An overview of SU(5) grand unification

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A Quick Review of the Standard Model (1)

In 1954, C.N. Yang and Robert Mills gave us the tools necessary to build quantum field theories out of Lagrangians that were invariant under any compact semisimple lie group, G. A combination of estimated guesses and experiment has informed us as to what Nature's choice of G might be. Whatever physics is out there at higher energy scales, in the low energy limit, the group of choice appears to be

$$G_{SM} = \mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$

In an analogy with electromagnetism, we insert a kinetic (field strength) term for each gauge boson corresponding to each normal subgroup of G_{SM} into the Lagrangian. We guess that there is also a term for each fermion, written as a contraction of 4-component spinors below.

$$\mathscr{L}_{Dirac} = \bar{f}(i\partial\!\!\!/ - m_f)f \tag{1}$$

A Quick Review of the Standard Model (2)

In QED and QCD, one introduces the covariant derivative at this point, giving rise to interactions between the gauge bosons and fermions, and rectifying the gauge-covariance of $\mathscr{L}_{Dirac}$. In the present case, however, the fermion mass term $-m_f\bar{f}f = -m_f(\bar{f}_L f_R + \bar{f}_R f_L)$ is not gauge invariant, since f_R and f_L transform differently under G_{SM}. We conclude that our guess for the fermion part of the Lagrangian was, at least partially, incorrect. To fix this problem, one supposes the existence of a complex scalar field Φ, that is a doublet under $\mathrm{SU}(2)_L$, and insert Yukawa interactions between Φ and the fermions into $\mathscr{L}$. Now we have non-kinetic fermion terms that are manifestly gauge singlets, but they are not mass terms. In order to go meaningfully from $\mathscr{L}$ to matrix elements in scattering processes, we assumed a unique vanishing vacuum expectation value, which Φ does not possess in its most general (renormalizable) manifestation in the Lagrangian. We therefore redefine Φ by a simple shift so that the VEV is indeed 0, and this hides the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry under the guise of $U(1)_{EM}$, giving mass to the fermions. The kinetic term for Φ, $\mathscr{L}_{\Phi,kinetic} = |D_\mu \Phi|^2$ will contain interactions that, after symmetry breaking, give us terms that look like mass eigenstates for linear combinations of the components of the gauge fields; these are the massive gauge bosons of the standard model: the W^+, W^-, and Z. The missing, orthogonal mass eigenstate must simply have a coefficient of 0, and we identify this as the photon. The shifted complex scalar field is redefined in terms of a real scalar h, the Higgs boson. All of these particles' existence and behaviours as per the predictions of the standard model have been experimentally verified to extreme accuracy as of 2016.

What is a GUT?

A Grand Unified Theory is a theory that unifies the strong, weak, and electromagnetic interactions under a single gauge coupling. The motivation for such a theory, besides the usual endeavour to compactify and generalize, is that at a certain energy scale, the gauge couplings of these three interactions become very near to each other. Underneath this scale, then, the gauge symmetry group of a GUT should break up into some combination of the groups that comprise G_{SM}. Therefore, whatever we guess the GUT gauge group, G_{GUT}, to be, it had better contain G_{SM} as a subgroup. The smallest such group that achieves this without creating obvious problems from the start is $G_{GUT} \equiv \mathrm{SU}(5)$.

Generators and gauge bosons, choosing a convenient basis (1)

$\mathrm{SU}(n)$ has n^2-1 generators, which means that a quantum field theory with an $\mathrm{SU}(5)$ symmetry will contain 24 massless gauge bosons before symmetry breaking. Call these generators $L^a,\ a=1,2,\ldots,24$, and let us use a 5-dimensional representation. We should choose a basis for the generators that makes it easy to identify the vector bosons V_μ^a. We can choose the first 8 generators $(a=1,\ldots,8)$, so that only $\mathrm{SU}(3)$ acts on the first three rows and columns, and the ninth and tenth generators so that the non-diagonal generators of $\mathrm{SU}(2)$ act only on the fourth and fifth columns:

$$L^{a=1,\ldots,8} \equiv \begin{pmatrix} & & & 0 & 0 \\ & \lambda^a & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} L^9 \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} L^{10} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & i & 0 \end{pmatrix}$$

Then $V_\mu^{a=1,\ldots,8}$ are the vector fields of this theory that should be associated with the gluons, and $\frac{1}{\sqrt{2}}(V_\mu^9 \pm iV_\mu^{10})$ should be associated with the $W^\pm$ bosons.

