Two-Higgs-Doublet Model

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Overview

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2. 2HDM Formalism
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   - 2HDM VEVs
   - Characters in the theory
3. Flavor Changing Neutral Currents
   - Higgs Basis
   - Yukawa Coupling
   - Removing FCNCs: Type 1 and Type II 2HDM
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   - Rare B meson decays
Motivation

- Super symmetry needs two Higgs doublets
- SM cannot explain the observed baryon asymmetry, possible in 2HDM
- CP Violations
- Candidates for dark matter
- Many more..
Part I: Review of SM
SM Higgs

- In the SM, we have one complex scalar doublet
  \[ \Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \] (1)

- The scalar potential is given by
  \[ V = -\mu^2 \Phi^\dagger \Phi + \frac{1}{2} \lambda \left( \Phi^\dagger \Phi \right)^2 \]

where \( \mu^2 \) and \( \lambda \) are real.
We assume that the scalar potential has a non-zero vev:

\[ \langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (2) \]

Expanding about the vev

\[ \Phi = \left( \frac{G^+}{(v + H + iG^0)/\sqrt{2}} \right) \quad (3) \]

where \( G^- \) is a complex field and \( H, G^0 \) are real fields

The potential becomes

\[ V = -\frac{\mu^4}{2\lambda} + \mu^2 H^2 + \mathcal{L}_{\text{Int}} \]

Only \( H \) gets a mass and it’s mass matrix, \( \frac{\partial^2 V}{\partial H \partial \bar{H}} = 2\mu^2 \) is trivial
SM Higgs (Continued)

- We thus have a charged goldstone boson: $G^+$, a neutral goldstone boson: $G^0$ and a Higgs: $H$
- This scalar field couples to the gauge fields via the covariant derivative

$$D_\mu = \partial_\mu + igT^aA^a_\mu + ig'\frac{1}{2}YB_\mu$$

- This gives us the $W'$s, the $Z$ and the photon field

$$W^\pm = \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu); \quad Z^0_\mu = \frac{gA^3_\mu - g'B_\mu}{\sqrt{g^2 + g'^2}}; \quad A_\mu = \frac{g'A^3_\mu + gB_\mu}{\sqrt{g^2 + g'^2}}$$

- We then couple the scalar field to the fermions to give them mass via Yukawa couplings

$$\mathcal{L}_{\text{Yukawa}} = -\lambda_e \bar{\psi}_L^2 \Phi \psi_E - \lambda_u \bar{\psi}_Q^1 \Phi \psi_u - \lambda_d \bar{\psi}_Q^2 \Phi \psi_d - \cdots$$
Part 2: 2HDM Formalism
Two-Higgs Doublet Model (2HDM)

- Now we add a second Higgs doublet
- The most general, renormalizable two-doublet scalar potential is [1]

\[
V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left( m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.C.} \right) \\
\quad + \frac{1}{2} \lambda_1 \left( \Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2} \lambda_2 \left( \Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) \\
\quad + \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \left[ \frac{1}{2} \lambda_5 \left( \Phi_1^\dagger \Phi_2 \right)^2 \\
\quad + \lambda_6 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_1^\dagger \Phi_2 \right) + \lambda_7 \left( \Phi_2^\dagger \Phi_2 \right) \left( \Phi_1^\dagger \Phi_2 \right) + \text{H.C.} \right]
\]

- where \( m_{11}^2, m_{22}^2 \) and \( \lambda_{1,2,3,4} \) are real and \( m_{12}^2 \) and \( \lambda_{5,6,7} \) are in general complex
Two-Higgs Doublet Model (2HDM)

The potential can be written more compactly as

\[ V = \sum_{a,b=1}^{2} \mu_{ab} \left( \Phi_a^\dagger \Phi_b \right) + \frac{1}{2} \sum_{a,b,c,d=1}^{2} \lambda_{ab,cd} \left( \Phi_a^\dagger \Phi_b \right) \left( \Phi_c^\dagger \Phi_d \right) \]  \hspace{1cm} (9)

where

\[ \mu_{11} = m_{11}^2 \]
\[ \mu_{12} = -m_{12}^2 \]
\[ \mu_{21} = -\left( m_{12}^2 \right)^* \]
\[ \mu_{22} = m_{22}^2 \]

\[ \lambda_{11,11} = \lambda_1 \]
\[ \lambda_{11,22} = \lambda_{22,11} = \lambda_3 \]
\[ \lambda_{11,12} = \lambda_{12,11} = \lambda_6 \]
\[ \lambda_{12,12} = \lambda_5 \]
\[ \lambda_{22,12} = \lambda_{12,22} = \lambda_7 \]
\[ \lambda_{22,22} = \lambda_2 \]
\[ \lambda_{12,2} = \lambda_{22,11} = \lambda_4 \]
\[ \lambda_{21,21} = \lambda_5^* \]
\[ \lambda_{11,21} = \lambda_{21,11} = \lambda_6^* \]
\[ \lambda_{22,21} = \lambda_{21,22} = \lambda_7^* \]
There are three types of vevs in the 2HDM:

