MIDTERM EXAM INSTRUCTIONS: You have 90 minutes to complete this exam. This is a closed book exam, although you are permitted to consult two sheets of handwritten material. No other consultations or collaborations are permitted during the exam. At the end of the exam please hand in all written work including your sheets of handwritten notes.

PART I: *Choose the best answer among the choices given. Justify your choice with a short computation. (Is that your final answer?) No credit will be given without an accompanying explanation. Each question is worth seven points.*

1. A partition divides a container into two parts of equal volume. One part contains 1 mole of O_2 gas and the other part contains 4 moles of N_2 gas. The partition is then removed and the gases mix. If the system is held at constant temperature T, what is the change in the Helmholtz free energy due to the mixing process? (The gas constant $R \equiv N_A k$, where N_A is Avogadro's number.)

(a) 0

(b) $-5RT \ln 2$

(c) $+5RT \ln 2$

(d) $-RT(\ln 5 + 4 \ln \frac{5}{4})$

(e) $+RT(\ln 5 + 4 \ln \frac{5}{4})$

2. The chemical potential of a photon gas is zero. This statement is

(a) always true.

(b) sometimes true.

(c) never true.

3. The walls of an evacuated insulated enclosure are in equilibrium at a temperature 300 K with the radiant energy enclosed. The volume of the enclosure is 1 m^3. The total number of microstates corresponding to the macrostate just described is about:

(a) 10^{26}

(b) 10^{200}

(c) 10^{100000}

(d) $10^{10^{10}}$

(e) $10^{10^{15}}$

4. A dilute quantity of O_2 gas is observed at $T = 10,000$ K. The heat capacities of the gas at constant pressure and constant volume are measured. The ratio $\gamma \equiv C_P/C_V$ is found to be:

(a) 5/3

(b) 7/5

(c) 4/3

(d) 9/7

(e) 5/4

5. The thermodynamic identity states that $dE = TdS - PdV + \mu dN$. Which of the following statements are true?

(a) This identity applies to any changes between equilibrium states.

(b) This identity applies only to *reversible* changes between equilibrium states.

(c) $P = -(\partial E/\partial V)_{S,N}$.

(d) $P = -(\partial E/\partial V)_{T,\mu}$.

6. A gas of hydrogen bromide is in thermal equilibrium at temperature T. It is known that the characteristic rotational temperature is $\theta_r = 12.2$ K, where $k\theta_r \equiv \hbar^2/2I$ and I is the moment of inertia of the molecule. Suppose that the corresponding populations of HBr molecules with angular momentum $J = 2$ and $J = 3$ are equal. The value of the temperature must be in the following range

(a) between 10 K and 20 K

(b) between 20 K and 40 K

(c) between 100 K and 120 K

(d) between 200 K and 240 K

(e) No such temperature range exists.

7. About 300,000 years after the bing bang birth of the universe, the temperature of the universe dropped to 3000 K. At this point, free electrons and nuclei combined to form neutral hydrogen and helium. We say that the matter and radiation have decoupled, since neutral matter scatters radiation much less than free charged particles do. In particular, the existing blackbody radiation of photons could subsequently expand freely, uncoupled to matter (to a very good approximation). Thus, photons that make up the cosmic microwave background radiation observed today have traveled unimpeded for more than 10 billion years! Assuming an *adiabatic* expansion of the universe during this time, by what factor (roughly) has the universe expanded since matter and radiation decoupled? (It is enough to consider the relative volume occupied by the expanding photon gas.)

(a) 10^3

(b) 10^6

(c) 10^9

(d) 10^{10}

(e) 10^{12}

PART II: PROBLEMS. *In order to earn total credit for a problem solution, you must show all work involved in obtaining the solution. The point value of each problem is indicated in the square brackets below.*

1. [27] The lower 10–15 km of the atmosphere, the troposphere, is certainly not isothermal. A semi-realistic model of the troposphere describes it as being in a convective steady state at constant entropy. In such an equilibrium state, Pv^γ is independent of the altitude, where $v \equiv V/N$ is the specific volume of the gas. This model is called the adiabatic atmosphere. Assume that the atmosphere is composed entirely of N_2 gas, which may be treated as an ideal gas.

