Two-Higgs-Doublet Model

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Overview

Motivation

- Super symmetry needs two Higgs doublets
- SM cannot explain the observed baryon asymmetry, possible in 2HDM
- CP Violations
- Candidates for dark matter
- Many more..

Part I: Review of SM

SM Higgs

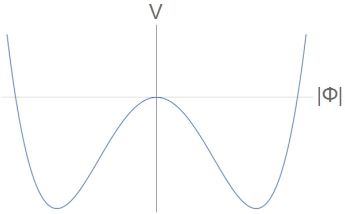
- In the SM, we have one complex scalar doublet

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{1}$$

- The scalar potential is given by

$$V = -\mu^2 \Phi^\dagger \Phi + \frac{1}{2}\lambda \left(\Phi^\dagger \Phi\right)^2$$

where μ^2 and λ are real.



SM Higgs (Continued)

- We assume that the scalar potential has a non-zero vev:

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{2}$$

- Expanding about the vev

$$\Phi = \begin{pmatrix} G^+ \\ (v + H + iG^0)/\sqrt{2} \end{pmatrix} \tag{3}$$

where G^- is a complex field and H, G^0 are real fields

- The potential becomes

$$V = -\frac{\mu^4}{2\lambda} + \mu^2 H^2 + \mathcal{L}_{\mathrm{Int}}$$

- Only H gets a mass and it's mass matrix, $\frac{\partial^2 V}{\partial H \partial H} = 2\mu^2$ is trivial

SM Higgs (Continued)

- We thus have a charged goldstone boson: G^+, a nuetral goldstone boson: G^0 and a Higgs: H
- This scalar field couples to the gauge fields via the covariant derivative

$$\mathcal{D}_\mu = \partial_\mu + igT^a A^a_\mu + ig'\frac{1}{2}YB_\mu \tag{4}$$

- This gives us the $W's$, the Z and the photon field

$$W^\pm = \frac{1}{\sqrt{2}}\left(A^1_\mu \mp iA^2_\mu\right); \quad Z^0_\mu = \frac{gA^3_\mu - g'B_\mu}{\sqrt{g^2+g'^2}}; \tag{5}$$

$$A_\mu = \frac{g'A^3_\mu + gB_\mu}{\sqrt{g^2+g'^2}} \tag{6}$$

- We then couple the scalar field to the fermions to give them mass via Yukawa couplings

$$\mathcal{L}_{\text{Yukawa}} = -\lambda_e \bar{\psi}_{L_2}\Phi\psi_E - \lambda_u\bar{\psi}_{Q_1}\Phi\psi_u - \lambda_d\bar{\psi}_{Q_2}\Phi\psi_d - \cdots \tag{7}$$

Part 2: 2HDM Formalism

Two-Higgs Doublet Model (2HDM)

- Now we add a second Higgs doublet
- The most general, renormalizable two-doublet scalar potential is [1]

$$
\begin{aligned}
V = {} & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.C.}\right) \\
& + \frac{1}{2}\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{1}{2}\lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) \\
& + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left[\frac{1}{2}\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2\right. \\
& \left. + \lambda_6\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_1^\dagger\Phi_2\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\left(\Phi_1^\dagger\Phi_2\right) + \text{H.C.}\right]
\end{aligned}
\tag{8}
$$

- where m_{11}^2, m_{22}^2 and $\lambda_{1,2,3,4}$ are real and m_{12}^2 and $\lambda_{5,6,7}$ are in general complex

Two-Higgs Doublet Model (2HDM)

- The potential can be written more compactly as

$$V = \sum_{a,b=1}^{2} \mu_{ab} \left(\Phi_a^\dagger \Phi_b\right) + \frac{1}{2} \sum_{a,b,c,d=1}^{2} \lambda_{ab,cd} \left(\Phi_a^\dagger \Phi_b\right) \left(\Phi_c^\dagger \Phi_d\right) \tag{9}$$

where

$$\begin{aligned} \mu_{11} &= m_{11}^2 \\ \mu_{12} &= -m_{12}^2 \\ \mu_{21} &= -\left(m_{12}^2\right)^* \\ \mu_{22} &= m_{22}^2 \end{aligned}$$

$$\begin{aligned} \lambda_{11,11} &= \lambda_1 & \lambda_{22,22} &= \lambda_2 \\ \lambda_{11,22} = \lambda_{22,11} &= \lambda_3 & \lambda_{12,2} = \lambda_{22,11} &= \lambda_4 \\ \lambda_{12,12} &= \lambda_5 & \lambda_{21,21} &= \lambda_5^* \\ \lambda_{11,12} = \lambda_{12,11} &= \lambda_6 & \lambda_{11,21} = \lambda_{21,11} &= \lambda_6^* \\ \lambda_{22,12} = \lambda_{12,22} &= \lambda_7 & \lambda_{22,21} = \lambda_{21,22} &= \lambda_7^* \end{aligned}$$