- "Normal" vacua (neutral, CP conserving, $v = \sqrt{v_1^2 + v_2^2} \approx 246\,\text{GeV}$)

\[
\langle \Phi_1 \rangle = \left( \begin{array}{c} 0 \\ \frac{v_1}{\sqrt{2}} \end{array} \right) \quad \text{and} \quad \langle \Phi_2 \rangle = \left( \begin{array}{c} 0 \\ \frac{v_2}{\sqrt{2}} \end{array} \right) \tag{10}
\]

- CP breaking vacua: where the vevs have a relative phase

\[
\langle \Phi_1 \rangle = \left( \begin{array}{c} 0 \\ \frac{\bar{v}_1 e^{i\theta}}{\sqrt{2}} \end{array} \right) \quad \text{and} \quad \langle \Phi_2 \rangle = \left( \begin{array}{c} 0 \\ \frac{\bar{v}_2}{\sqrt{2}} \end{array} \right) \tag{11}
\]

- Charged vacua:

\[
\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \alpha \\ v_1' \end{array} \right) \quad \text{and} \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_2' \end{array} \right) \tag{12}
\]

(These vevs are bad: they give the photon a mass)
Higgs Masses

- For illustration and simplicity, we will work with a CP conserving vacua and a scalar potential which is CP conserving, in which the quartic terms obeys a $Z_2$ symmetry:
  \[ Z_2 : \quad \Phi_1 \rightarrow -\Phi_1 \quad \text{and} \quad \Phi_2 \rightarrow \Phi_2 \]  

- Under these assumptions, the scalar potential is
  \[ V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left( \Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \]  
  \[ + \frac{1}{2} \lambda_1 \left( \Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2} \lambda_2 \left( \Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) \]  
  \[ + \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \frac{\lambda_5}{2} \left( \left( \Phi_1^\dagger \Phi_2 \right)^2 + \left( \Phi_2^\dagger \Phi_1 \right)^2 \right) \]  

- Expanding about these vev's the fields are
  \[ \Phi_a = \begin{pmatrix} \phi_a^+ \\ (\nu_a + \rho_a + i\eta_a)/\sqrt{2} \end{pmatrix}, \quad a \in \{1, 2\} \]
Higgs Masses (continued)

- There are 8 fields. Three fields get "eaten" up by the $W^\pm_\mu$ and the $Z^0_\mu$
- The remaining content is: two neutral scalars, a pseudo scalar and a charged scalar.
- To determine mass matrices, one needs to enforce extrema of the potential: \( \frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = 0 \), which occurs only when

\[
0 = m_{11}^2 v_1 - \text{Re} (m_{12}^2) v_2 + v_1^3 \lambda_1 \frac{\lambda_1}{2} + v_1 v_2^2 \lambda_{345} \frac{\lambda_{345}}{2} \tag{16}
\]

\[
0 = m_{22}^2 v_2 - \text{Re} (m_{12}^2) v_1 + v_2^3 \lambda_2 \frac{\lambda_2}{2} + v_2 v_1^2 \lambda_{345} \frac{\lambda_{345}}{2} \tag{17}
\]

where \( \lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5 \)
Charged Higgs

- For the charged scalars, the mass matrix is

\[
\left( M_{\phi^\pm}^2 \right)_{ij} = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j^+} = \begin{pmatrix}
 m_{12}^2 - (\lambda_4 + \lambda_5) v_1 v_2 \\
 m_{12}^2 - (\lambda_4 + \lambda_5) v_1 v_2
\end{pmatrix}
\begin{pmatrix}
 \frac{v_2}{v_1} & -1 \\
 -1 & \frac{v_1}{v_2}
\end{pmatrix}
\]

- This matrix has one zero eigenvalue corresponding to the charged Goldstone boson (which gets eaten by the $W$). The mass of the charged Higgs is

\[
H^+ = \frac{v_2 \phi_1^+ - v_1 \phi_2^+}{\sqrt{v_1^2 + v_2^2}}
\]

\[
M_{H^+}^2 = \frac{v^2}{v_1 v_2} \left[ m_{12}^2 - v_1 v_2 (\lambda_4 + \lambda_5) \right]
\]
Pseudo Scalar Higgs

- The mass matrix for the pseudo scalar Higgs is

\[
(M_A^2)_{ij} = \frac{\partial^2 V}{\partial \eta_i \partial \eta_j} = \left( \frac{m_{12}^2}{v_1 v_2} - \lambda_5 \right) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix}
\]