We get one diagonal generator out from whichever L^a we embed σ^3 in, two from the Gell-Mann embeddings, L^3 and L^8 above. Therefore 1 diagonal generator remains. The unique traceless generator that satisfies the normalization $\mathrm{Tr}[L^a L^b] = 2\delta^{ab}$ is L^{12} below.

$$L^{11} \equiv \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 1 & 0 \\ & & & 0 & -1 \end{pmatrix} \quad L^{12} \equiv \frac{1}{\sqrt{15}} \begin{pmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & 3 & \\ & & & & 3 \end{pmatrix}$$

Notice that $L^1 1$ we will relate with the W^3, and $L^1 2$ with the hyperphoton, B. Soon we will build the photon out of these fields, and be able to make a prediction for the weak mixing angle.

The other generators: the X and Y boson families (1)

The remaining generators of $\mathrm{SU}(5)$ do not correspond to any of the subgroups of G_{SM}. They all take a similar form, which resembles either σ^1 or $\pm\sigma^2$ upon squinting, for example

$$L^{13} \equiv \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} L^{14} \equiv \begin{pmatrix} 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Notice the pattern: they are (anti)symmetric matrices that don't take values on the $3 \times 3 2 \times 2$ block-diagonal. It pays to define these generators as linear combinations of matrices Δ^j_k, whose entries are all zero except that in the jth row, kth column, which is defined to be unity. Then, the relation can be inverted; for example,

$$\Delta^4_1 = \frac{1}{2}(L^{13} - iL^{14})$$

$$\Delta^1_4 = \frac{1}{2}(L^{13} + iL^{14})$$

Let us relabel once more, and call X^i_μ the gauge bosons corresponding to $\Delta^i_4, i = 1, 2, 3$, and $\bar{X}^i_\mu$ those corresponding to Δ^4_i. To Δ^5_i and Δ^i_5 we relate what we'll call Y bosons, $\bar{Y}^i_\mu$ and Y^i_μ respectively. That is, for example,

$$X^1_\mu = \frac{1}{2}(V^{13} - iV^{14})$$
$$\bar{X}^1_\mu = \frac{1}{2}(V^{13} + iV^{14})$$

The field strength tensor

As in QCD, it is convenient to define

$$V_\mu \equiv \frac{1}{\sqrt{2}} V_\mu^a L^a$$

So that we may form

$$\mathscr{L}_{gauge,kinetic} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a = -\frac{1}{4} \mathrm{Tr}(F^2_{\mu\nu})$$

$$F_{\mu\nu} = \partial_\mu V_\nu - \frac{i}{\sqrt{2}} g V_\mu V_\nu - \partial_\nu V_\mu + \frac{i}{\sqrt{2}} g V_\nu V_\mu$$

We have, suppressing ubiquitous Lorentz indices,

$$V_\mu = \begin{pmatrix} G^1_1 - \frac{2}{\sqrt{30}} B & G^1_2 & G^1_3 & \bar{X}^1 & \bar{Y}^1 \\ G^2_1 & G^2_2 - \frac{2}{\sqrt{30}} B & G^2_3 & \bar{X}^2 & \bar{Y}^2 \\ G^3_1 & G^3_2 & G^3_3 - \frac{2}{\sqrt{30}} B & \bar{X}^3 & \bar{Y}^3 \\ X^1 & X^2 & X^3 & \frac{1}{\sqrt{2}} W^3 + \frac{3}{\sqrt{30}} B & W^+ \\ Y^1 & Y^2 & Y^3 & W^- & -\frac{1}{\sqrt{2}} W^3 + \frac{3}{\sqrt{30}} B \end{pmatrix}$$