2HDM VEVs

- There are three types of vevs in the 2HDM
- "Normal" vacua (neutral, CP conserving, $v = \sqrt{v_1^2 + v_2^2} \approx 246 GeV$)

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix} \quad \text{and} \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix} \tag{10}$$

- CP breaking vacua: where the vevs have a relative phase

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ \frac{\bar{v}_1 e^{i\theta}}{\sqrt{2}} \end{pmatrix} \quad \text{and} \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ \frac{\bar{v}_2}{\sqrt{2}} \end{pmatrix} \tag{11}$$

- Charged vacua:

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \alpha \\ v_1' \end{pmatrix} \quad \text{and} \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2' \end{pmatrix} \tag{12}$$

(These vevs are bad: they give the photon a mass)

Higgs Masses

- For illustration and simplicity, we will work with a CP conserving vacua and a scalar potential which is CP conserving, in which the quartic terms obeys a Z_2 symmetry:

$$Z_2 : \quad \Phi_1 \to -\Phi_1 \qquad \text{and} \qquad \Phi_2 \to \Phi_2 \tag{13}$$

- Under these assumptions, the scalar potential is

$$\begin{aligned} V = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \\ & + \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) \\ & + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \frac{\lambda_5}{2} \left(\left(\Phi_1^\dagger \Phi_2 \right)^2 + \left(\Phi_2^\dagger \Phi_1 \right)^2 \right) \end{aligned} \tag{14}$$

- Expanding about these vev's the fields are

$$\Phi_a = \begin{pmatrix} \phi_a^+ \\ (v_a + \rho_a + i\eta_a)/\sqrt{2} \end{pmatrix}, \qquad a \in \{1, 2\} \tag{15}$$

Higgs Masses (continued)

- There are 8 fields. Three fields get "eaten" up by the $W_\mu^\pm$ and the Z_μ^0
- The remaining content is: two neutral scalars, a pseudo scalar and a charged scalar.
- To determine mass matricies, one needs to enfore extrema of the potential: $\frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = 0$, which occurs only when

$$0 = m_{11}^2 v_1 - \mathrm{Re}\left(m_{12}^2\right) v_2 + v_1^3 \frac{\lambda_1}{2} + v_1 v_2^2 \frac{\lambda_{345}}{2} \tag{16}$$

$$0 = m_{22}^2 v_2 - \mathrm{Re}\left(m_{12}^2\right) v_1 + v_2^3 \frac{\lambda_2}{2} + v_2 v_1^2 \frac{\lambda_{345}}{2} \tag{17}$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$

Charged Higgs

- For the charged scalars, the mass matrix is

$$\left(M^2_{\phi^\pm}\right)_{ij} = \frac{\partial^2 V}{\partial\phi_i^- \partial\phi_j^+} = \left[m_{12}^2 - (\lambda_4 + \lambda_5)\, v_1 v_2\right] \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix}$$

- This matrix has one zero eigenvalue corresponding to the charged Goldstone boson (which gets eaten by the W). The mass of the charged Higgs is

$$H^+ = \frac{v_2\phi_1^+ - v_1\phi_2^+}{\sqrt{v_1^2 + v_2^2}}$$

$$M_{H^+}^2 = \frac{v^2}{v_1 v_2}\left[m_{12}^2 - v_1 v_2(\lambda_4 + \lambda_5)\right]$$

Pseudo Scalar Higgs

- The mass matrix for the pseudo scalar Higgs is

$$\left(M_A^2\right)_{ij} = \frac{\partial^2 V}{\partial \eta_i \partial \eta_j} = \left(\frac{m_{12}^2}{v_1 v_2} - \lambda_5\right)\begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix}$$

- Diagonalizing this, we find one massless pseudo scalar (which gets eaten by the Z boson) and a massive pseudo scalar with mass M_A^2 :

$$A = \frac{v_2\eta_1 - v_1\eta_2}{\sqrt{v_1^2 + v_2^2}}$$

$$M_A^2 = (v_1^2 + v_2^2)\left(\frac{m_{12}^2}{v_1 v_2} - \lambda_5\right)$$

Neutral Scalar Higgs

- Lastly we have the mass matrix for the neutral scalar Higgs

$$\left(M_h^2\right)_{ij} = \frac{\partial^2 V}{\partial \rho_i \partial \rho_j} = \begin{pmatrix} A & C \\ C & B \end{pmatrix}$$

where

$$A = m_{11}^2 + \frac{3\lambda_1}{2} v_1^2 + \frac{\lambda_{345}}{2} v_2^2 \tag{18}$$

$$B = m_{22}^2 + \frac{3\lambda_2}{2} v_2^2 + \frac{\lambda_{345}}{2} v_1^2 \tag{19}$$

$$C = -m_{12}^2 + \lambda_{345} v_1 v_2 \tag{20}$$

- This can be diagonalized by and angle α, resulting in the physical fields h and H:

$$H = (-\rho_1 \cos\alpha - \rho_2 \sin\alpha) \qquad h = (\rho_1 \sin\alpha - \rho_2 \cos\alpha) \tag{21}$$

