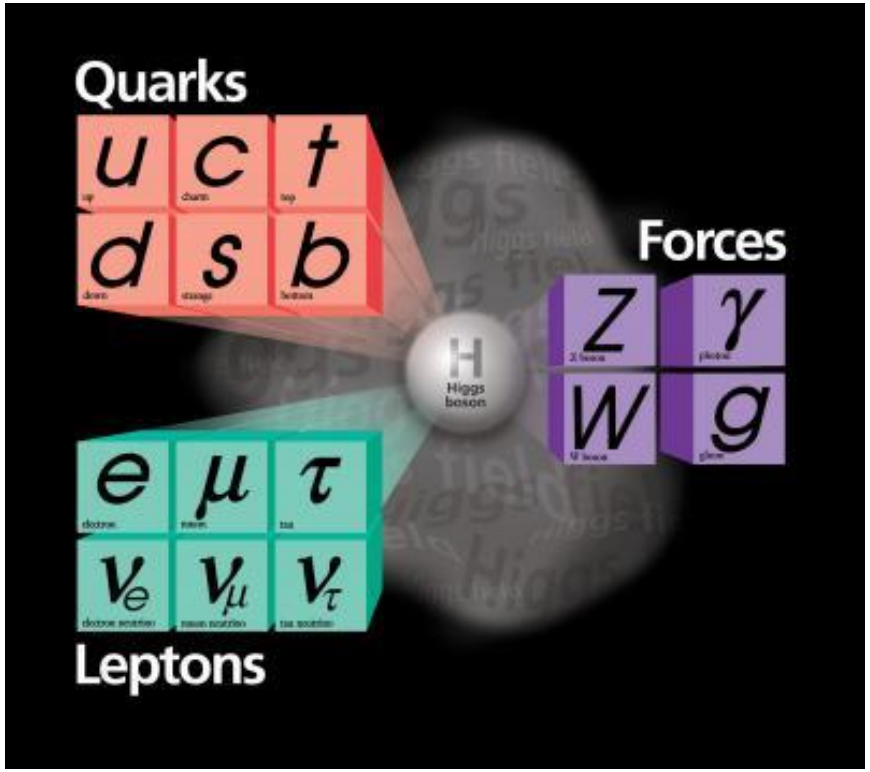
Research on the Theory of the TeV energy scale (Terascale)

Howard Haber
SCIPP Theory
March 13, 2025

For further details, check out my webpage:
http://scipp.ucsc.edu/~haber/

The Standard Model (SM) of Particle Physics



The elementary particles consists of three generations of spin-1/2 quarks and leptons, the gauge bosons of SU(3)xSU(2)xU(1), and the Higgs boson.

Technically, massive neutrinos require an extension of the Standard Model, but most likely the relevant scale of the new physics lies way beyond the terascale.

On July 4, 2012, the discovery of a new boson is announced which may be the long sought after Higgs boson.

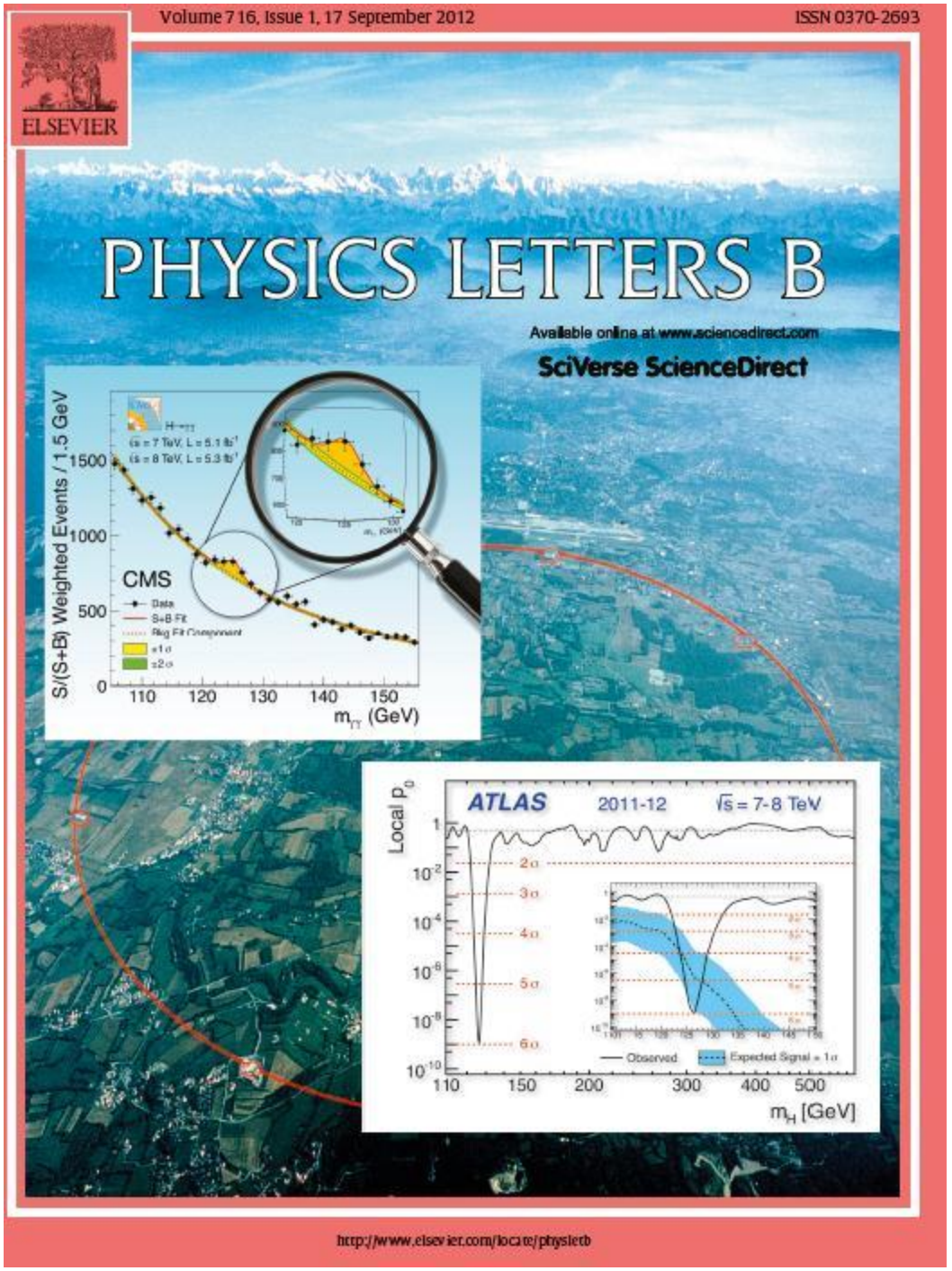
The discovery papers are published two months later In Physics Letters B.

ATLAS Collaboration:

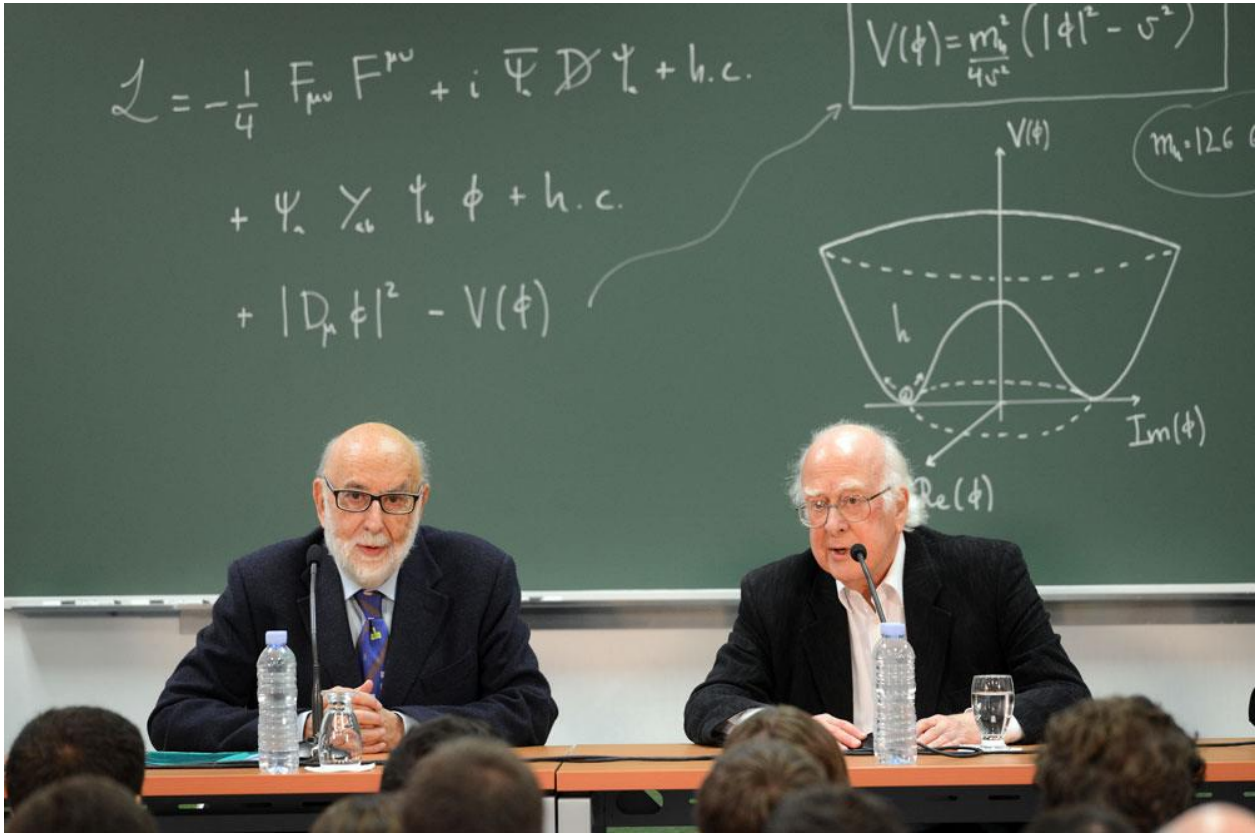
Physics Letters B716 (2012) 1—29

CMS Collaboration:

Physics Letters B716 (2012) 30—61



Winners of the 2013 Nobel Prize in Physics



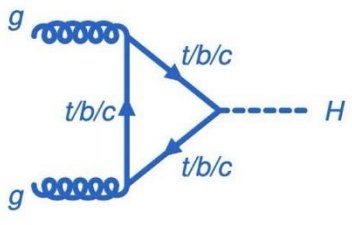
François Englert

and

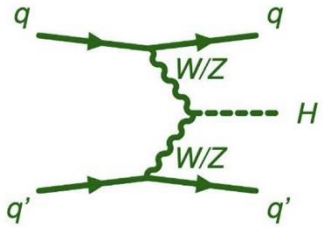
Peter Higgs

Higgs boson production and decay mechanisms

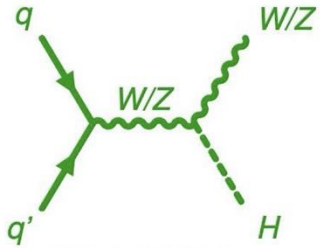
Higgs boson production



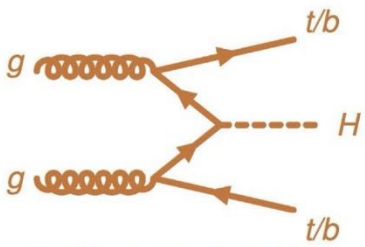
ggF (87%)



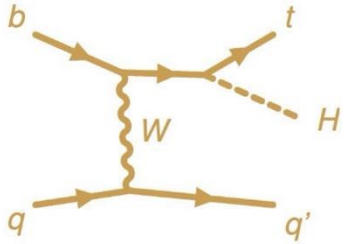
VBF (7%)



VH (4%)

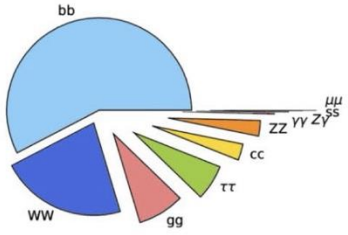


ttH/bbH (2%)

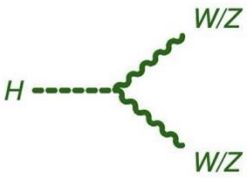


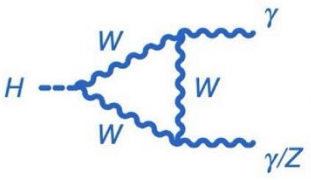
tH (0.1%)