The most natural spinor representation of $\mathrm{SU}(5)$ is the **5**, that is, the 5-dimensional representation $\psi = (a^1\ a^2\ a^3\ a^4\ a^5)^\mathsf{T}$. The covariant derivative

$$D_\mu = \partial_\mu - \frac{i}{2} g V_\mu$$

acts on these 5-component spinors to give interactions with the gauge bosons of the theory. Recall that the $\mathrm{SU}(2)$ generators embedded within the $\mathrm{SU}(5)$ generators $L^{a=9,10,11}$ (contained in the hidden implicit sum V_μ) are the only generators that act on the bottommost components of ψ, a^4, and a^5. Nothing like $\mathrm{SU}(2)$ operates on the upper three components of ψ, making a^4 and a^5 together an $\mathrm{SU}(2)$ doublet, and $(a^1\ a^2\ a^3)$ an $\mathrm{SU}(2)$ singlet. Conversely, and by the same logic, $(a^1\ a^2\ a^3)$ is an $\mathrm{SU}(3)$ triplet, and $(a^1\ a^2)$ an $\mathrm{SU}(3)$ singlet. We can therefore identify the top part of ψ with quarks, and the bottom part of ψ with leptons.

The photon and the weak mixing angle (1)
What about the photon?

If we have successfully embedded $\mathrm{SU}(2)\times\mathrm{U}(1)$ in $\mathrm{SU}(5)$, then the photon is made up of a linear combination of the gauge fields associated with $\mathrm{SU}(5)$ the generators L^{11} and L^{12}, as mentioned previously in passing. Because the generators are all traceless, so too must be the charge operator, which is a linear combination of traceless generators; that is, the sum of all charges in any given representation is 0. For a 5-spinor above, that means

$$\sum_j Q_{a_j} = 3Q_{quark} + Q_{neutrino} + Q_{chargedlepton} = 0 \Rightarrow Q_{quark} = -\frac{1}{3}e$$

And so we identify a^1, a^2 and a^3 with the down quark color triplet, and the charge operator as

$$Q = \mathrm{diag}(-1/3, -1/3, -1/3, 1, 0) = \frac{1}{2}\left(L^{11} + \sqrt{\frac{5}{3}}L^{12}\right)$$

What about the other fermions? Well, they must transform in a different representation. It turns out they transform in a 10-dimensional representation.

The covariant derivative terms that act on the fields W_μ^3 and B_μ look like

$$D_\mu = \partial_\mu - i\frac{g}{2}\left(W_\mu L^{11} + B_\mu L^{12}\right)$$

As in the standard model, we may rewrite these in terms of two new fields Z_μ and A_μ, the two pairs of fields being related by a rotation matrix parametrized by the weak mixing angle, θ_W.

$$\begin{aligned} D_\mu &= \partial_\mu - i\frac{g}{2}\left[(\sin\theta_W L^{11} + \cos\theta_W L^{12})A_\mu + (\cos\theta_W L^{11} - \sin\theta_W L^{12})Z_\mu\right] \\ &\equiv \partial_\mu - i(eQA_\mu + g_W Q_Z Z_\mu) \end{aligned} \quad (2)$$

This definition encompasses the relation $e = g\sin\theta_W$. Recall that this was also derived in the standard model by the action of the covariant derivative on fermions, which in the case of $\mathrm{SU}(5)$ clearly breaks up into the groups we are familiar with. We can therefore read off $e = g\sin\theta_W$ from, for example, $D_\mu \nu_e = \partial_\mu \nu_e + 0$, just as we did before, where now the neutrino is just the component of a doublet within a quintuplet rather than just a doublet. The action of $\mathrm{SU}(2)$ on the lepton doublet within the 5-component spinor doesn't get mixed up with the other groups, so nothing is different.