- The masses of the two neutral scalar Higges are [2]

$$M_h = v^2\left[\lambda - \frac{\hat{\lambda}\cos(\beta-\alpha)}{\sin(\beta-\alpha)}\right] \tag{22}$$

$$M_H = v^2\left[\lambda + \frac{\hat{\lambda}\sin(\beta-\alpha)}{\cos(\beta-\alpha)}\right] \tag{23}$$

where β is defined by $\tan\beta \equiv v_2/v_1$ and

$$\lambda = \lambda_1\cos^4(\beta) + \lambda_2\sin^2(\beta) + \frac{1}{2}\lambda_{345}\sin^2(2\beta) \tag{24}$$

$$\hat{\lambda} = \frac{1}{2}\sin(2\beta)\left[\lambda_1\cos^2(\beta) - \lambda_2\sin^2(\beta) - \lambda_{345}\cos(2\beta)\right] \tag{25}$$

Part 3: Flavor Changing Neutral Currents

Higgs Basis

- To examine FCNCs it is convenient to work in the Higgs Basis
- To get to the Higgs Basis, we rotate our basis $\{\Phi_1, \Phi_2\}$ such that $\langle\Phi_1\rangle \neq 0$ and $\langle\Phi_2\rangle = 0$. Assuming the general vevs of the form

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ v_1/\sqrt{2} \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ v_2 e^{i\delta}/\sqrt{2} \end{pmatrix} \tag{26}$$

- The Higgs basis is achieved by rotating via a unitary matrix U

$$H_a = \sum_{b=1}^{2} U_{ab}\Phi_b = \frac{e^{-i\delta/2}}{v} \begin{pmatrix} v_1 e^{i\delta/2} & v_2 e^{-i\delta/2} \\ -v_2 e^{i\delta/2} & v_1 e^{-i\delta/2} \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \tag{27}$$

where $v = \sqrt{v_1^2 + v_2^2}$. This is the *Higgs Basis*.

- Expanding about the vev the fields are

$$H_1 = \begin{pmatrix} G^+ \\ (v + H + iG^0)/\sqrt{2} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ (R + iI)/\sqrt{2} \end{pmatrix} \tag{28}$$

where R, L, H are neutral, G^0, G^+ and are Goldstones

Coupling to Fermions: Yukawa Couplings

- The most general gauge invariant coupling we can have between spinors and the Higges are

$$\mathcal{L}_{\mathrm{Y}} = -\sum_{j=1}^{2}\left[\bar{Q}_L\left(\Phi_j Y_j^D n_R + \tilde{\Phi}_j Y_j^U p_R\right) + \bar{L}_L \Phi_j Y_j^e \ell_R\right] + \text{H.C.} \quad (29)$$

  where Q_L, L_L, n_R, p_R and ℓ_R are flavor three vectors and $\tilde{\Phi}_j = i\sigma_2\Phi_j^*$ and Y_j^D's and Y_j^U's are 3×3 coupling matrices.
- Rotating into the Higgs basis and defining mass matrices M_n and N_n

$$\Phi_1 = \frac{1}{v}\left(\tilde{v}_1 H_1 + \tilde{v}_2^* H_2\right) \qquad \Phi_2 = \frac{1}{v}\left(\tilde{v}_1^* H_1 - \tilde{v}_2 H_2\right)$$

$$M_n = \frac{1}{\sqrt{2}}\left(\tilde{v}_1 V_1^d + \tilde{v}_2 Y_2^d\right) \quad N_n = \frac{1}{\sqrt{2}}\left(\tilde{v}_2^* Y_1^d - \tilde{v}_1^* Y_2^d\right)$$

Yukawa Couplings (Continued)

- In terms of the mass matrices, M_n and N_n, we have

$$\sum_{j=1}^{2} \bar{Q}_L \Phi_j Y_j^d n_R = \frac{\sqrt{2}}{v} \bar{Q}_L (M_n H_1 + N_n H_2) n_R \tag{30}$$