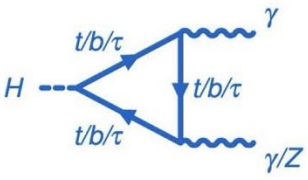
Higgs boson decay channels

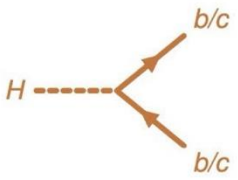


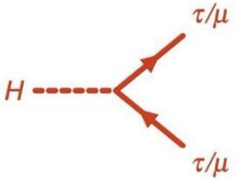
Higgs boson decay BR



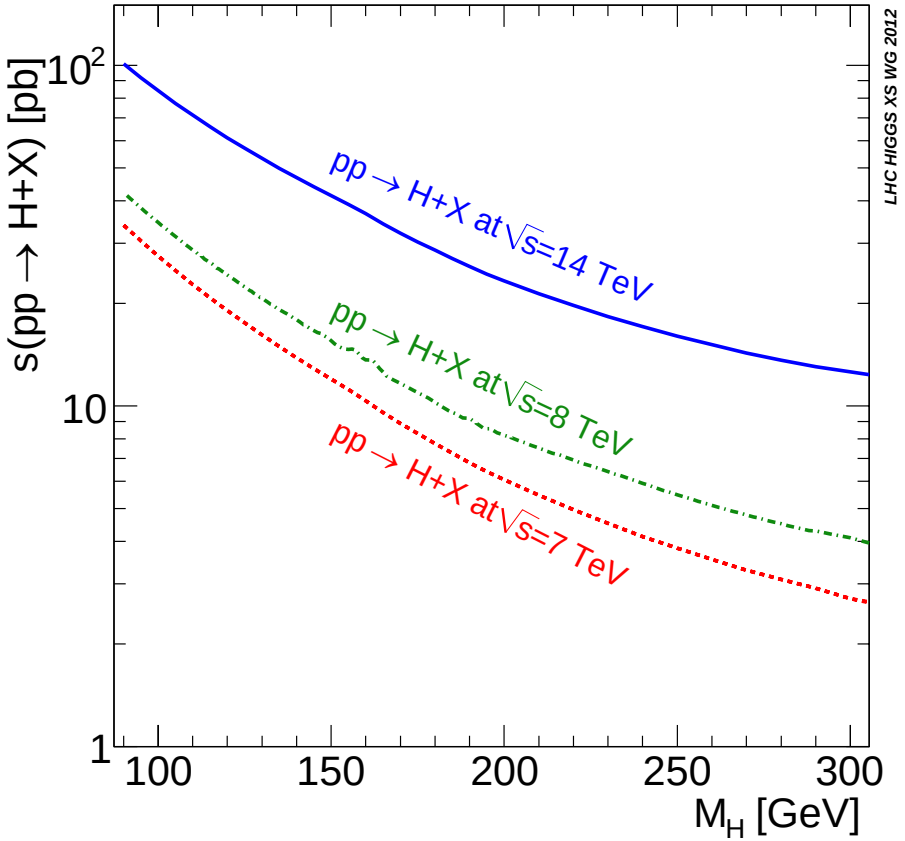


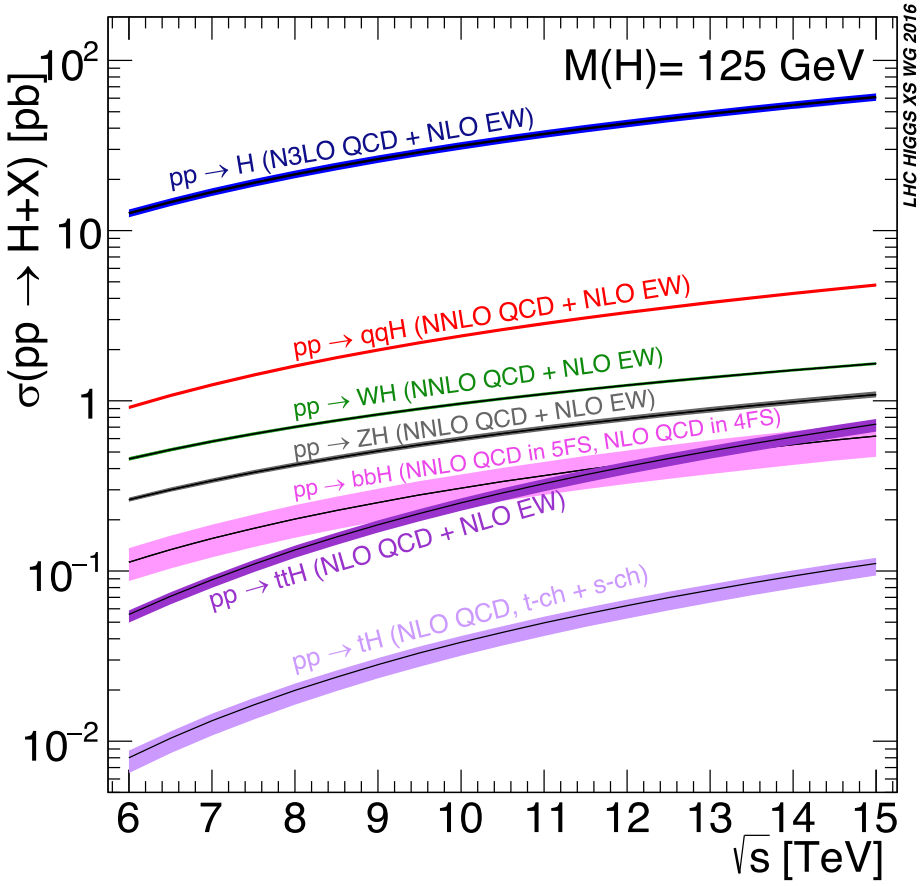






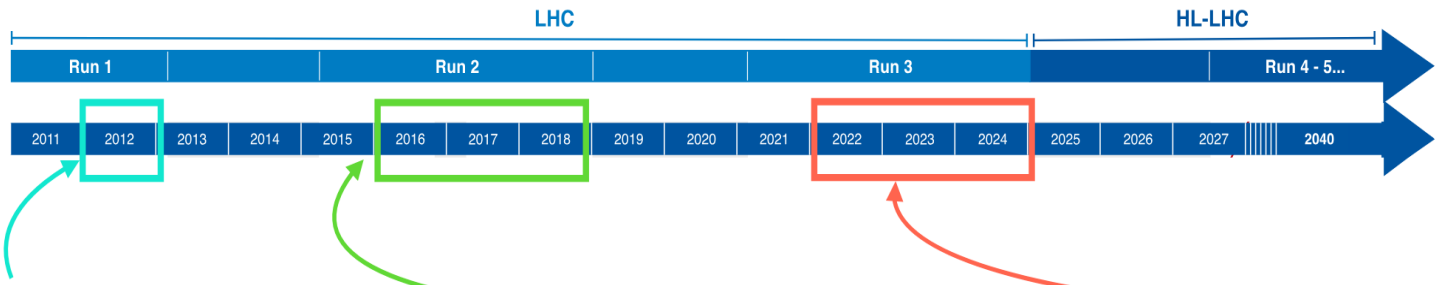
Higgs boson production cross sections at a pp collider





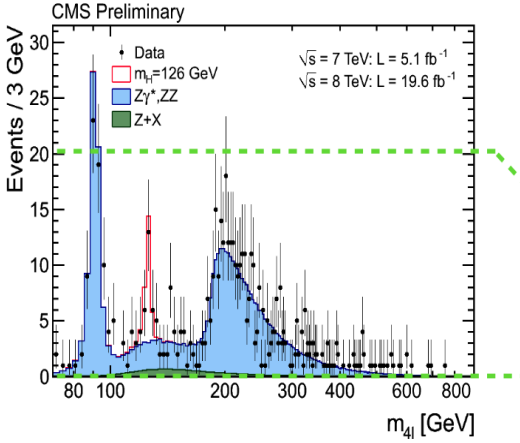
With nearly 140 fb^{-1} of data delivered by the LHC in Run 2 to both ATLAS and CMS in 2015—2018 at a center of mass energy of 13 TeV, roughly 7.5 million Higgs bosons per experiment were produced, assuming the Higgs mass is 125 GeV.

LHC / HL-LHC Plan



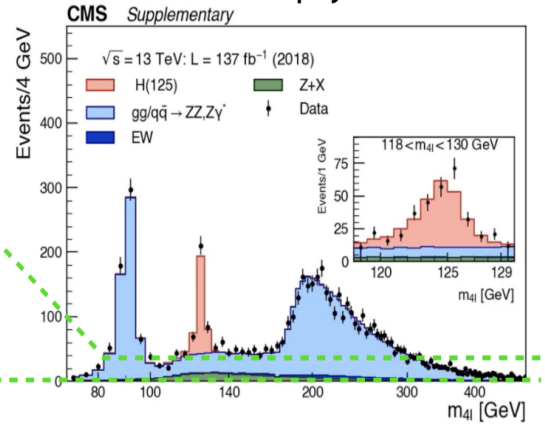
Run I

Discovery of a new particle



Run-II

Precision physics era



Run-III

The Higgs realm

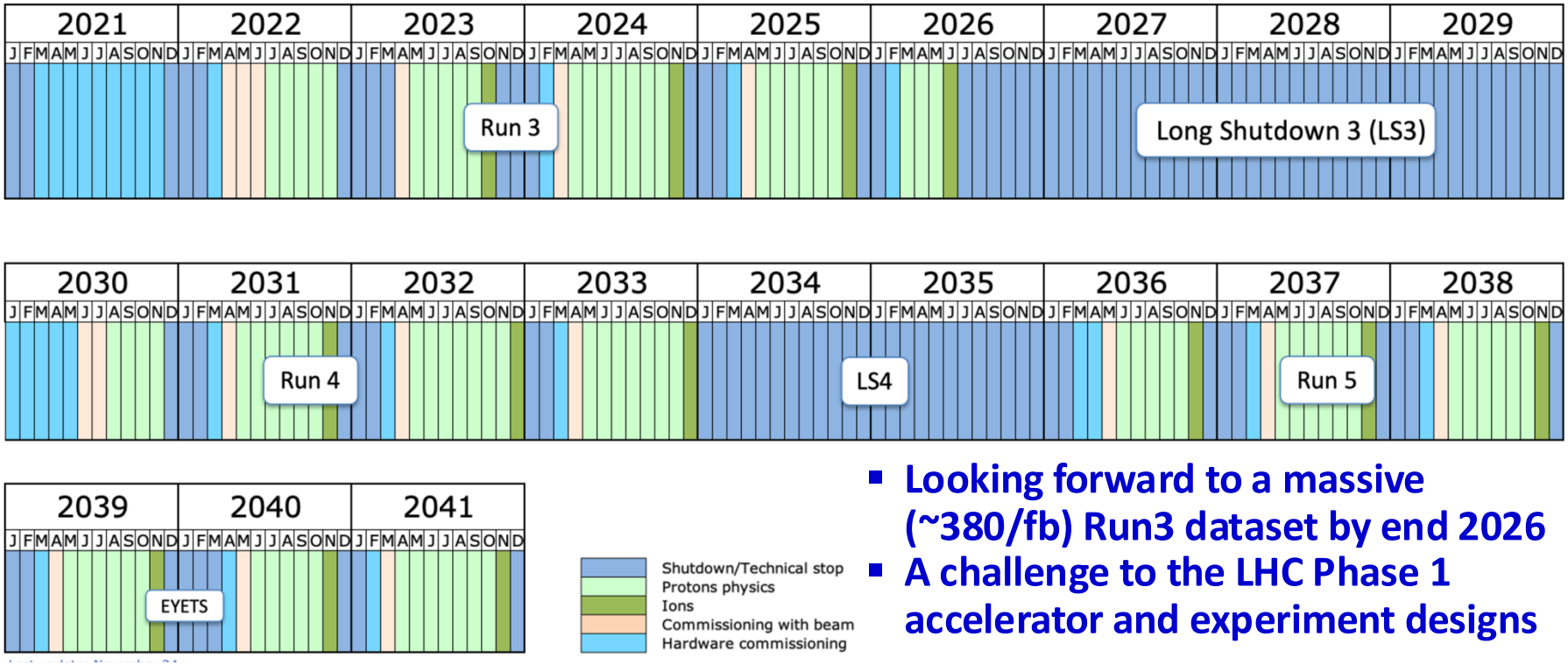
- Access to higher $\sqrt{s}$
- Improved analysis techniques
- Stress test of the SM predictions

Taken from Harvey Newman's Chair Report at the USLUA Annual Meeting (12/17/24)