The photon and weak mixing angle (3)

Comparing both sides of equation (2), we can see that

$$eQ = \frac{g}{2}\left(\sin\theta_W L^{11} + \cos\theta_W L^{12}\right)$$
$$\sin\theta_W \operatorname{diag}(-1/3,-1/3,-1/3,1,0) =$$
$$\frac{1}{2}\left[\sin\theta_W \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 1 & 0 \\ & & & 0 & -1 \end{pmatrix} + \cos\theta_W \begin{pmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & 3 & \\ & & & & 3 \end{pmatrix}\right]$$

This is a general relation giving 5 redundant equations. Reading off just the last corner of each matrix after dividing everything by $\sin\theta_W$, one has

$$0 = -\frac{1}{2} + \frac{1}{2}\sqrt{\frac{3}{5}}\cot\theta_W$$

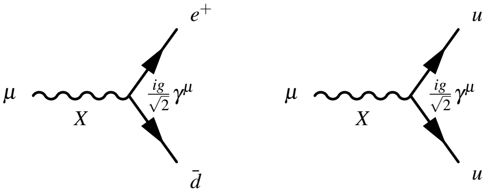
and so this theory predicts

$$\tan\theta_W = \sqrt{\frac{3}{5}},\ \sin\theta_W = \sqrt{\frac{3}{8}}$$

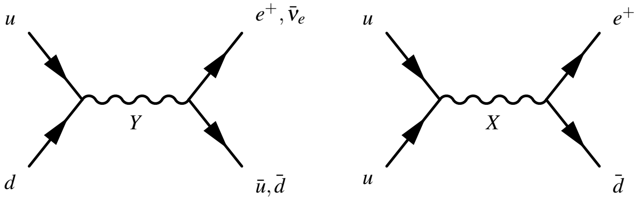
and, using the above in $e = g\sin\theta_W$,

$$g = \sqrt{\frac{8}{3}}e$$

Interactions: X and Y bosons, leptoquarks, $B - L$ conservation (1)



When we actually expand out $\bar{\psi}\not{D}\psi$, we can read off the feynman rules for the fermion interactions. The new bosons will allow quarks and leptons to interact directly. The X boson couples, for example, down quarks to electrons, and up quarks to up quarks, as shown above. An interesting feature of all these interactions of the X and Y bosons is that they preserve baryon number minus lepton number, while violating both of those symmetries separately, both of which were present (if not by accident) in the standard model. Examples of processes that violate B and L, but preserve $B - L$ are proton decay and neutron-antineutron oscillations, phenomena which have been the focus of experimentalists for decades now.



Shown above are some tree-level processes in the Georgi-Glashow model. Notice that it is because of diagrams like the above and interactions like on the page previous that proton decay, e.g. via $p^+ \to e^+\pi^0$ ($uud \to e^+u\bar{d}$) is possible. Using the Feynman rules from before, we know that each vertex will give something $\propto g^2$, and the propgators will be inversely proportional to m^2, so, crudely,

$$\mathcal{M}_{p^+\to e^+\pi^0} \approx \frac{g^2_{X,Y}}{m^2_{X,Y}} \Rightarrow \tau_{p^+} \approx \frac{m^2_{X,Y}}{g^4_{X,Y} m^5_{p^+}}$$

The extremely stringent limits currently placed on the proton lifetime constrain the masses of the X and Y bosons to be absurdly large. If they exist, they are morbidly obese particles with $m_{X,Y} \gtrsim 10^{15}$GeV.

Spontaneous symmetry breaking in $\mathrm{SU}(5)$ (1)
The dream scheme

There is, in fact, more than one way to break down $\mathrm{SU}(5)$ in such a way that we get phenomenologically acceptable results. I will proceed to break the symmetry in a two-step fashion,

$$\mathrm{SU}(5) \to \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \to \mathrm{SU}(3) \times \mathrm{U}(1)_{EM}$$

Using this method, the first stage of breaking is achieved through a duododecuplet (a 24-plet), $\Sigma_{a=1,\ldots,24}$. The covariant derivative that acts on Σ must then be in the adjoint (24-dimensional) representation,

$$D_\mu \Sigma = \partial_\mu \Sigma - i\frac{g}{2} V_\mu^k F^k \Sigma$$

i.e. $F^k_{aa'}$ are twenty-four 24×24 matrices that form a represenation of $\mathrm{SU}(5)$.

