Supersymmetry Highlights

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Outline

- SHO Example
- Why SUSY?
- SUSY Fields
- Superspace Lagrangians
- SUSY QED
- MSSM

Warm Up
A Hint Of SUSY...

- Remember Quantum Simple Harmonic Oscillator...
- Canonical Coordinates w/Commutation Relation

$$[x, p] = i$$

- Input the standard Hamiltonian

$$H = \frac{1}{2}p^2 + \frac{1}{2}x^2$$

Creation/Annihilation Ops

$$
\begin{aligned}
H &= \frac{1}{2}p^2 + \frac{1}{2}x^2 \\
&= \frac{1}{2}(p^2 + x^2) \\
&= \frac{1}{2}(x - ip)(x + ip) - \frac{i}{2}[x, p] \\
&= a^\dagger a + \frac{1}{2}
\end{aligned}
$$

Fermions?

- Anticommutator instead of commutator

$$\{x, p\} = i$$

- No terms like x^2 or p^2 allowed in H
  - Try... $H = ixp$

Fermion Creation \Annihilation Ops

$$
\begin{aligned}
H &= ixp \\
&= \frac{i}{2}(xp - px) + \frac{i}{2}\{x, p\} \\
&= \frac{1}{2}(x - ip)(x + ip) + \frac{i}{2}\{x, p\} \\
&= \quad b^\dagger b - \frac{1}{2}
\end{aligned}
$$

SUSY Transformations

- Boson (x,p) Fermion (ψ,π) SHOs

$$H = \frac{1}{2}x^2 + \frac{1}{2}p^2 + i\psi\pi = a^\dagger a + b^\dagger b$$

- Construct operators which commute with H

$$Q = iab^\dagger, \;\; Q^\dagger = -iba^\dagger$$

  - Bosonic Modes -> Fermionic Modes, And Visa Versa [1]

- Generators of SUSY Transformations

SUSY Transformations

$$Q = iab^\dagger, \;\; Q^\dagger = -iba^\dagger$$

$$N_F = \{0, 1\}$$

$$Q\left|N_B, N_F\right\rangle = \sqrt{(N_B)(N_F+1)}\left|N_B - 1, N_F + 1\right\rangle$$

$$Q^\dagger\left|N_B, N_F\right\rangle = \sqrt{(N_B+1)(N_F)}\left|N_B + 1, N_F - 1\right\rangle$$

$$(QQ^\dagger + Q^\dagger Q)\left|N_B, N_F\right\rangle = (N_B + N_F)\left|N_B, N_F\right\rangle$$

↑ Use N_F={0,1}

More On Q

- From Previous Slide

$$H = \{Q, Q^\dagger\}$$

- The Vacuum Satisfies

$$Q|0\rangle = Q^\dagger|0\rangle = 0$$

- Therefore The Vacuum Energy Is

$$\langle 0|H|0\rangle = \langle 0|\{Q, Q^\dagger\}|0\rangle = 0$$

Principle Of SUSY

- Equal Number Bosonic And Fermionic Degrees Of Freedom (DoFs)
- Find Conserved Charges Which Generate SUSY Transformations
- Expect Nice Cancellations
  - e.g Vacuum Energy In SHO Example

Why SUSY?

Why?

- Fermion Loops Come With Extra Minus Signs
  - This Can Cause Important Cancellations
  - Similar To Ghosts In Non-Abelian Gauge Theory

Quintessential Example: Higgs Mass

- Fermion Loops => Quadratic Divergences
- New Scalar -> Same Divergence With The Opposite Sign!
    - Dirac fermion (4 DoFs) => Two Complex Scalars

Explicit Loop Calculation

$$\mathcal{L} \supset -\lambda H \bar{f} f$$

$$-(-i\lambda)^2 \mathrm{tr} \int \frac{d^4k}{(2\pi)^4} \frac{i(\not{k} + m)}{k^2 - m^2} \frac{i(\not{k} + \not{p} + m)}{(k+p)^2 - m^2} \to -4\lambda^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}$$