- In general, we can bi-diagonalize M_n by rotating the flavor basis' of Q_L and n_R

$$Q_L = U_L Q_L' \quad n_R = U_R^n n_R'$$

- We when have new mass matrices

$$M_d = U_L^\dagger M_n U_R^n \qquad \text{and} \qquad N_d = U_L^\dagger N_n U_R^n \tag{31}$$

where M_d is diagonal: $M_d = \text{diag}(m_d, m_s, m_b)$

- In general, N_d will not be diagonal. If it is not, there will be **flavor changing neutral currents**

Yukawa Couplings (Continued)

- If FCNCs were allowed, we could have, for example, $H \to d\bar{s}$ which would give rise to K-$\bar{K}$ oscillations at tree level [1]

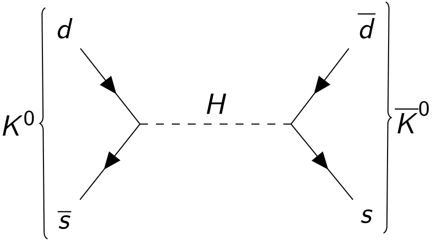


Figure: FCNC lead to effects such a Kaon osc. at tree level

- This affects the rate of Kaon oscillation and thus require the neutral flavor changing mediator to have a mass of 10 TeV
- Theories with FCNC can still are still viable

Removing FCNCs: Type I and II 2HDMs

- It is possible to rid our theory of FCNC by requiring that all particles with same quantum numbers couple to the one Higgs (for example $M_n \propto N_n$)
- There are two ways to do this, called: Type I and Type II models
- Type I: all quarks couple to the same Higgs doublet. This can be achieved by imposing a Z_2 symmetry:

$$Z_2 : \Phi_1 \to -\Phi_1 \qquad \text{and} \qquad \Phi_2 \to \Phi_2 \tag{32}$$

- Type II: all $Q = 2/3$ couple one doublet (say Φ_1) and all $Q = -1/3$ quarks couple the other double Φ_2. This type is enforced using

$$\Phi_1 \to -\Phi_1 \qquad \text{and} \qquad d_R^i \to -d_R^i \tag{33}$$

The type II model uses the same Yukawa couplings as MSSM

Yukawa Couplings in Type I and II models

- Recall that the neutral scalars and pseudo scalar are given by

$$H = (-\rho_1 \cos\alpha - \rho_2 \sin\alpha) \qquad h = (\rho_1 \sin\alpha - \rho_2 \cos\alpha) \tag{34}$$

$$A = \frac{1}{\sqrt{v_1^2 + v_2^2}}(v_2\eta_1 - v_1\eta_2) \tag{35}$$

- In the Type I model ($\tan\beta \equiv v_2/v_1$)

	h	A	H
up-type quarks	$\cos\alpha/\sin\beta$	$\cot\beta$	$\sin\alpha/\sin\beta$
down-type quarks and leptons	$\cos\alpha/\sin\beta$	$-\cot\beta$	$\sin\alpha/\sin\beta$

- In the Type II model

	h	A	H
up-type quarks	$\cos\alpha/\sin\beta$	$\cot\beta$	$\sin\alpha/\sin\beta$
down-type quarks and leptons	$-\sin\alpha/\cos\beta$	$-\tan\beta$	$\cos\alpha/\cos\beta$

Charged Higgs Couplings in Type I and II models

- The charges Higgs Largrangian is

$$\mathcal{L}_{H^{\pm}} = -\frac{\sqrt{2}}{v} H^{+} \left[V_{ij} \bar{u}_i \left(X_u m_{u_i} P_L + X_d m_{d_j} P_R \right) d_j + m_\ell X_\ell \bar{\nu}_\ell P_L \ell \right] + \text{H.C.} \quad (36)$$

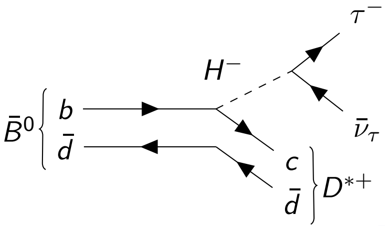
where V_{ij} is the CMK matrix

- The couplings X_u, X_d and X_ℓ are

	Type I	Type II
X_u	$\cot\beta$	$\cot\beta$
X_d	$\cot\beta$	$\tan\beta$
X_ℓ	$\cot\beta$	$-\tan\beta$

Application

- The 2HDM makes testable predictions such as the decay width of B mesons ($\bar{B} \to D^{*}\tau^{-}\nu$ or $\bar{B} \to D\tau^{-}\nu$)
- In the SM, theses decay are mediated via W's
- SM doesn't predict correct decay rate of the $\bar{B}$
- In the 2HDM, this process can also occur through a charge Higgs boson
- It is still unclear if the 2HDM can explain the discrepancy in the decay rate of the $\bar{B}$ [3]



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