

1. Consider the power series,

$$f(x) = \sum_{n=2}^{\infty} (-1)^n \frac{x^n}{n^2 - n}, \quad (1)$$

where x is a real variable.

(a) Compute the radius of convergence of the power series $f(x)$.

(b) Determine whether the power series given in eq. (1) is absolutely convergent, conditionally convergent, or divergent at the point $x = 1$.

HINT: The method of partial fractions is your friend (after factoring the denominator).

(c) If $f(1)$ is convergent, determine its value.

2. Consider the real valued function:

$$g(x) = \left(\frac{3}{x^3} - \frac{1}{x} \right) \sin x - \frac{3}{x^2} \cos x.$$

(a) Compute $\lim_{x \rightarrow 0} g(x)$.

(b) Find the *behavior* of $g(x)$ as $x \rightarrow 0$.

3. Evaluate the following quantities:

(a) $(-1)^i$

(b) $\text{Im} [ix + \sqrt{1 - x^2}]^{-1}$, where x is a real number and $|x| < 1$

(c) $\text{Arg}(\sin i)$

Be sure to indicate all possible values if the quantity in question is multivalued. Simplify your expressions as much as possible.

4. Find all complex number solutions z to the equation, $z^3 = i$.

5. Consider the system of equations:

$$x_1 + 3x_2 - x_3 = 4,$$

$$x_1 + 2x_2 + x_3 = 2,$$

$$3x_1 + 7x_2 + x_3 = c,$$

where c is some unspecified real number. There exists one value of c for which there are solutions to the above system of equations. Find that value of c and determine the allowed solutions.