2 Complex Scalars

$$\mathcal{L} \supset -\lambda^2 H^2 s^\dagger s$$

↓

$$2 \times (-2i\lambda^2) \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2} \to +4\lambda^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}$$

SUSY Fields

Chiral Superfield

- Complex Scalar (sfermion)
- One Weyl Fermion (fermion)
  - Bosonic DoF = 2
  - Fermionic DoF = 2

Lagrangian

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi + \chi^\dagger i \bar{\sigma}^\mu \partial_\mu \chi$$

$$\delta\phi = \epsilon\chi \quad \delta\phi^\dagger = \epsilon^\dagger \chi^\dagger$$

$$\delta\chi_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi \quad \delta\chi^\dagger_{\dot\alpha} = i(\epsilon\sigma^\mu)_{\dot\alpha} \partial_\mu \phi^\dagger$$

$$\delta\mathcal{L} = \text{total divergence}$$

Vector Superfields

- Gauge Invariant Real Vector (Gauge Boson)
- Weyl Fermion (Gaugino)
  - Bosonic DoF = 2
  - Fermionic DoF = 2

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i\lambda^\dagger \bar{\sigma}^\mu \partial_\mu \lambda$$

$$\delta A^\mu = \epsilon \sigma^\mu \lambda^\dagger - \epsilon^\dagger \bar{\sigma}^\mu \lambda$$

$$\delta \lambda_\alpha = \frac{i}{2} (\sigma^\mu \bar{\sigma}^\nu \epsilon)_\alpha F_{\mu\nu}$$

$$\delta \lambda^\dagger_{\dot{\alpha}} = \frac{i}{2} (\epsilon^\dagger \bar{\sigma}^\mu \sigma^\nu)_{\dot{\alpha}} F_{\mu\nu}$$

$$\delta \mathcal{L} = \text{total divergence}$$

Superspace

Superspace

- SUSY Transformations Can Be Unified
- Define Superspace Coordinates
  - $(x, \theta^a, \theta^\dagger_{\dot{a}})$; a=1,2
- The θ coordinates are Grassmann Numbers

Superfields

- Superfields Are Taylor Expansions In $\theta, \theta^\dagger$
- General Superfield:

$$S(x,\theta,\theta^\dagger) = a + \theta\xi + \theta^\dagger\chi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger c$$

$$+\theta^\dagger\bar{\sigma}^\mu\theta v_\mu + \theta^\dagger\theta^\dagger\theta\eta + \theta\theta\theta^\dagger\zeta^\dagger + \theta\theta\theta^\dagger\theta^\dagger d$$

Supertransformations

- Combine All Supertransformations [2]

$$\delta_\epsilon S = -i(\epsilon Q + \epsilon^\dagger Q^\dagger)S$$

$$= \left(\epsilon^\alpha \frac{\partial}{\partial\theta^\alpha} + \epsilon^\dagger_{\dot\alpha} \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}} + i[\epsilon\sigma^\mu\theta^\dagger + \epsilon^\dagger\bar\sigma^\mu\theta]\partial_\mu\right) S$$

$$= S(x^\mu + i\epsilon\sigma^\mu\theta^\dagger + i\epsilon^\dagger\bar\sigma^\mu\theta, \theta + \epsilon, \theta^\dagger + \epsilon^\dagger) - S(x^\mu\theta, \theta^\dagger)$$

- Supertransformations -> Translation In Superspace

Vacuum Energy

- Generically, We Can Write The Hamiltonian As [2]

$$\langle \Omega | H | \Omega \rangle \propto \langle \Omega | \{ Q_\alpha, Q^\dagger_{\dot{\beta}} \} | \Omega \rangle$$