Long Term LHC Schedule (to 2041): Update Sept. 2024

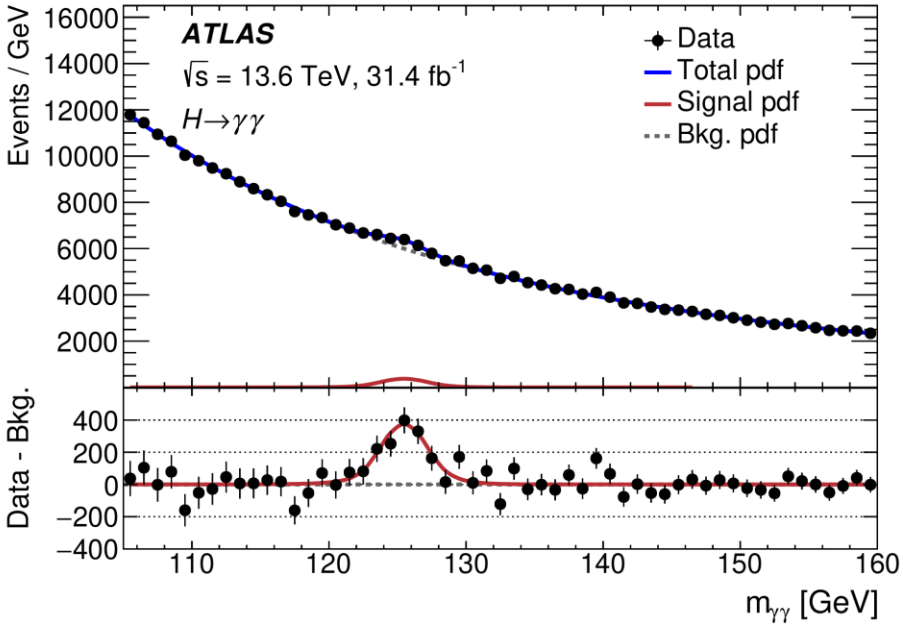
Towards HL LHC: Challenge and Opportunity

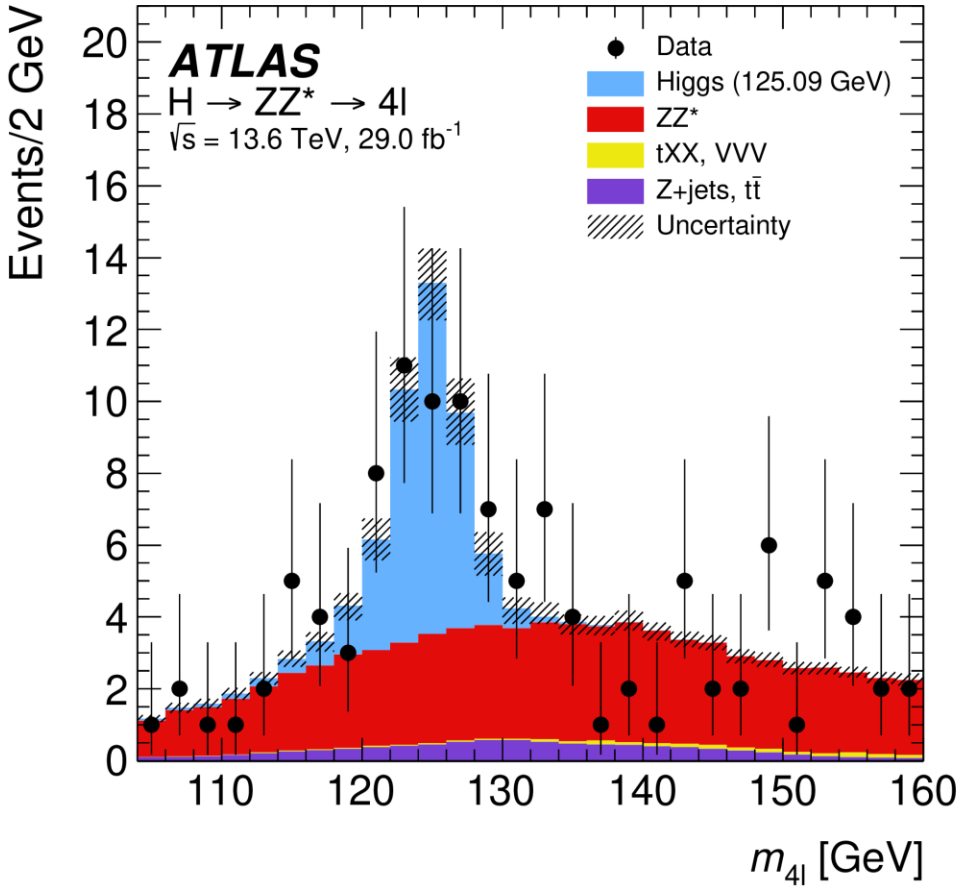
- Short YETS 25/26
- Extend Run 3 to end June 2026
- Start LHC LS3 July 2026
- Start final Hardware Commissioning January 2030
- First beam June 2030
- LS3 - beam to beam: 3 years 11 months, 47 months



- Looking forward to a massive (~380/fb) Run3 dataset by end 2026
- A challenge to the LHC Phase 1 accelerator and experiment designs

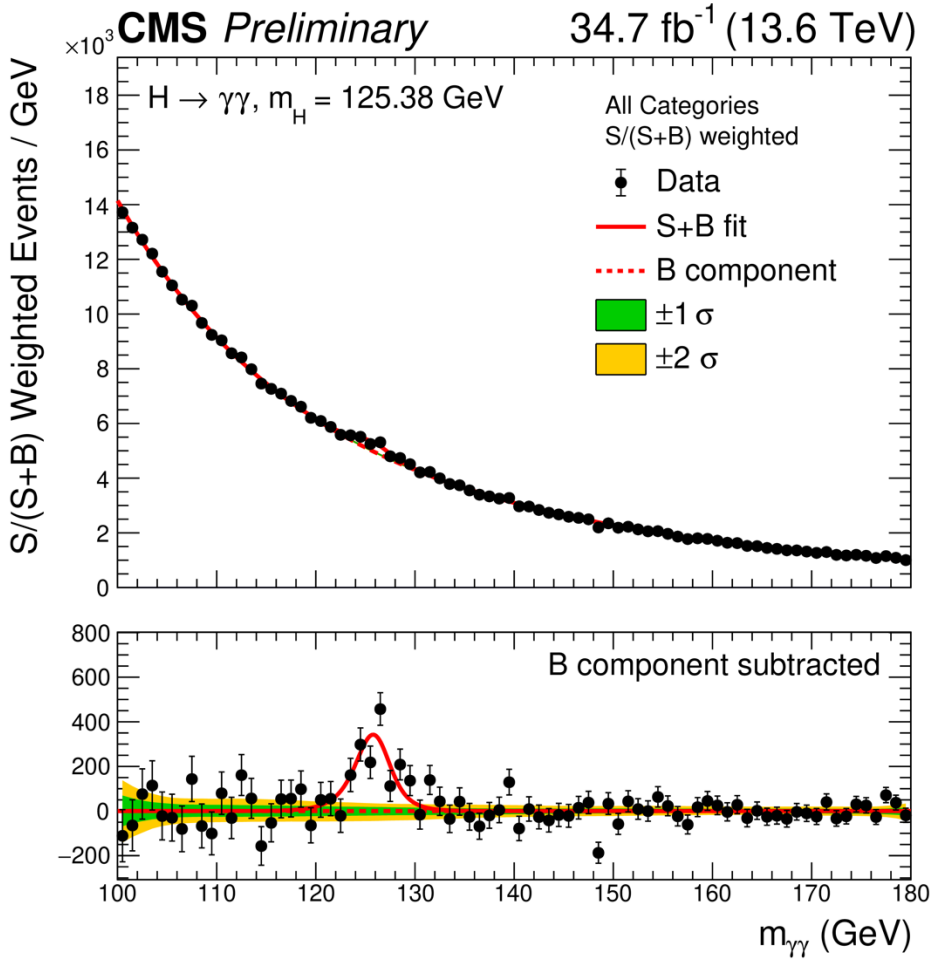
ATLAS Run 3 observations of the Higgs boson



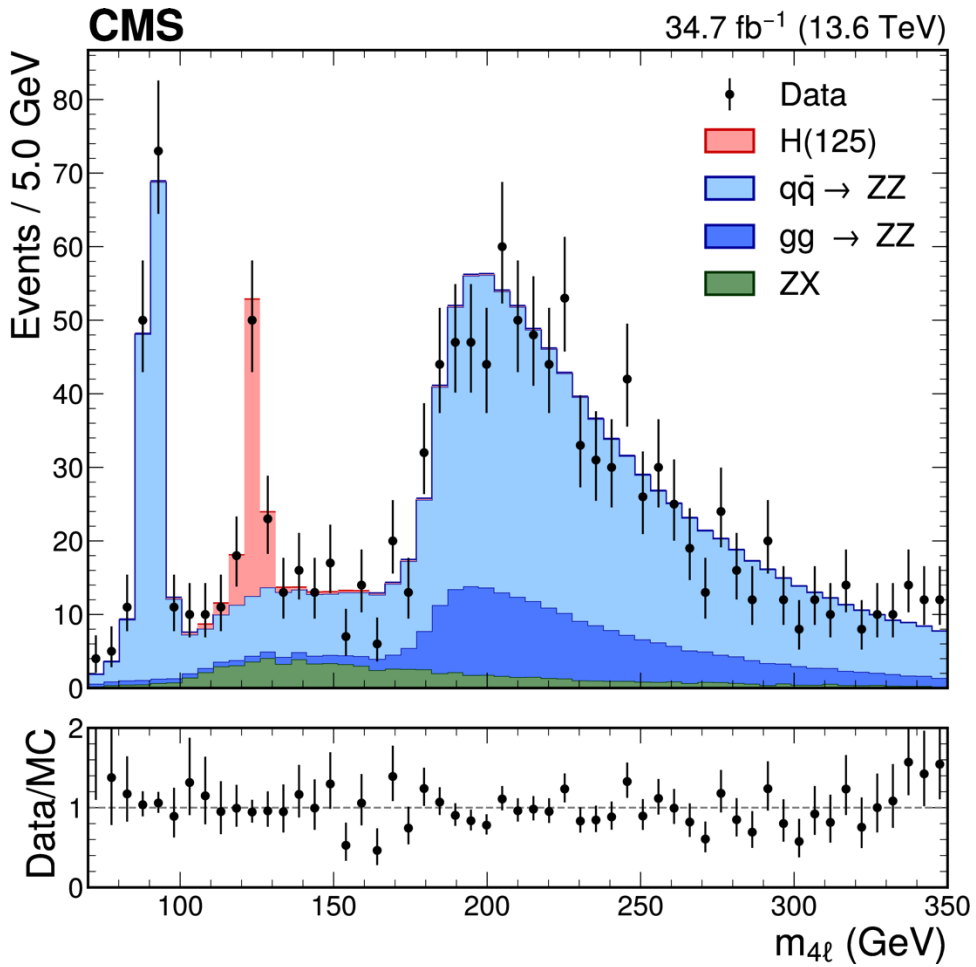


Taken from [Eur. Phys. J. C 84 (2024) 78]

CMS Run 3 observations of the Higgs boson



Taken from CMS-PAS-HIG-23-014



Taken from CERN-EP-2024-336

Higgs boson decay channels observed at the LHC

Higgs boson decay mode	Branching ratio (for $m_h = 125$ GeV)
$h^0 \rightarrow \mathbf{b\bar{b}}$	0.582
$h^0 \rightarrow \boldsymbol{\tau^+ \tau^-}$	6.27×10^{-2}
$h^0 \rightarrow \boldsymbol{\ell^+ \ell^- \nu\nu}$ ($\boldsymbol{\ell}$ = **e** or $\boldsymbol{\mu}$)	1.06×10^{-2}
$h^0 \rightarrow \boldsymbol{\gamma\gamma}$	2.27×10^{-3}
$h^0 \rightarrow \boldsymbol{\ell^+ \ell^- \ell^+ \ell^-}$ ($\boldsymbol{\ell}$ = **e** or $\boldsymbol{\mu}$)	1.24×10^{-4}
$h^0 \rightarrow Z\boldsymbol{\gamma} \rightarrow \boldsymbol{\ell^+ \ell^- \gamma}$ ($\boldsymbol{\ell}$ = **e** or $\boldsymbol{\mu}$)	1.03×10^{-4}
$h^0 \rightarrow \boldsymbol{\mu^+ \mu^-}$	2.18×10^{-4}

Taken from https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CERNYellowReportPageBR#Branching_Ratios

Remarks:

