DUE: TUESDAY, MARCH 5, 2024

1. Jackson, problem 9.12

HINT: The charge density can be expressed as $\rho(\vec{\boldsymbol{x}},t)=\rho_0\Theta(R(\theta)-r)$, where the step function $\Theta(x)=1$ for $x>0$ and $\Theta(x)=0$ for $x<0$. The constant ρ_0 can be determined in terms of the total charge Q which is conserved and hence time-independent. Find the relation between ρ_0 and Q assuming that $\beta\ll 1$. Then writing $\beta=\beta_0 e^{-i\omega t}$, expand the expression for $\rho(\vec{\boldsymbol{x}},t)$ to linear order in β_0. You can now use this expression to evaluate the electric multipole moments. You can also evaluate the current density $\vec{\boldsymbol{J}}(\vec{\boldsymbol{x}},t)$ by making use of the continuity equation. This will be needed to evaluate the magnetic multipole moments.

2. Jackson, problem 9.17

3. Jackson, problem 12.1

HINT: An alternative approach to the Lagrangian formalism for a relativistic charged particle is to treat the 4-vector of position x_μ and the 4-velocity u_μ as Lagrangian coordinates. Then the Euler-Lagrange equations have the obvious covariant form:

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial u_\mu}\right)-\frac{\partial L}{\partial x_\mu}=0\,,$$

where L is a Lorentz-invariant Lagrangian and τ is the proper time.

4. Jackson, problem 12.3

5. Jackson, problem 12.11

HINT 1: In part (a), instead of making use of the parenthetical hint provided by Jackson, you should proceed as follows. Begin your analysis with the Thomas equation, which is given in eq. (11.170) of Jackson. Note that the Thomas equation is an equation of the form

$$\frac{d\vec{\boldsymbol{s}}}{dt}=\vec{\boldsymbol{s}}\times\vec{\boldsymbol{\omega}}\,,$$

where $\vec{\boldsymbol{s}}$ is the spin vector and $\vec{\boldsymbol{\omega}}$ is a vector whose magnitude is the spin precession frequency.

HINT 2: Note that the energy values given in part (c) correspond to the total relativistic energies.