Just like the matrix of bosons we saw before, we can represent the 24 degrees of freedom in Σ as a traceless 5×5 matrix, which we can write as a linear combination of generators of $\mathrm{SU}(5)$.

$$\Sigma \equiv L^a \Sigma^a$$

In that case, it transforms differently, i.e. like the adjoint representation of the group, and so we can write the action of the covariant derivative as a commutator, as we have seen before,

$$D_\mu \Sigma = \partial_\mu \Sigma - i\frac{g}{2}[V^a_\mu L^a, \Sigma]$$

and write the kinetic term like we did for the vector bosons:

$$\mathscr{L}_{\Sigma,kinetic} = \mathrm{Tr}[(D_\mu \Sigma)^\dagger (D_\mu \Sigma)]$$

Spontaneous symmetry breaking in $\mathrm{SU}(5)$ (3)

Expanding out the previous two equations, we see that after Σ acquires a vacuum expectation value, (call it $\langle\Sigma\rangle$), $\mathscr{L}_{\Sigma,kinetic}$ gets terms like

$$\frac{g^2}{4}\,\mathrm{Tr}\left([V_\mu,\langle\Sigma\rangle]^2\right) \equiv m^2_{ab}V^a_\mu V^{\mu b}$$

Since is this is just the first stage of breaking, we don't want the Goldstone bosons relating to G_{SM} to acquire mass yet, so we must have a Σ that is traceless but gives mass only to those $V_\mu^{a=13,\ldots,24}$. Notice that a matrix of the form $\mathrm{diag}(a,a,a,b,b)$ will commute with all $L^{a=1,\ldots,12}$, and so a $\langle\Sigma\rangle$ of this form will not give mass to the first twelve bosons we'd like to associate with the second breaking of symmetry. Therefore, since we want $\langle\Sigma\rangle$ to be diagonal, the fact that it must be traceless then fixes

$$\langle\Sigma\rangle = v\,\mathrm{diag}\left(1,1,1,-\frac{3}{2},-\frac{3}{2}\right) = -\frac{\sqrt{15}}{2}vL^{12}$$

up to some constant v. If one now plugs in the V_μ boson matrix from before into the topmost equation of this slide, one finds, via brute force,

$$\frac{g^2}{4}\left([V,\langle\Sigma\rangle]\right)^2 = -\frac{1}{2}\frac{25g^2v^2}{8}\begin{pmatrix} X_1\bar{X}_1+Y_1\bar{Y}_1 & X_2\bar{X}1+Y_2\bar{Y}_1 & X_3\bar{X}_1+Y_3\bar{Y}_1 & 0 & 0\\ X_1\bar{X}_2+Y_1\bar{Y}_2 & X_2\bar{X}_2+Y_2\bar{Y}_2 & X_3\bar{X}_2+Y_3\bar{Y}_2 & 0 & 0\\ X_1\bar{X}_3+Y_1\bar{Y}_3 & X_2\bar{X}_3+Y_2\bar{Y}_3 & X_3\bar{X}_3+Y_3\bar{Y}_3 & 0 & 0\\ 0 & 0 & 0 & X_i\bar{X}_i & X_i\bar{Y}_i\\ 0 & 0 & 0 & \bar{X}_iY_i & Y_i\bar{Y}_i \end{pmatrix}$$

Taking the trace of the above, then, we find

$$m_X^2 = m_Y^2 = \frac{25}{8} g^2 v^2$$

We need a potential which can lead us to the form of the VEV we had just a second ago. This involves discussions of renormalizability, so I will quote the result

$$V(\Sigma) = -\mu^2 \,\mathrm{Tr}(\Sigma^2) + \frac{a}{4} \left[\mathrm{Tr}(\Sigma^2)\right]^2 + \frac{b}{2} \,\mathrm{Tr}(\Sigma^4)$$

The unique minimum to the above is the one we want, so long as $b > 0, \mu^2 > 0, a > -7b/5$. Plugging this in and setting $V = 0$, one has

$$\mu^2 = \left(\frac{15}{2} a + \frac{7}{2} b \right) v^2$$

And so we have effectively hidden the $\mathrm{SU}(5)$ symmetry, since now the gauge invariance in the Yukawa interaction terms for the X and Y bosons with Σ is no longer rectified by a simultaneous transformation in Σ. What remains unbroken are the terms related to the twelve “original" generators of G_{SM}, as we had hoped.

