DUE: TUESDAY, MARCH 19, 2024

You must submit the final homework to my office or to my ISB mailbox by 5pm on Tuesday March 19 in order to earn the proper credit. Solutions will then be posted on the class website.

FINAL EXAM ALERT: The final exam will be held in person on Wednesday March 20 from 4–7 pm in ISB 231. During the exam, you may refer to Jackson's text, the class handouts and solution sets, your own personal notes, and one additional textbook of your choosing (should you feel the need). You may also consult a reference for integrals or other mathematical facts. You may use your laptop to access the links provided on the class webpage. Use of a calculator (if needed) is encouraged. However, you should *not* collaborate with anyone else during the exam.

1. Jackson, problem 14.4

2. Jackson, problem 14.5

3. In class, we showed that the angular distribution of the power radiated by a charge e moving along a trajectory $\vec{\boldsymbol{r}}(t)$ at velocity $c\vec{\boldsymbol{\beta}}(t) \equiv d\vec{\boldsymbol{r}}(t)/dt$ is given by:

$$\frac{dP}{d\Omega} = \lim_{r\to\infty} \frac{cr^2}{4\pi} \int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} d\omega''\, \vec{\boldsymbol{E}}^*_{\omega'}(\vec{\boldsymbol{x}}) \cdot \vec{\boldsymbol{E}}_{\omega''}(\vec{\boldsymbol{x}})\, e^{i(\omega'-\omega'')t}\,,$$

where r is the distance of the observer from the origin and the Fourier coefficient of the electric field vector is given by

$$\boldsymbol{E}_\omega(\vec{\boldsymbol{x}}) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} dt\, \vec{\boldsymbol{E}}(\vec{\boldsymbol{x}}, t) e^{i\omega t}$$

(a) Derive the following expression for the Fourier coefficient,

$$\boldsymbol{E}_\omega(\vec{\boldsymbol{x}}) = \frac{-ie\,\omega\, e^{i\omega r/c}}{2\pi rc} \int_{-\infty}^{\infty} dt\ \hat{\boldsymbol{n}}\times(\hat{\boldsymbol{n}} \times \vec{\boldsymbol{\beta}})\, e^{i\omega(t-\hat{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}(t)/c)}\,,$$

where $\hat{\boldsymbol{n}}$ is a unit vector pointing from the charge to the observer.[1]

[1] As noted by Jackson below his eq. (14.62), assuming that the observation point $\vec{\boldsymbol{x}}$ is located very far away from the region of space where the acceleration occurs, the unit vector $\hat{\boldsymbol{n}}$ can be very well approximated as being constant in time.

(b) Suppose that the motion of the radiating particle repeats itself with periodicity T. Using the results of part (a) and the Poisson sum formula, solve Jackson problem 14.13, where the power distribution time-averaged over one cycle can be written as:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{1}{T}\int_0^T dt\, \frac{dP}{d\Omega} = \sum_{m=1}^{\infty} \frac{dP_m}{d\Omega}\,.$$

HINT for part (a): Use the same integration by parts technique employed by Jackson in obtaining his eq. (14.67), and assume that Jackson's justification for dropping the boundary term is valid.

HINT for part (b): Convert the integral over t in part (a) into a more useful form by dividing up the range of integration into the intervals $mT \leq t \leq (m+1)T$, for $m = 0, \pm 1, \pm 2, \ldots$, and then apply the Poisson sum formula. A statement of this formula along with its derivation is given in Section 5 of the class handout entitled *Generalized Functions for Physics.*

4. Jackson, problem 13.9

5. Jackson, problem 10.1