- Diagonalizing this, we find one massless pseudo scalar (which gets eaten by the $Z$ boson) and a massive pseudo scalar with mass $M_A^2$:

\[
A = \frac{v_2 \eta_1 - v_1 \eta_2}{\sqrt{v_1^2 + v_2^2}} \\
M_A^2 = (v_1^2 + v_2^2) \left( \frac{m_{12}^2}{v_1 v_2} - \lambda_5 \right)
\]
Neutral Scalar Higgs

- Lastly we have the mass matrix for the neutral scalar Higgs

\[
(M_h^2)_{ij} = \frac{\partial^2 V}{\partial \rho_i \partial \rho_j} = \begin{pmatrix} A & C \\ C & B \end{pmatrix}
\]

where

\[
A = m_{11}^2 + \frac{3\lambda_1}{2} v_1^2 + \frac{\lambda_{345}}{2} v_2^2 \\
B = m_{22}^2 + \frac{3\lambda_2}{2} v_2^2 + \frac{\lambda_{345}}{2} v_1^2 \\
C = -m_{12}^2 + \lambda_{345} v_1 v_2
\]

- This can be diagonalized by an angle \(\alpha\), resulting in the physical fields \(h\) and \(H\):

\[
H = (-\rho_1 \cos \alpha - \rho_2 \sin \alpha) \\
h = (\rho_1 \sin \alpha - \rho_2 \cos \alpha)
\]
The masses of the two neutral scalar Higgses are [2]

\[ M_h = v^2 \left[ \lambda - \hat{\lambda} \frac{\cos(\beta - \alpha)}{\sin(\beta - \alpha)} \right] \] (22)

\[ M_H = v^2 \left[ \lambda + \hat{\lambda} \frac{\sin(\beta - \alpha)}{\cos(\beta - \alpha)} \right] \] (23)

where \( \beta \) is defined by \( \tan \beta \equiv v_2/v_1 \) and

\[ \lambda = \lambda_1 \cos^4(\beta) + \lambda_2 \sin^2(\beta) + \frac{1}{2} \lambda_{345} \sin^2(2\beta) \] (24)

\[ \hat{\lambda} = \frac{1}{2} \sin(2\beta) \left[ \lambda_1 \cos^2(\beta) - \lambda_2 \sin^2(\beta) - \lambda_{345} \cos(2\beta) \right] \] (25)
Part 3: Flavor Changing Neutral Currents
To examine FCNCs it is convenient to work in the Higgs Basis. To get to the Higgs Basis, we rotate our basis \( \{ \Phi_1, \Phi_2 \} \) such that \( \langle \Phi_1 \rangle \neq 0 \) and \( \langle \Phi_2 \rangle = 0 \). Assuming the general vevs of the form

\[
\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 / \sqrt{2} \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 e^{i\delta} / \sqrt{2} \end{pmatrix}
\] (26)

The Higgs basis is achieved by rotating via a unitary matrix \( U \)

\[
H_a = \sum_{b=1}^{2} U_{ab} \Phi_b = \frac{e^{-i\delta/2}}{v} \begin{pmatrix} v_1 e^{i\delta/2} & v_2 e^{-i\delta/2} \\ -v_2 e^{i\delta/2} & v_1 e^{-i\delta/2} \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}
\] (27)

where \( v = \sqrt{v_1^2 + v_2^2} \). This is the Higgs Basis.

Expanding about the vev the fields are

\[
H_1 = \begin{pmatrix} G^+ \\ (v + H + iG^0)/\sqrt{2} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ (R + iL)/\sqrt{2} \end{pmatrix}
\] (28)

where \( R, L, H \) are neutral, \( G^0, G^+ \) and are Goldstones.
The most general gauge invariant coupling we can have between spinors and the Higgs are

\[ L_Y = - \sum_{j=1}^{2} \left[ \bar{Q}_L \left( \Phi_j Y^D_j n_R + \tilde{\Phi}_j Y^U_j p_R \right) + \bar{L}_L \Phi_j Y^e_j \ell_R \right] + \text{H.C.} \quad (29) \]

where \( Q_L, L_L, n_R, p_R \) and \( \ell_R \) are flavor three vectors and \( \tilde{\Phi}_j = i\sigma_2 \Phi_j^* \) and \( Y^D_j \)'s and \( Y^U_j \)'s are \( 3 \times 3 \) coupling matrices.