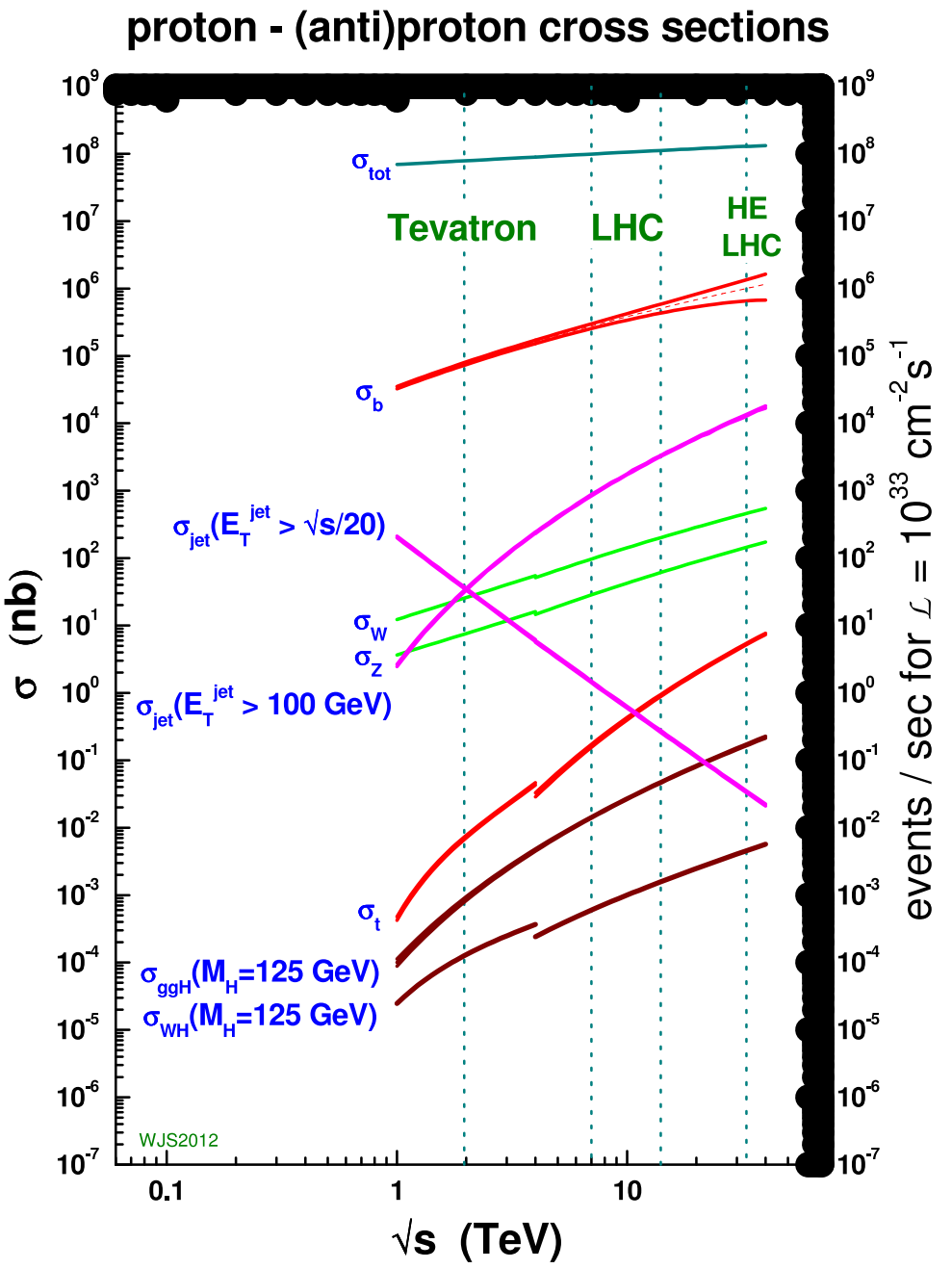
1. $h^0 \rightarrow WW^*$ is observed primarily via the $\boldsymbol{\ell^+ \nu \ell^- \nu}$ ($\boldsymbol{\ell}$ = **e** or $\boldsymbol{\mu}$) final state.
2. $h^0 \rightarrow ZZ^*$ is observed primarily via the $\boldsymbol{\ell^+\ell^- \ell^+\ell^-}$ ($\boldsymbol{\ell}$ = **e** or $\boldsymbol{\mu}$) final state.

In the decays to the diboson final state, kinematics dictates that one of the vector bosons is off-shell (i.e., “virtual”) and is thus indicated by a superscript star.

Question: why not search inclusively for Higgs bosons that decay into a pair of b-quarks?

Answer: The Standard Model background is overwhelming. There are more than 10^7 times as many b-quark pairs produced in proton-proton collisions as compared to b-quark pairs that arise from a decaying Higgs boson.

Nevertheless, the observation of H → bb in the VH channel was confirmed by ATLAS and CMS in 2018!



Summary of ATLAS Higgs boson data from Run 2 at the LHC

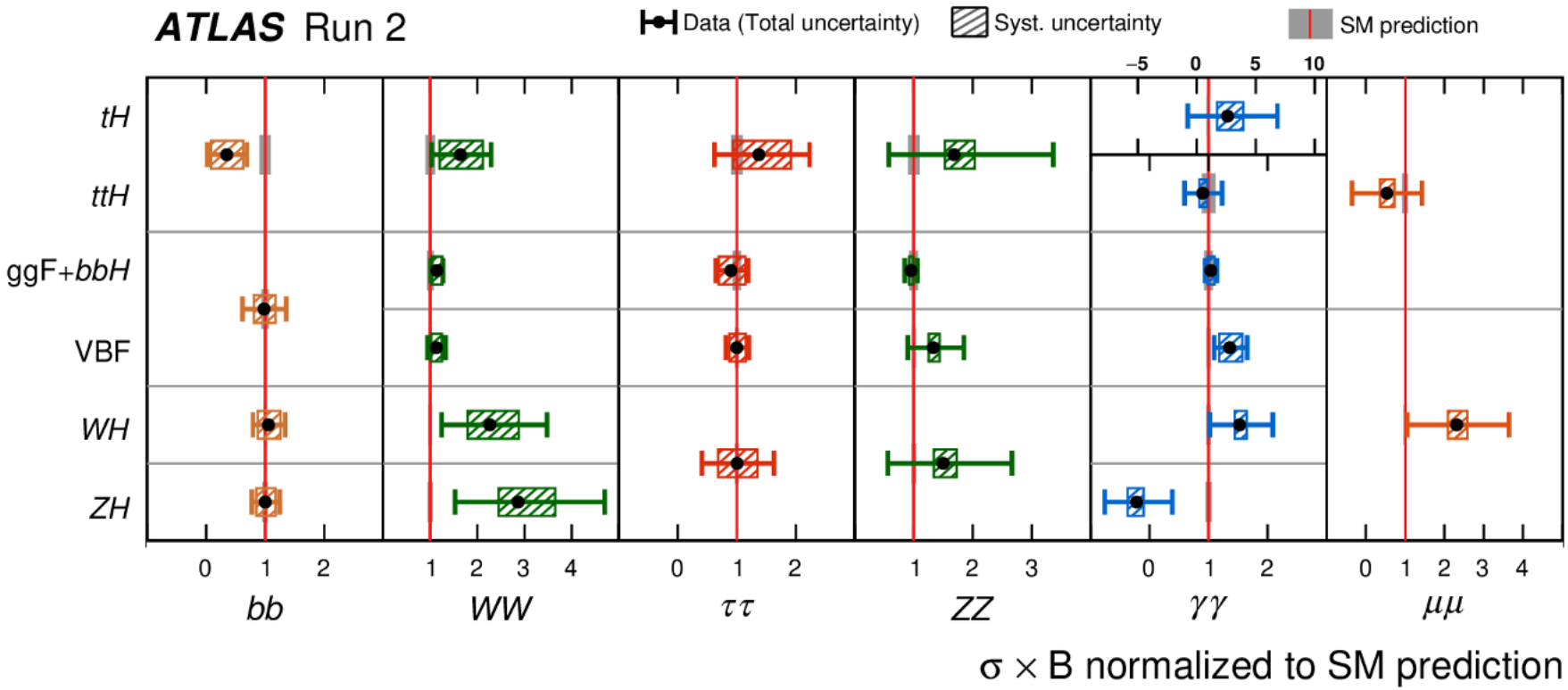
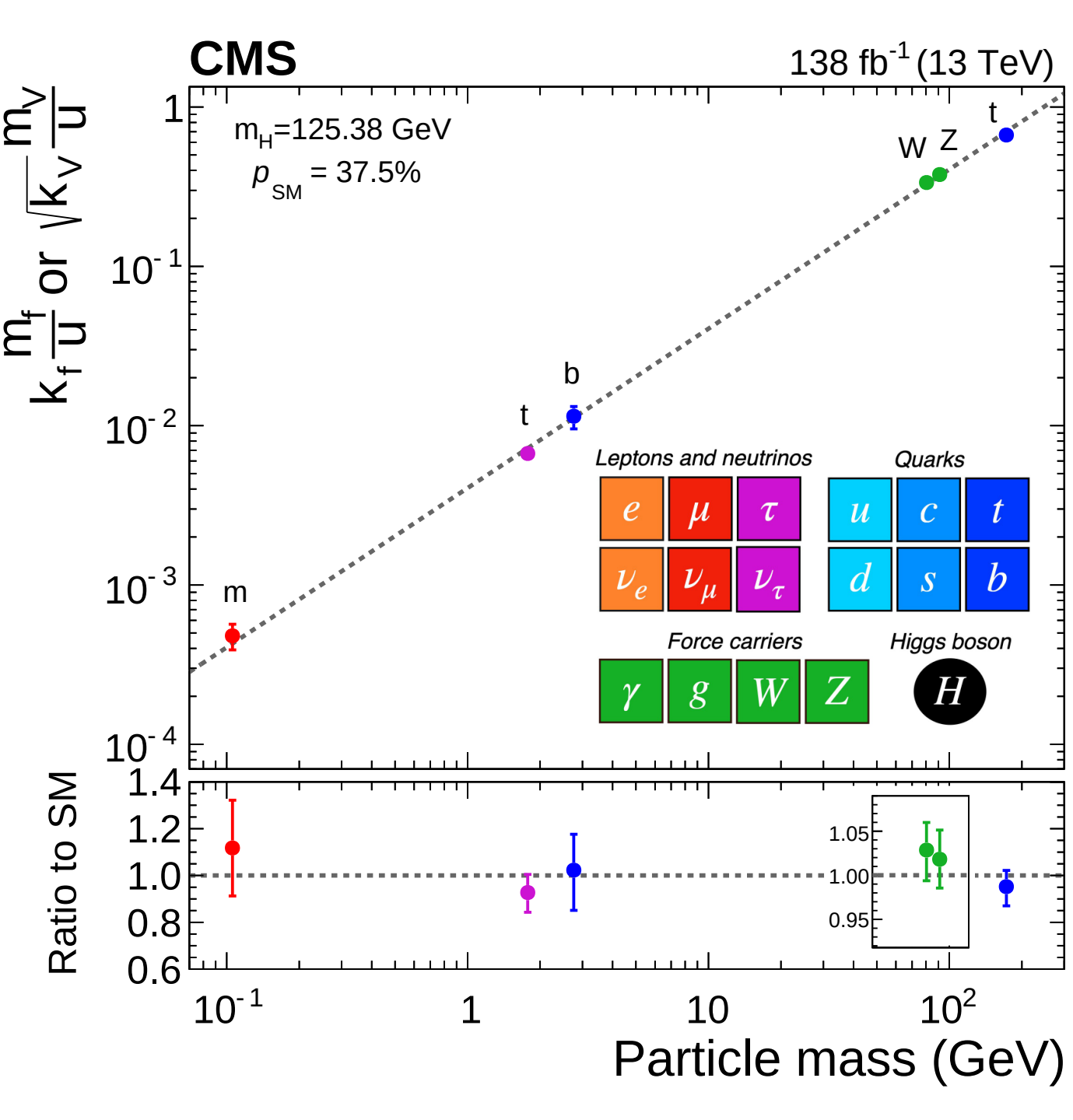
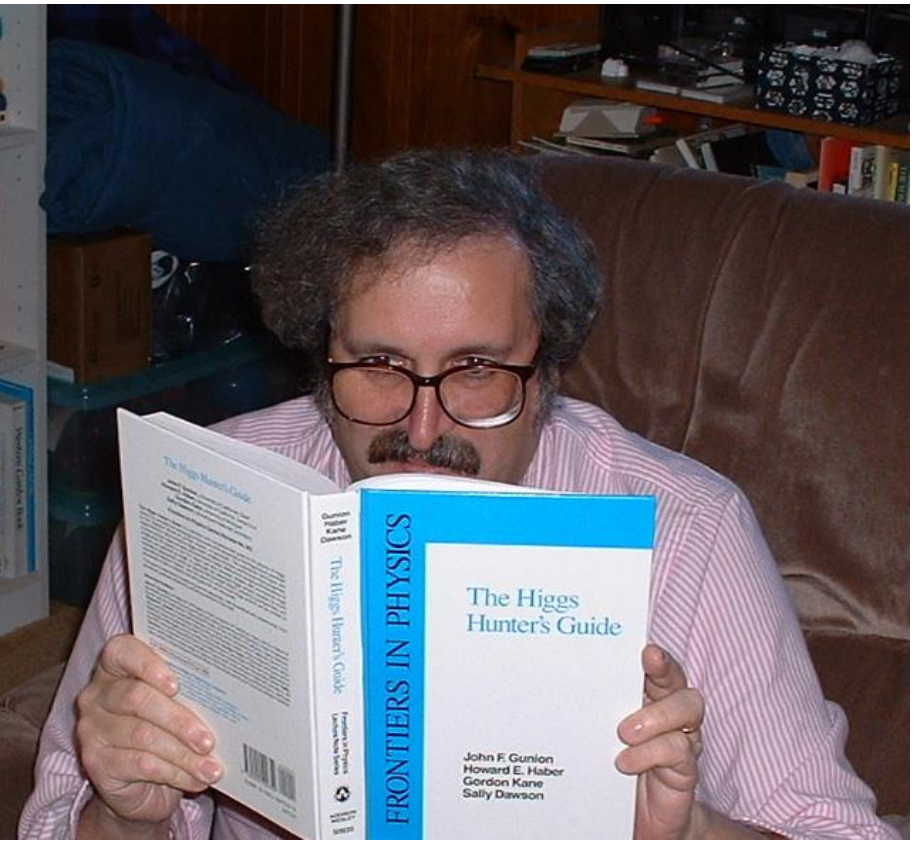


Fig. 3 | Ratio of observed rate to predicted standard model event rate for different combinations of Higgs boson production and decay processes. The horizontal bar on each point denotes the 68% confidence interval. The narrow grey bands indicate the theory uncertainties in the standard model (SM) cross-section multiplied by the branching fraction predictions. The p value for compatibility of the measurement and the SM prediction is 72%. $\sigma_i B_f$ is normalized to the SM prediction. Data are from ATLAS Run 2.