Spontaneous symmetry breaking in $\mathrm{SU}(5)$ (5)

We now need to break the symmetry again, down into $SU(3) \times \mathrm{U}(1)_{EM}$. The simplest possibility turns out to be a 5-component Higgs field $H = (h^1\ h^2\ h^3\ h^+\ -h^0)^T$. In analogy with the 5-spinor we chose earlier, we'd like the upper three components to be a triplet under $\mathrm{SU}(3)$ (and a singlet under $\mathrm{SU}(2)$), and the bottom two to be a doublet under $\mathrm{SU}(2)$ (and a singlet under $\mathrm{SU}(3)$) (just like the SM Higgs is a doublet under $\mathrm{SU}(2)$ and a singlet under $\mathrm{U}(1)$, the group we want to break into). We introduce an analogous potential for the Higgs field to the SM case,

$$V(H) = -\frac{v'^2}{2}|H|^2 + \frac{\lambda}{4}(|H|^2)^2;\ v'^2, \lambda > 0$$

Right now the W and Z bosons are still massless, but when we add a kinetic term for H to the lagrangian, it interacts with the remaining twelve massless bosons via the covariant derivative. When we break this symmetry down by giving H a VEV, these bosons acquire mass. The equation above defines a VEV which we can take to be in the neutral direction

$$\langle -h^0 \rangle = v_0$$
$$\langle H \rangle = (0\,0\,0\,0\,v_0)^T,\ v'^2 = \lambda v_0^2$$

This produces the pattern of symmetry breaking we'd like, with

$$m_W^2 = m_Z^2 \cos^2\theta_W = \frac{1}{4} g_W^2 v_0^2$$

Interpreting the theory (1)

Pros

- The main attractive quality of the Georgi-Glashow model is that it successfully unifies the electroweak and strong gauge couplings of the Standard Model and gives the correct mass terms to particles, as we saw for the W and Z bosons.
- It explains charge quantization, an issue which the Standard Model has been thought to be silent about (although some research claims that it accounts for this just fine with things like non-linear sigma models). Even if we turn out to be underestimating the SM, the quantization of charge in SU(5) is natural and simple.
- It is renormalizable.
- It generalizes the Standard Model in a most intuitive, minimal fashion; a "simple as possible, but no simpler" approach. It elegantly echoes results of standard model calculations while predicting new phenomena.

Cons

- Predicts many unobserved phenomena such as proton decay and other baryon- and lepton-number violating processes. Predicts particles so heavy that we can't hope to detect them directly any time soon.
- Suffers from something called the doublet-triplet splitting problem, which is a hierarchy problem in which the higgs triplet part of the 5-component higgs needs to be 10^{14} times as massive as the higgs doublet. This is an arguably unnatural difference in mass scales.
- Naively predicts an incorrect value for the weak mixing angle. Even when the scale-dependence of the couplings is considered, the predicted value is about 0.214. Very close, but not within the experimental range.

Interpreting the theory (2)

I leave you with a quote by Lee Smolin,

After some [thirty] years, we are still waiting. No protons have decayed. We have been waiting long enough to know that SU(5) grand unification is wrong. It's a beautiful idea, but one that nature seems not to have adopted. Page 64.

Indeed, it would be hard to underestimate the implications of this negative result. SU(5) is the most elegant way imaginable of unifying quarks with leptons, and it leads to a codification of the properties of the standard model in simple terms. Even after [thirty] years, I still find it stunning that SU(5) doesn't work. Page 65.

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