Rotating into the Higgs basis and defining mass matrices \( M_n \) and \( N_n \)

\[
\Phi_1 = \frac{1}{v} \left( \tilde{\nu}_1 H_1 + \tilde{\nu}_2^* H_2 \right) \quad \Phi_2 = \frac{1}{v} \left( \tilde{\nu}_1^* H_1 - \tilde{\nu}_2 H_2 \right) \\
M_n = \frac{1}{\sqrt{2}} \left( \tilde{\nu}_1 V_1^d + \tilde{\nu}_2 Y_2^d \right) \quad N_n = \frac{1}{\sqrt{2}} \left( \tilde{\nu}_2^* Y_1^d - \tilde{\nu}_1^* Y_2^d \right)
\]
Yukawa Couplings (Continued)

- In terms of the mass matrices, $M_n$ and $N_n$, we have

$$\sum_{j=1}^{2} \bar{Q}_L \Phi_j Y^d_j n_R = \frac{\sqrt{2}}{v} \bar{Q}_L (M_n H_1 + N_n H_2) n_R$$  \hspace{1cm} (30)

- In general, we can bi-diagonalize $M_n$ by rotating the flavor basis' of $Q_L$ and $n_R$

$$Q_L = U_L Q'_L \quad n_R = U^R_R n'_R$$

- We when have new mass matrices

$$M_d = U^\dagger_L M_n U^R_R \quad \text{and} \quad N_d = U^\dagger_L N_n U^R_R$$  \hspace{1cm} (31)

where $M_d$ is diagonal: $M_d = \text{diag}(m_d, m_s, m_b)$

- In general, $N_d$ will not be diagonal. If it is not, there will be flavor changing neutral currents
Yukawa Couplings (Continued)

- If FCNCs were allowed, we could have, for example, \( H \rightarrow d\bar{s} \) which would give rise to \( K-\bar{K} \) oscillations at tree level [1].

\[
\begin{align*}
K^0 & \rightarrow H \rightarrow d\bar{s} \\
\bar{K}^0 & \rightarrow H \rightarrow \bar{d}s
\end{align*}
\]

**Figure:** FCNC lead to effects such as Kaon oscillations at tree level.

- This affects the rate of Kaon oscillation and thus require the neutral flavor changing mediator to have a mass of 10 TeV.
- Theories with FCNC can still are still viable.
Removing FCNCs: Type I and II 2HDMs

- It is possible to rid our theory of FCNC by requiring that all particles with same quantum numbers couple to the one Higgs (for example $M_n \propto N_n$)
- There are two ways to do this, called: Type I and Type II models
- Type I: all quarks couple to the same Higgs doublet. This can be achieved by imposing a $Z_2$ symmetry:
  \[ Z_2 : \Phi_1 \rightarrow -\Phi_1 \quad \text{and} \quad \Phi_2 \rightarrow \Phi_2 \]  
  (32)
- Type II: all $Q = 2/3$ couple one doublet (say $\Phi_1$) and all $Q = -1/3$ quarks couple the other double $\Phi_2$. This type is enforced using
  \[ \Phi_1 \rightarrow -\Phi_1 \quad \text{and} \quad d_R^i \rightarrow -d_R^i \]  
  (33)

The type II model uses the same Yukawa couplings as MSSM
Yukawa Couplings in Type I and II models

- Recall that the neutral scalars and pseudo scalar are given by

\[
H = (-\rho_1 \cos \alpha - \rho_2 \sin \alpha) \quad h = (\rho_1 \sin \alpha - \rho_2 \cos \alpha) \quad (34)
\]

\[
A = \frac{1}{\sqrt{v_1^2 + v_2^2}}(v_2 \eta_1 - v_1 \eta_2) \quad (35)
\]

- In the Type I model \((\tan \beta \equiv v_2/v_1)\)

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<th>(h)</th>
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<tbody>
<tr>
<td>up-type quarks</td>
<td>(\cos \alpha/\sin \beta)</td>
<td>(\cot \beta)</td>
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<td>(-\cot \beta)</td>
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- In the Type II model

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The charges Higgs Lagrangian is

\[ \mathcal{L}_{H^\pm} = -\frac{\sqrt{2}}{v} H^+ \left[ V_{ij} \bar{u}_i \left( X_u m_{ui} P_L + X_d m_{dj} P_R \right) d_j + m_\ell X_\ell \bar{\nu}_\ell P_L \ell \right] + \text{H.C.} \]  

(36)

where \( V_{ij} \) is the CMK matrix.

The couplings \( X_u, X_d \) and \( X_\ell \) are

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<td>\cot \beta</td>
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<td>( X_\ell )</td>
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<td>- \tan \beta</td>
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The 2HDM makes testable predictions such as the decay width of $B$ mesons ($\bar{B} \rightarrow D^{*}\tau^{-}\nu$ or $\bar{B} \rightarrow D\tau^{-}\nu$).

In the SM, these decay are mediated via $W$'s.

SM doesn't predict correct decay rate of the $\bar{B}$.

In the 2HDM, this process can also occur through a charge Higgs boson.

It is still unclear if the 2HDM can explain the discrepancy in the decay rate of the $\bar{B}$ [3].
References