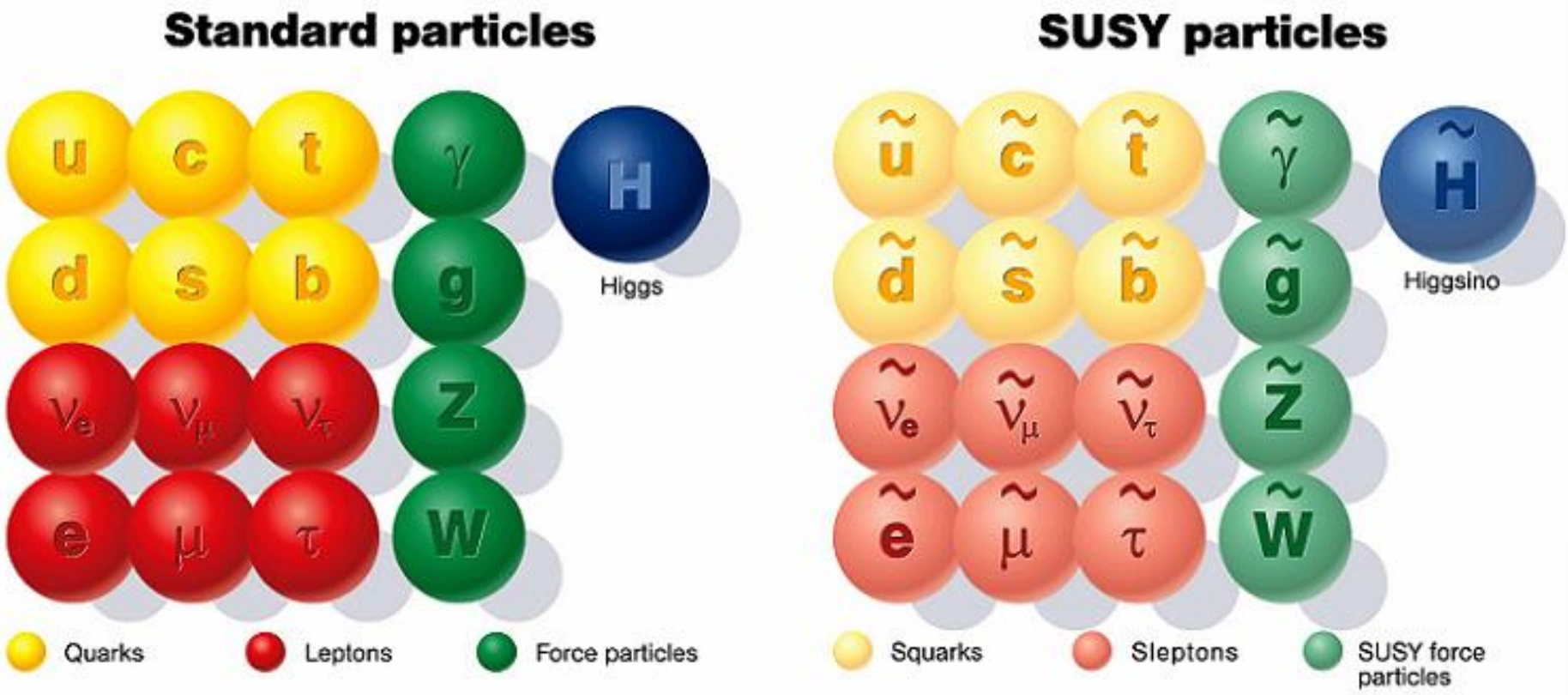


Reduced Higgs coupling modifiers compared to their corresponding prediction from the Standard Model (SM). The error bars represent 68% CL intervals for the measured parameters. In the lower panel, the ratios of the measured coupling modifiers values to their SM predictions are shown.
[Taken from: Nature 607 (2022) 60]

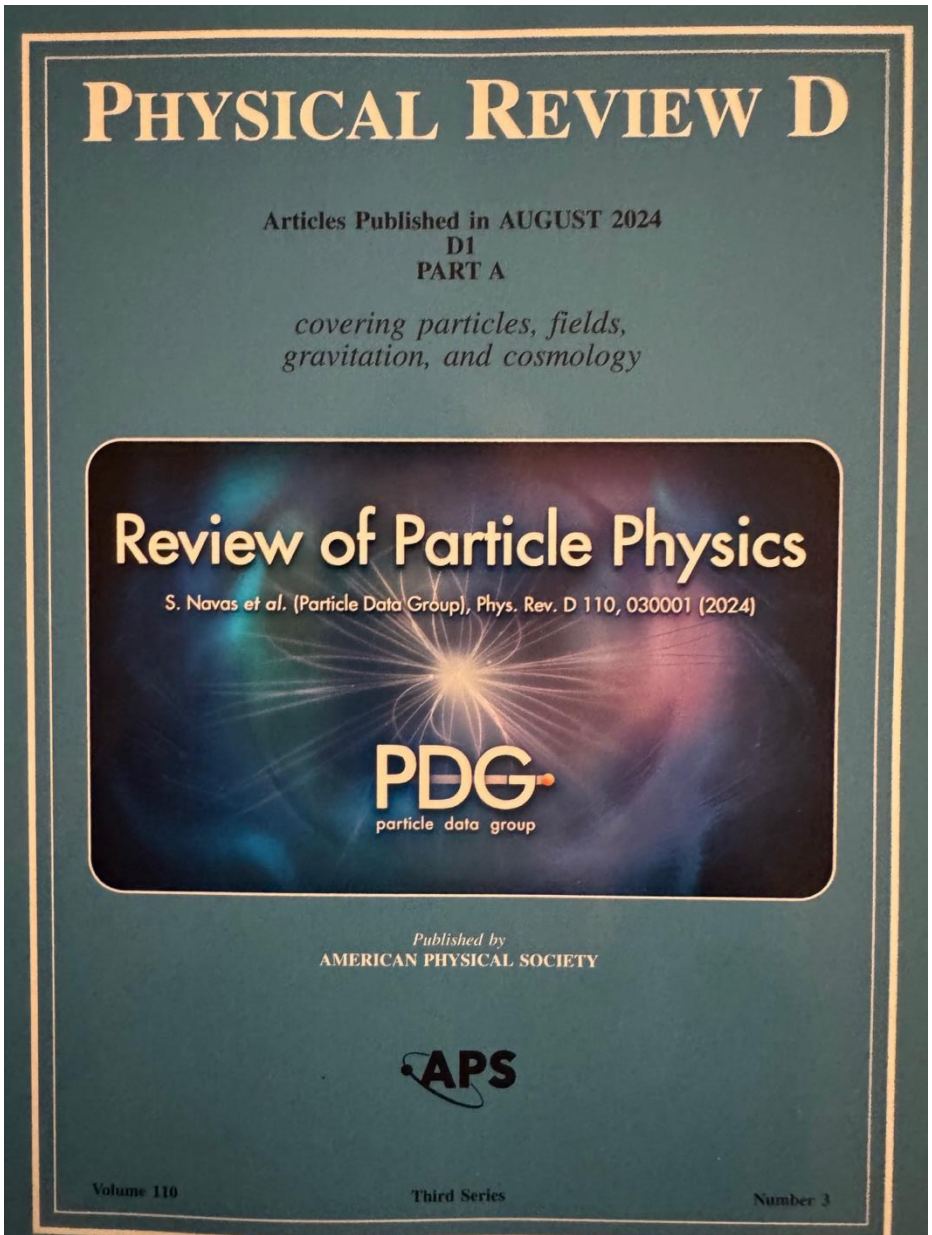
Research program 1: theory and phenomenology of Higgs bosons



Research program 2: theory and phenomenology of TeV-scale supersymmetry (SUSY)



As members of the Particle Data Group, B.C. Allanach and I are co-authors of the biennial Supersymmetry Theory review.



88. Supersymmetry, Part I (Theory)

Revised August 2023 by B.C. Allanach (DAMTP, Cambridge U.) and H.E. Haber (UC Santa Cruz).

88.1 Introduction

Supersymmetry (SUSY) is a generalization of the space-time symmetries of quantum field theory that transforms fermions into bosons and vice versa [1]. The existence of such a non-trivial extension of the Poincaré symmetry of ordinary quantum field theory was initially surprising, and its form is highly constrained by theoretical principles [2]. SUSY also provides a framework for the unification of particle physics and gravity [3–5] at the Planck energy scale, $M_{\rm P} \sim 10^{19}$ GeV, where the gravitational interactions become comparable in strength to the gauge interactions.

less, with some restrictions on the dimension-three ter
dated in Ref. [10]. The impact of the soft terms become
at energy scales much larger than the size of the SUS
masses. Thus, a theory of weak-scale supersymmetry
effective scale of supersymmetry breaking is tied to
electroweak symmetry breaking, provides a natural fra
the origin and the stability of the gauge hierarchy [6–

At present, there is no unambiguous experimental e
the breakdown of the SM at or below the TeV scal
pectations for new TeV-scale physics beyond the SM
primarily on three theoretical arguments. First, in a
an elementary scalar field of mass m and interaction
(*e.g.*, a quartic scalar self-coupling, the square of a gau
or the square of a Yukawa coupling), the stability with
quantum corrections requires the existence of an en
roughly of order $(16\pi^2/\lambda)^{1/2}m$, beyond which new p
enter [12]. A significantly larger energy cutoff would
unnatural fine-tuning of parameters that govern the ef
energy theory. Applying this argument to the SM
expectation of new physics at the TeV scale [9].

Second, the unification of the three SM gauge cou
very high energy close to the Planck scale is possible if
beyond the SM (which modifies the running of the gaug
above the electroweak scale) is present. The minimal
metric extension of the SM, where superpartner mass
a few TeV, provides an example of successful gauge co
fication [13].

Third, the existence of dark matter that makes u
mately one quarter of the energy density of the unive
be explained within the SM of particle physics [14]. R
a stable weakly-interacting massive particle (WIMP)
and interaction rate are governed by new physics asso
the TeV-scale can be consistent with the observed den
matter (this is the so-called WIMP miracle, which is
Ref. [15]). The lightest supersymmetric particle (LSP
is a promising (although not the unique) candidate f
matter [16–20]. Further aspects of dark matter can
Sec. 27.

