algebras and Representation Theory, by James E. Humphreys

Lie Algebras, by Nathan Jacobson

An Introduction to Lie Groups and Lie Algebras, by Alexander Kirillov, Jr. Lie Groups Beyond an Introduction (2nd edition), by Anthony W. Knapp Groups and Symmetries: From Finite Groups to Lie Groups, by Yvette Kosmann-Schwarzbach Theory of Continuous Groups, by Charles Loewner Clifford Algebras and Spinors, by Pertti Lounesto Topological Groups: An Introduction, by Nelson G. Markley Lie Theory and Special Functions, by Willard Miller, Jr. The Unitary and Rotation Groups, by Frank Murnaghan *Linear Representations of the Lorentz Group*, by M.A. Naimark Theory of Group Representations, by M.A. Naimark and A.I. Stern Lectures on Real Semisimple Lie Algebras and Their Representations, by Arkady L. Onishchik Lie Groups and Algebraic Groups, by Arkady L. Onishchik and Ernest B. Vinberg The Lie Algebras su(N)—An Introduction, by Walter Pfeifer Topological Groups, by L.S. Pontryagin Clifford Algebras and the Classical Groups, by Ian R. Porteous Lie Groups: A Problem-Oriented Introduction via Matrix Groups, by Harriet Pollatsek Topological Groups, by L.S. Pontryagin Lectures in Geometry, Semester V: Lie Groups and Lie Algebras, by M. Postnikov Lie Groups: An Approach through Invariants and Representations, by Claudio Procesi Geometry of Lie Groups, by Boris Rosenfeld Lie Groups, An Introduction Through Linear Groups, by Wulf Rossmann An Introduction to the Theory of Groups (4th edition), by Joseph J. Rotman The Symmetric Group, by Bruce E. Sagan Introduction to Lie Groups and Lie Algebras, by Arthur A. Sagle and Ralph E. Walde Representing Finite Groups-A Semisimple Introduction, by Ambar N. Sengupta Compact Lie Groups, by Mark R. Sepanski Representations of Finite and Compact Groups, by Barry Simon *Naive Lie Theory*, by John Stillwell Special Functions: A Group Theoretical Approach, by James D. Talman Matrix Groups for Undergraduates (2nd edition), by Kristopher Tapp Lie Groups, Lie Algebras, and Their Representations, by V.S. Varadarajan An Introduction to Clifford Algebras and Spinors, by Jayme Vaz, Jr. and Roldão Da Rocha, Jr. *Linear Representations of Groups*, by Ernest B. Vinberg Group and Representation Theory, by J.D. Vergados Lie Algebras, by Zhe-Xian Wan Compact Lie Groups and their Representations, by D.P. Zelobenko

Other selected references in group theory for particle physicists:

Group Theory in Particle, Nuclear, and Hadron Physics, by Syed Afsar Abbas Lectures on Group Theory and Particle Theory, by H. Bacry

Group Theory for the Standard Model of Particle Physics and Beyond, by Ken J. Barnes

Symmetries and Group Theory in Particle Physics: An Introduction to Space-Time and Internal Symmetries, by Giovanni Costa and Gianluigi Fogli

Mathematical Gauge Theory: With Applications to the Standard Model of Particle Physics, by Mark J.D. Hamilton

Symmetries And Conservation Laws In Particle Physics: An Introduction To Group Theory for Particle Physicists, by Stephen Haywood

An Introduction to Non-Abelian Discrete Symmetries for Particle Physicists, by H. Ishimori et al. Symmetry and the Standard Model: Mathematics and Particle Physics, by Matthew Robinson Group Theory for High Energy Physicists, by Mohammad Saleem and Muhammad Rafique Group Theory in Subnuclear Physics, by Fl. Stancu