(a) Show that at any altitude in the troposphere,

$$\frac{dT}{dP} = \left(\frac{\gamma - 1}{\gamma}\right) \frac{T}{P} \, .$$

(b) Show that dT/dz is a constant, where T is the temperature at an altitude z. *HINT*: The differential equation for the pressure is the same as in the case of the isothermal atmosphere. (Why?) However, in this case T is not constant. Use the chain rule to relate dT/dz and dP/dz. Your final answer for dT/dz should depend on γ, $m(N_2)$, and the acceleration due to gravity g.

(c) What value should you use for γ? By how many degrees Celsius does the temperature drop from sea level to the top of Mount Everest (altitude 8848 m)?

2. [24] The velocity of longitudinal sound waves in liquid ^{4}He at temperatures below 0.6 K is 238.3 m s^{-1}. It is an experimental fact that there are no transverse sound waves in the liquid. The mass density of liquid ^{4}He is 0.145 g cm^{-3}.

(a) Calculate the Debye temperature.

(b) Applying the Debye theory of heat capacity of solids to liquid helium, compute the heat capacity per kg. Compare your result to the experimental value (in units of J kg^{-1} K^{-1}) of $C_V = 20.4 \times T^3$, where T is given in degrees K.

NOTE: Although it may appear odd to apply a theory of specific heats in solids to liquid helium, the results of part (b) suggest that phonons are the most important excitations in liquid ^{4}He below 0.6 K. Liquids differ from solids in one important way: they typically can support only longitudinal vibrations. Thus the phonon polarization factor, which was 3 in the calculations presented in class, is 1 here. This does not modify the Debye temperature, but does modify the energy and specific heat. (Why?)

1. For your convenience, I list some of the fundamental constants and mass parameters below:

$$
\begin{aligned}
k &= 1.381 \times 10^{-23} \text{ J K}^{-1} \\
\sigma_B &= 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \\
h &= 6.626 \times 10^{-34} \text{ J s} \\
\hbar &= 1.0546 \times 10^{-34} \text{ J s} \\
c &= 2.998 \times 10^{8} \text{ m s}^{-1} \\
N_A &= 6.022 \times 10^{23} \text{ mole}^{-1} \\
R &= 8.315 \text{ J mole}^{-1} \text{ K}^{-1} \\
g &= 9.8 \text{ m s}^{-2} \\
m(\text{N}_2) &= 4.653 \times 10^{-26} \text{ kg} \\
m(^4\text{He}) &= 6.649 \times 10^{-27} \text{ kg} \\
1 \text{ a.m.u.} &= 1.6605 \times 10^{-27} \text{ kg}
\end{aligned}
$$

where k is Boltzmann's constant, N_A is Avagadro's number, g is the acceleration due to gravity, and a.m.u. is an atomic mass unit. The Stefan-Boltzmann constant is related to other fundamental constants via

$$\sigma_B \equiv \frac{\pi^2 k^4}{60 \hbar^3 c^2}\,, \tag{1}$$

and the gas constant is given by $R \equiv N_A k$.

2. Given the partition function of any system, you may compute all relevant thermodynamic quantities. For your convenience, I note the following two results.

(a) For the ideal gas,

$$Z = \frac{(V\lambda_{\rm th}^{-3})^N}{N!} \, ,$$

where the thermal wavelength is given by

$$\lambda_{\rm th} \equiv \left(\frac{2\pi\hbar^2}{mkT} \right)^{1/2} \, .$$

In evaluating $\ln\, Z$, it is sufficient to use $\ln N! \simeq N \ln N - N$ in the case of $N \gg 1$.

(b) For the photon gas,

$$\ln\, Z = \frac{4\sigma_B V T^3}{3kc} \, ,$$

where σ_B is given by eq. (1).

3. For a photon gas, the number of modes with frequency between ν and $\nu + d\nu$ is given by $D(\nu)d\nu$, where

$$D(\nu) = \frac{8\pi V \nu^2}{c^3} \, .$$

For a phonon gas, replace c with the speed of sound v_s and correct for the number of phonon polarization states.