88.2 Structure of the MSSM

The minimal supersymmetric extension of the SM (M
sists of the fields of the two-Higgs-doublet extension of
the corresponding superpartner fields [21–25]. A part
superpartner together form a supermultiplet. The co
field content of the supermultiplets of the MSSM and
quantum numbers are shown in Table 88.1. The elec
$Q = T_3 + \frac{1}{2}Y$ is determined in terms of the third co
the weak isospin (T_3) and the U(1) weak hypercharge

The gauge supermultiplets consist of the gluons and
fermionic superpartners and the SU(2)×U(1) gauge
their gaugino fermionic superpartners. The matter
plets consist of three generations of left-handed quarks

Published by Cambridge University Press on June 7, 2023

"The new book by Dreiner, Haber, and Martin is a must have for folks who are interested in beyond the Standard Model phenomenology. It contains innumerable lessons for performing quantum field theory calculations both at the conceptual and technical level, by way of many concrete examples within the Standard Model and its supersymmetric extension. I expect this will become a go-to reference for everyone from graduate students to seasoned researchers."
Prof. Tim Cohen, CERN/EPFL and the University of Oregon

"The book gives a self-contained description of the Standard Model of particle physics and its supersymmetric extension. It is well suited for students, as well as experienced researchers in the field. Its unique feature is the comprehensive description of quantum field theory and its application to particle physics in the framework of two-component (Weyl) spinors. [···] The book will be of enormous help to all those that try to teach and try to learn the subject."
Prof. Hans-Peter Nilles, Universität Bonn

"This is a massive, definitive text on phenomenological supersymmetry in quantum field theory by three giants of the field. The book develops two-component spinor formalism and its practical use in amplitude computations with many phenomenological examples up to one loop order. Supersymmetric extensions of the Standard Model are also covered and many other gems besides."
Prof. Ben Allanach, University of Cambridge

Supersymmetry is an extension of the successful Standard Model of particle physics; it relies on the principle that fermions and bosons are related by a symmetry, leading to an elegant predictive structure for quantum field theory. This textbook provides a comprehensive and pedagogical introduction to supersymmetry and other aspects of particle physics at the high-energy frontier. Aimed at graduate students and researchers, it also discusses concepts of physics beyond the Standard Model, including extended Higgs sectors, grand unification, and the origin of neutrino masses.

Cover image: Sierralara/RooM/Getty Images

Designed by EMC Design Ltd

Dreiner, Haber and Martin

From Spinors to Supersymmetry

CAMBRIDGE UNIVERSITY PRESS

ISBN 978-0-521-80088-4

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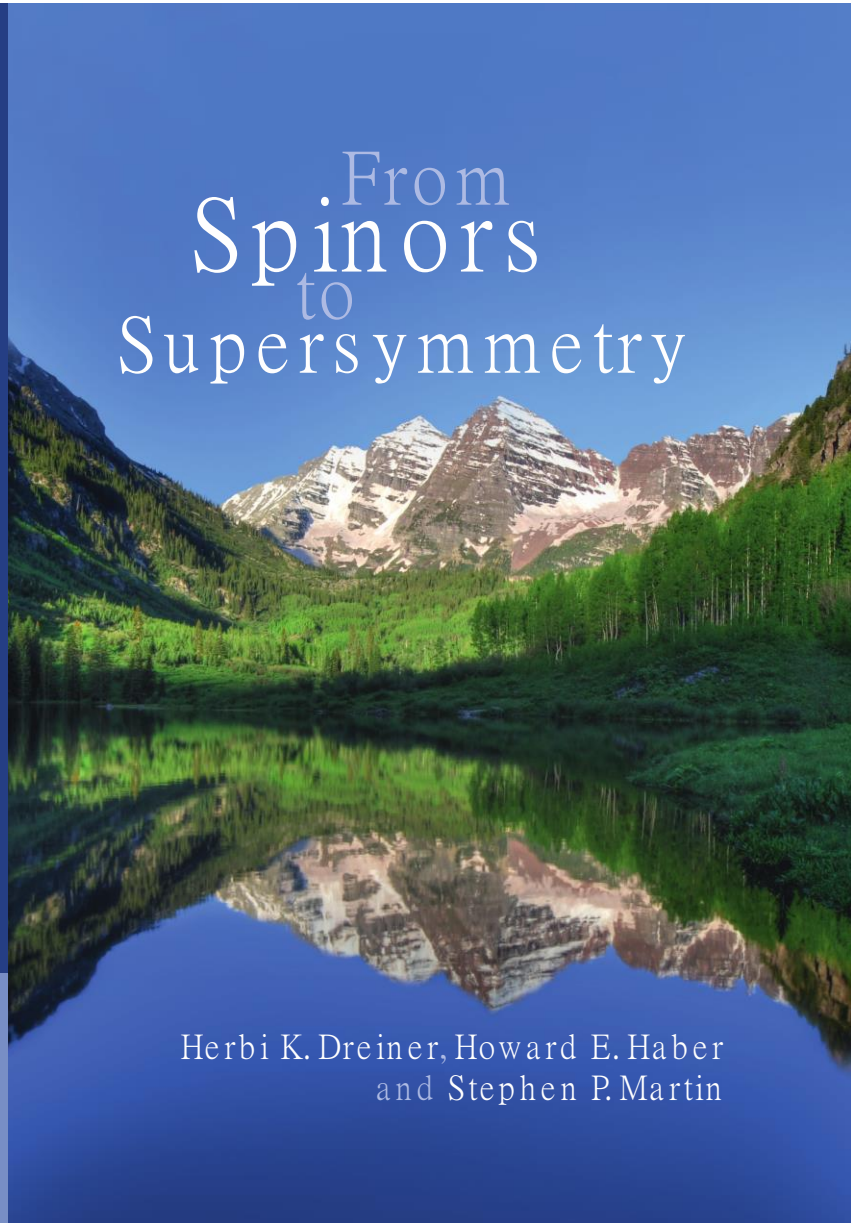
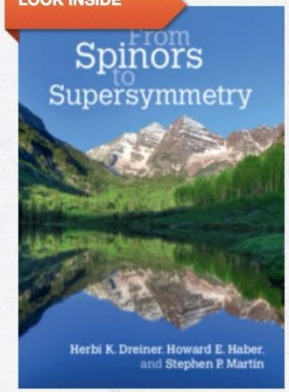


LOOK INSIDE



From Spinors to Supersymmetry

TEXTBOOK

AUTHORS:
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Howard E. Haber, University of California, Santa Cruz
Stephen P. Martin, Northern Illinois University

DATE PUBLISHED: June 2023
AVAILABILITY: Available
FORMAT: Hardback
ISBN: 9780521800884

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Description Contents Resources Courses About the Authors

Supersymmetry is an extension of the successful Standard Model of particle physics; it relies on the principle that fermions and bosons are related by a symmetry, leading to an elegant predictive structure for quantum field theory. This textbook provides a comprehensive and pedagogical introduction to supersymmetry and spinor techniques in quantum field theory. By utilising the two-component spinor formalism for fermions, the authors provide many examples of practical calculations relevant for collider physics signatures, anomalies, and radiative corrections. They present in detail the component field and superspace formulations of supersymmetry and explore related concepts, including the theory of extended Higgs sectors, models of grand unification, and the origin of neutrino masses. Numerous exercises are provided at the end of each chapter. Aimed at graduate students and researchers, this volume provides a clear and unified treatment of theoretical concepts that are at the frontiers of high energy particle physics.

Read more

Reviews & endorsements

'The new book by Dreiner, Haber, and Martin is a must have for folks who are interested in beyond the Standard Model phenomenology. It contains innumerable lessons for performing quantum field theory calculations both at the conceptual and technical level, by way of many concrete examples within the Standard Model and its supersymmetric extension. I expect this will become a go-to reference for everyone from graduate students to seasoned researchers.' Tim Cohen, CERN/EPFL and the University of Oregon


Cruz
UCSC
From
Spinors
to
Supersymmetry
Herbi K. Dreiner Howard E. Haber
and Stephen P. Martin

Research program 3: explorations of the Terascale at the LHC and at future colliders

- Studies of non-minimal Higgs sectors
- Precision measurements of new physics observables
- Distinguishing among different theoretical interpretations of new physics signals
- Using a future lepton collider as a precision Higgs factory
- Terascale footprints of lepton-number-violation
- New sources for CP-violation (Higgs and/or SUSY mediated)

Selected Publications (2023—2025)

RG-stable parameter relations of a scalar field theory in absence of a symmetry
H.E. Haber, and P. Ferreira, **arXiv:2502.11011 [hep-ph]**.

Correlating A→γγ with EDMs in the 2HDM in light of the diphoton excesses at 95 GeV and 152 GeV
S. Banik, G. Coloretti, A. Crivellin, and H.E. Haber, **arXiv:2412.00523 [hep-ph]**.

Explicit form for the most general Lorentz transformation revisited
H.E. Haber, Symmetry 2024, 16, 1155.

Classes of complete dark photon models constrained by Z-Physics
M. Bento, H.E. Haber, and J.P. Silva, Phys. Lett. B 850, 138501 (2024).

Tree-level Unitarity in $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ Models
M. Bento, H.E. Haber, and J.P. Silva, JHEP 10 (2023) 083.

Accommodating Hints of New Heavy Scalars in the Framework of the Flavor-Aligned Two-Higgs-Doublet Model
J.M. Connell, H.E. Haber, and P. Ferreira, Phys.Rev. D 108, 055031 (2023).

Major thrusts in phenomenological particle physics today

➢ What lies beyond the Standard Model and why haven't we seen it yet?

- New physics beyond the Standard Model (BSM) may be associated with a new heavy mass scale of order a few TeV or larger. If accessible at the LHC, not enough events have been produced yet (more luminosity needed). If the LHC is not energetic enough, one would need a higher energy collider facility.

- New BSM physics may be very weakly coupled to the Standard Model (SM). It could consist of completely new sectors of particles (e.g., the dark sector). The origin of dark matter could reside here. Many possibilities exist, so it is difficult to guess where the breakthrough will occur.

- If new BSM physics is completely neutral with respect to the SM, then it can only communicate with the SM via "portals" that consist of products of SM fields that have no net SM (color, weak or EM) charge.
  Examples: the Higgs portal $H^\dagger H$; the neutrino portal $H^\dagger LN$ (N could be a new sterile neutrino); or photon mixing $F_{\mu\nu}X^{\mu\nu}$ (where X is the dark photon).

Should we expect an extended Higgs sector beyond the SM?

- The fermion and gauge boson sectors of the SM are not of minimal form ("who ordered that?"). So, why should the spin-0 (scalar) sector be minimal?

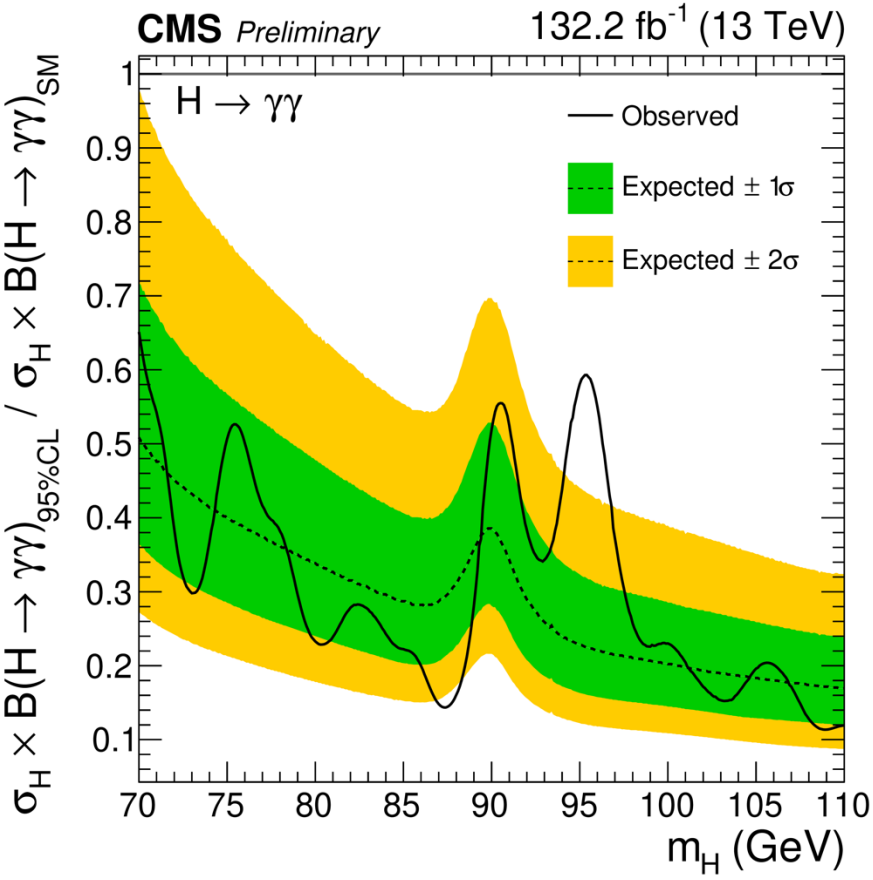
- Adding new scalar states can alleviate the metastability of the vacuum, allowing the Higgs-sector-extended SM to be valid all the way up to the Planck scale.

- Extended Higgs sectors can provide a dark matter candidate.

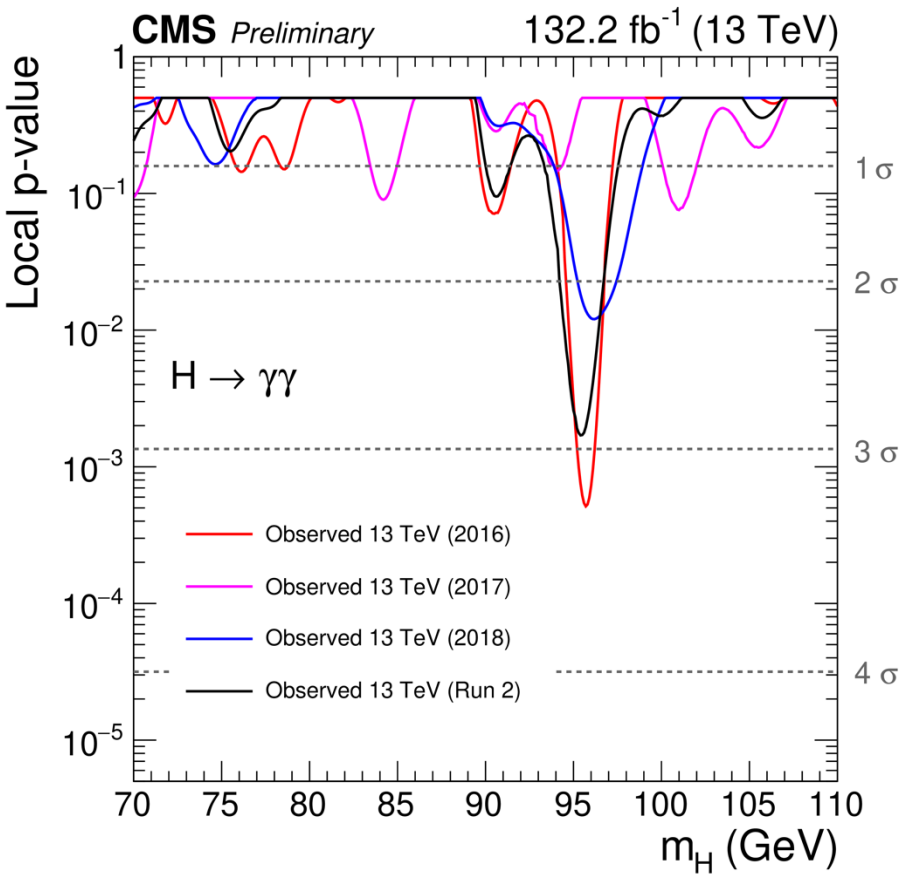
- Extended Higgs sectors can provide new sources of CP violation (which may be useful in baryogenesis).

- Models of physics beyond the SM often require additional scalar Higgs states. E.g., two Higgs doublets are required in the minimal supersymmetric extension of the SM (MSSM).

Evidence for a new Higgs scalar ?



Expected and observed exclusion limits (95% CL, in the asymptotic approximation) on the product of the production cross section and branching fraction into two photons for an additional SM-like Higgs boson, from the analysis of the combined data from 2016, 2017, and 2018. The inner and outer bands indicate the regions containing the distribution of limits located within ±1 and 2σ, respectively, of the expectation under the background-only hypothesis.



The observed local *p*-values for an additional SM-like Higgs boson as a function of m_H, from the analysis of the data from 2016, 2017, 2018, and their combination. Taken from CMS-PAS-HIG-20-002 (20 March 2023).

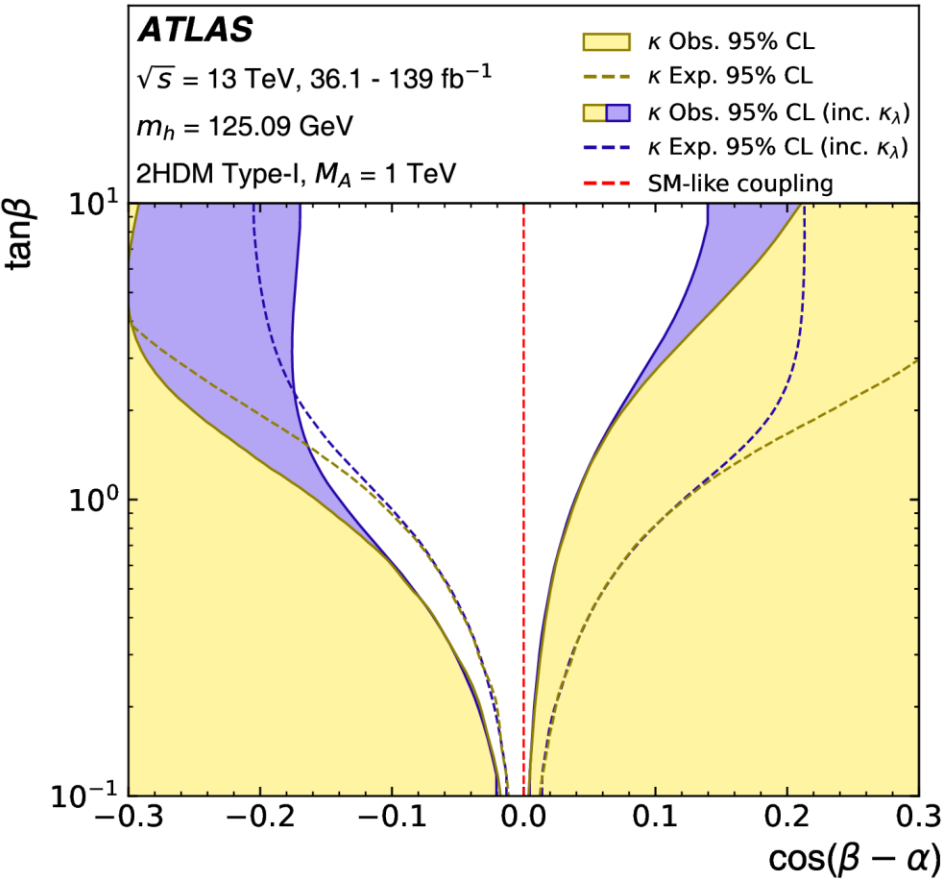
Why is the observed Higgs boson SM-like?

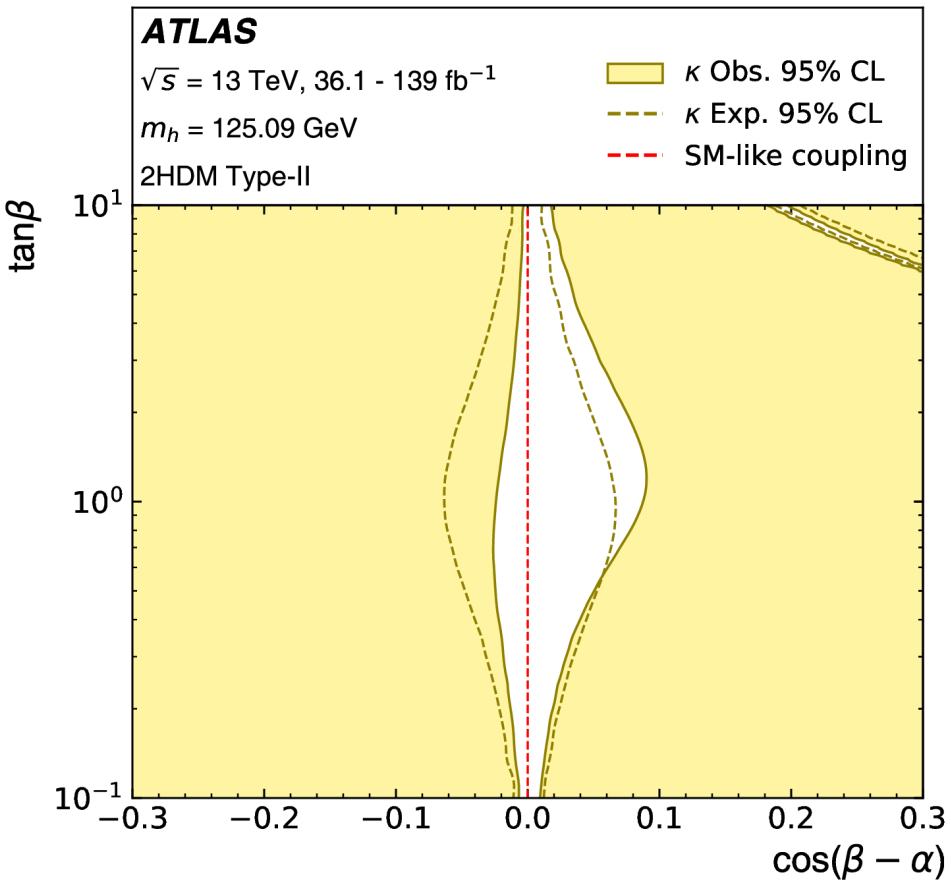
- There is no extended Higgs sector.

- All other scalars (apart from the SM-like Higgs boson) are very heavy
  - This is the decoupling limit.

- A neutral scalar field with the tree-level properties of the SM Higgs boson is an approximate mass eigenstate (due to suppressed mixing with other neutral scalar fields of the extended Higgs sector).
  - This is the Higgs field alignment limit.
  - The other physical scalars of the model may or may not be significantly heavier than the SM Higgs boson. That is, the decoupling limit is a special case of the Higgs field alignment limit.

Experimental constraints on the two Higgs doublet model (2HDM)





Regions excluded at 95% CL in the κ-framework-based approach by the measured rates of Higgs boson production and decays in the 2HDM with Type-I and Type-II Yukawa couplings, respectively. The dark yellow dashed lines show the borders of the corresponding expected exclusion regions for the SM hypothesis. Exact Higgs alignment corresponds to cos(β - α) = 0. Taken from the ATLAS Collaboration, JHEP 11 (2024) 097.

My most recent Ph.D. students and their projects

- 2HDM theory and phenomenology (with E. Shahly). Currently on leave but returning in April 2025. Expected Ph.D. in December 2025.

  - Neutral Higgs-mediated flavor violation in the lepton sector due to renormalization group running (with S. Gori and E. Shahly).
  - One-loop renormalization of the 2HDM in the Higgs basis.

- Phenomenological aspects of more general 2HDMs (with J.M. Connell). Received his Ph.D. in June 2024.

  - Explored some (local) 2—3σ deviations in LHC searches for new Higgs bosons, with implications for the flavor-aligned 2HDM. Results published in **Phys.Rev. D 108, 055031 (2023).**
  - Examined the structure of lepton flavor-changing neutral currents mediated by neutral Higgs bosons in extended Higgs models. Results to appear on the arXiv later this spring.

From a forthcoming paper in collaboration with Stefania Gori and Eric Shahly. Off-diagonal couplings of the neutral Higgs boson to $\tau\mu$ can be generated if flavor alignment is imposed at a very high energy scale Λ, due to renormalization group evolution from Λ down to the energy scale of electroweak physics (100 GeV).

4 Results

4.1 Lepton flavor violating decays of the SM-like Higgs boson

The partial widths for the decays of the SM-like Higgs field h into a pair of fermions are given below. Note that the color factor $N_C = 3$ for quarks, and $N_C = 1$ for leptons.

$$\Gamma(h \to f_i \overline{f_i}) = \frac{N_C G_F}{4\sqrt{2}\pi} m_h m_{f_i}^2 \left[\mathrm{Re}\left(s_{\beta-\alpha} + \epsilon_6 c_{\beta-\alpha} \frac{\rho_f^{ii}}{\kappa_f^{ii}} \right)^2 \left(1 - \frac{4m_{f_i}^2}{m_h^2} \right)^{3/2} + \mathrm{Im}\left(s_{\beta-\alpha} + \epsilon_6 c_{\beta-\alpha} \frac{\rho_f^{ii}}{\kappa_f^{ii}} \right)^2 \left(1 - \frac{4m_{f_i}^2}{m_h^2} \right)^{1/2} \right] \tag{4.1}$$

$$\Gamma(h \to f_i \overline{f_j}) = \Gamma(h \to f_j \overline{f_i}) = N_C \frac{m_h c_{\beta-\alpha}^2}{16\pi} \left(|\rho_f^{ij}|^2 + |\rho_f^{ji}|^2 \right) \times \left[1 - \left(\frac{m_{f_i} - m_{f_j}}{m_h} \right)^2 \right] \times \left[\left(1 - \frac{m_{f_i}^2 + m_{f_j}^2}{m_h^2} \right)^2 - \frac{4m_{f_i}^2 m_{f_j}^2}{m_h^4} \right]^{1/2} \quad (i \neq j) \tag{4.2}$$

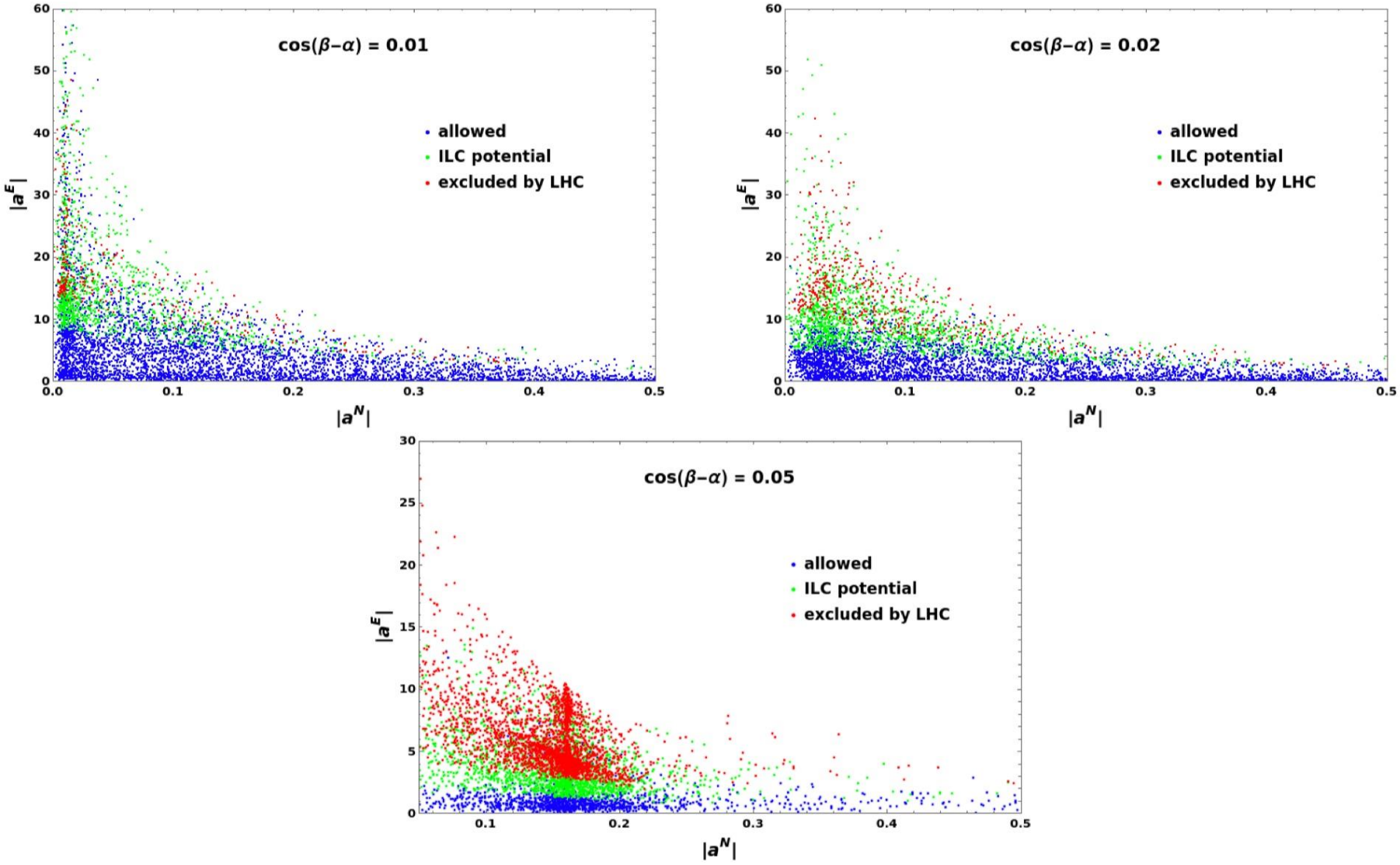


Figure 3: BR($h \to \mu\tau$) results for the case of $\cos(\beta - \alpha) = 0.01$ (left), 0.02 (right) and 0.05 (bottom) for fixed quark parameters $a^U = 0.1$ and $a^D = 1$. Green points indicate choices of the alignment parameters that lead to $h \to \mu\tau$ branching ratios that exceed the projected ILC upper bound of 2.3×10^{-4}, but are not yet excluded by LHC bounds. Red points are already excluded by LHC bounds and blue points remain unexcluded by both current experimental bounds and ILC projections.

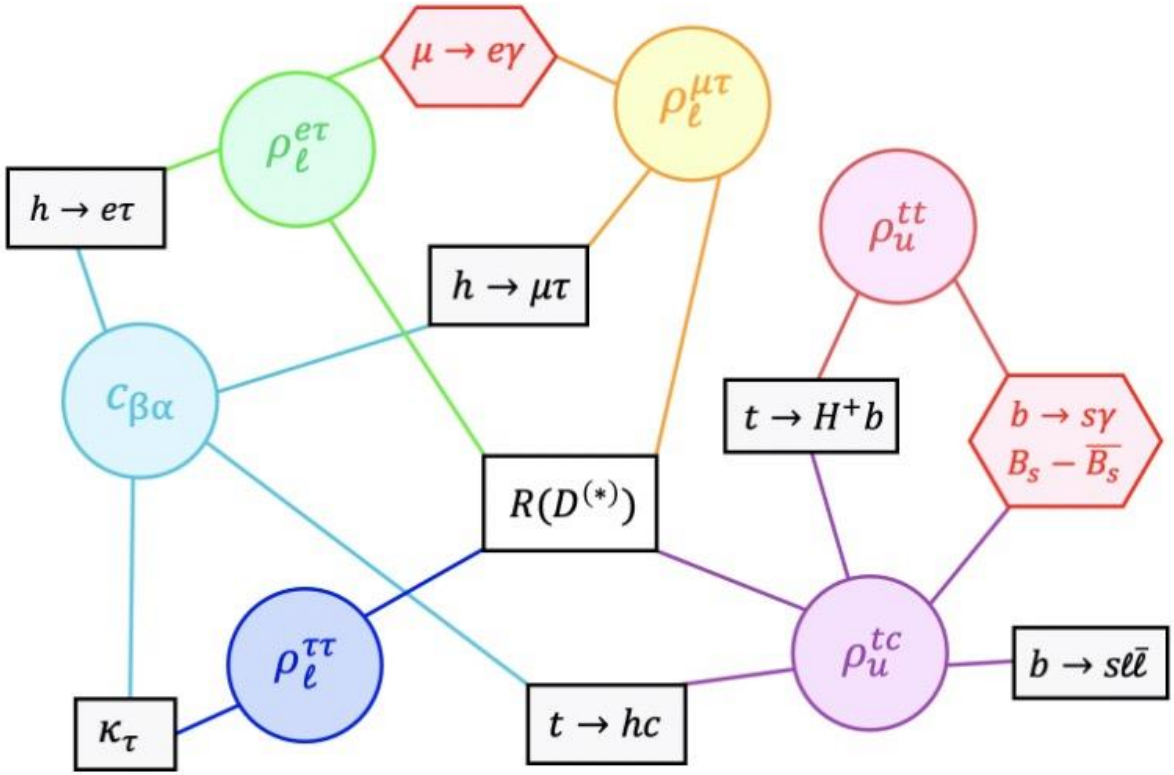


FIG. 3. Diagram showing the correlations between the free parameters (circles) of our model (except the Higgs masses) and the observables. Observables providing strong constraints are shown as red hexagons while the ones pointing towards a NP effect are shown as black rectangles.

Taken from A. Crivellin and S. Iguro, Phys. Rev. **D** 110, 015014 (2024).

From a forthcoming paper with Joseph Connell. Nondiagonal lepton—Higgs couplings are constrained by many observables. For example, consider $\tau \longrightarrow \mu\,\gamma$.

$$A(\tau \to \mu\gamma) \simeq \frac{1}{16\pi^2}\left(\sqrt{2}\sum_{\phi}\frac{g_{\phi\mu\tau}g_{\phi\tau\tau}}{m_\phi^2}\left(\ln\frac{m_\phi^2}{m_\tau^2}-\frac{3}{2}\right)+2\sum_{\phi,f}g_{\phi\mu\tau}g_{\phi ff}\frac{N_c Q_f^2\alpha}{\pi}\frac{1}{m_\tau m_f}f_\phi(\frac{m_f^2}{m_\phi^2})\right.$$
$$\left.-\sum_{\phi=h,H}g_{\phi\mu\tau}C_{\phi WW}\frac{g\alpha}{2\pi m_\tau m_W}\left[3f_\phi(\frac{m_W^2}{m_\phi^2})+\frac{23}{4}g(\frac{m_W^2}{m_\phi^2})+\frac{3}{4}h(\frac{m_W^2}{m_\phi^2})+m_\phi^2\frac{f_\phi(\frac{m_W^2}{m_\phi^2})-g(\frac{m_W^2}{m_\phi^2})}{2m_W^2}\right]\right)$$

We define three integrals for real positive values of z $[1,2]$:

$$g(z) = \tfrac{1}{2}z\int_0^1 \frac{dx}{x(1-x)-z}\ln\left[\frac{x(1-x)}{z}\right],$$

$$f(z) = \tfrac{1}{2}z\int_0^1 \frac{1-2x(1-x)}{x(1-x)-z}\ln\left[\frac{x(1-x)}{z}\right]dx,$$

$$h(z) = -\tfrac{1}{2}z\int_0^1 \frac{dx}{x(1-x)-z}\left\{1-\frac{z}{x(1-x)-z}\ln\left[\frac{x(1-x)}{z}\right]\right\}.$$

Then, one can derive the following expressions for $f(z)$ and $h(z)$ in terms of $g(z)$:

$$f(z) = z(2+\ln z)+(1-2z)g(z),$$

$$h(z) = \frac{z\left[2g(z)+\ln z\right]}{1-4z}.$$

An explicit expression for $g(z)$ is given by:

$$g(z) = \begin{cases} \dfrac{z}{\sqrt{1-4z}}\left\{\mathrm{Li}_2(x_+) - \mathrm{Li}_2(x_-) - \frac{1}{2}\ln z \ln\left(\dfrac{x_+}{x_-}\right)\right\}, & \text{for } 0 < z \leq \frac{1}{4}, \\ \dfrac{2z}{\sqrt{4z-1}}\,\mathrm{Cl}_2\left(2\sin^{-1}\dfrac{1}{2\sqrt{z}}\right), & \text{for } z > \frac{1}{4}, \end{cases} \tag{62}$$

where $x_\pm \equiv \frac{1}{2}\left[1 \pm \sqrt{1-4z}\,\right]$ and $0 \leq \sin^{-1}[1/(2\sqrt{z})] \leq \frac{1}{2}\pi$ (for $z \geq \frac{1}{4}$). In Fig. 1, we have employed Mathematica (Version 14.0) to produce plots of the functions $g(z)$, $f(z)$ and $-h(z)$ for $0.01 \leq z \leq 100$. This figure reproduces the results first shown in Fig. 3 of Ref. [2].

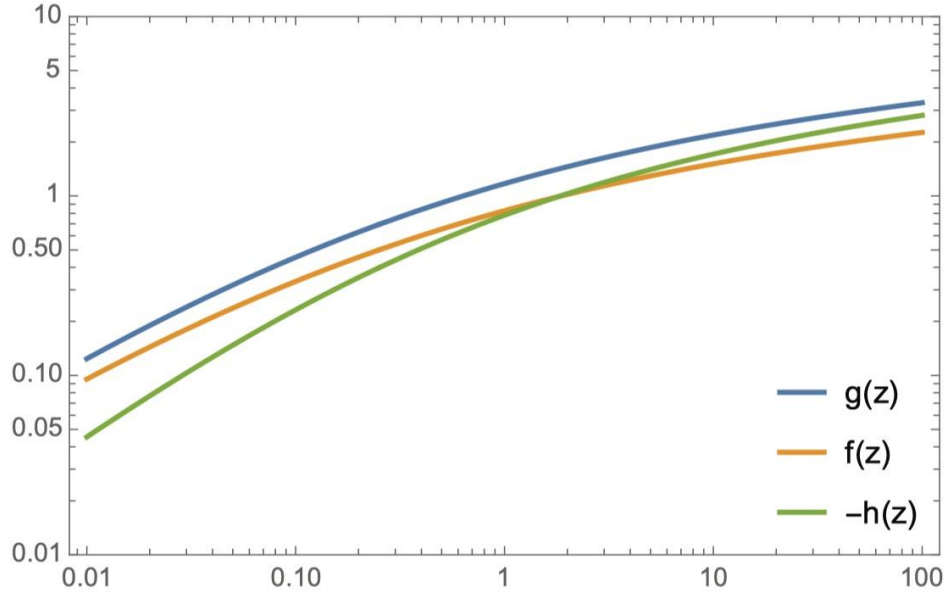


Figure 1: Plots of $g(z)$ given by eq. (62), $f(z)$ given by eq. (60), and $h(z)$ given by eq. (61) as a function of the variable z for $0.01 \leq z \leq 100$. These plots were produced using Version 14.0 of Mathematica.

Ongoing and Future Activities

- Higgs alignment at one loop (with Eric Shahly).
- Reassessing the Cheng-Sher ansatz for off-diagonal flavor couplings of neutral Higgs bosons (with Joseph Connell).
- Basis-invariant treatment of the 3HDM (with V. Keus).
- Extension of 2HDM symmetries of the scalar potential to the Yukawa sector (with Sergio Carrolo, Luis Lourenco, and J.P. Silva).
- Beyond the S, T, and U oblique parameters in extended electroweak models containing a dark Z boson.
- The anapole moment of fundamental particles (with H. Dreiner).