- In Unbroken SUSY, The Vacuum Satisfies

$$Q_\alpha | \Omega \rangle = Q^\dagger_{\dot{\beta}} | \Omega \rangle = 0$$

- Therefore We Have

$$\langle \Omega | H | \Omega \rangle = 0$$

Chiral Superfield

- Useful To Use New Coordinates

$$y = x^\mu + i\theta^\dagger \bar{\sigma}^\mu \theta$$

- We Can Write Chiral Superfield As

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$$

- F Is An Auxiliary Field -> No On Shell DoFs

Vector Superfield

- Impose That Superfield S Be Real

$$V = a + \theta\xi + \theta^\dagger\xi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger b^\dagger + \theta^\dagger\bar{\sigma}^\mu\theta v_\mu + \theta^\dagger\theta^\dagger\theta\eta + \theta\theta\theta^\dagger\eta^\dagger + \theta\theta\theta^\dagger\theta^\dagger d$$

- Super Gauge Transformations

$$V \rightarrow V + i(\Omega - \Omega^\dagger)$$

- Wess-Zumino Gauge

$$V_{WZ} = \theta^\dagger\bar{\sigma}^\mu\theta A_\mu + \theta^\dagger\theta^\dagger\theta\lambda + \theta\theta\theta^\dagger\lambda^\dagger + \frac{1}{2}\theta\theta\theta^\dagger\theta^\dagger D$$

  - Retains Ordinary Gauge Freedom

Lagrangians In Superspace

- Superspace -> Automatic SUSY Invariance
- S = Superfield, Φ = Chiral Superfield
  - The Following Are SUSY Invariant

$$\int d^2\theta d^2\theta^\dagger S$$ "D" Term

$$\int d^2\theta \Phi \qquad \int d^2\theta^\dagger \Phi^\dagger$$ "F" Term

D-Term Example

- D-Term Only Transforms By A Total Div.

- $$S(x, \theta, \theta^\dagger) = a + \theta\xi + \theta^\dagger\chi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger c$$
  $$+\theta^\dagger\bar{\sigma}^\mu\theta v_\mu + \theta^\dagger\theta^\dagger\theta\eta + \theta\theta\theta^\dagger\zeta^\dagger + \theta\theta\theta^\dagger\theta^\dagger d$$

- $$\left(\epsilon^\alpha \frac{\partial}{\partial\theta^\alpha} + \epsilon^\dagger_{\dot{\alpha}} \frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} + i[\epsilon\sigma^\mu\theta^\dagger + \epsilon^\dagger\bar{\sigma}^\mu\theta]\partial_\mu\right) S$$

- $\Rightarrow \delta d \propto \epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\eta + \epsilon\sigma^\mu\partial_\mu\zeta^\dagger$

Super Potential

- Product Of Chiral Superfields = Chiral Superfield
- Construct Super Potential W

$$W = \frac{1}{2} M_{ij} \Phi_i \Phi_j + \frac{1}{6} y_{ijk} \Phi_i \Phi_j \Phi_k$$

- Use F Term In Lagrangian Plus h.c

$$\mathcal{L} \supset [W]_F + \mathrm{h.c}$$

SUSY QED

SUSY QED

- Two Chiral Superfields, Opposite Charge

$$\mathcal{L}_{\rm SQED} = [\Phi_1^\dagger e^V \Phi_1]_D + [\Phi_2^\dagger e^{-V} \Phi_2]_D$$

$$+M[\Phi_1 \Phi_2]_F + M[\Phi_1^\dagger \Phi_2^\dagger]_F$$

- D-Terms = Super-Gauge Invariant Kinetic Terms

$$\Phi_1 \to e^{i\Omega}\Phi_1, \;\; \Phi_2 \to e^{-i\Omega}\Phi_2, \;\; V \to V - i(\Omega - \Omega^\dagger)$$

- F-Terms = Mass Terms

SUSY QED

$$\mathcal{L}_{\mathrm{SQED}} =$$

$$- \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}i\bar{\Lambda}\gamma^{\mu}\partial_{\mu}\Lambda + \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - M)\Psi$$

$$- \phi_1^{\dagger}(\partial^2 + M^2)\phi_1 - \phi_2^{\dagger}(\partial^2 + M^2)\phi_2$$

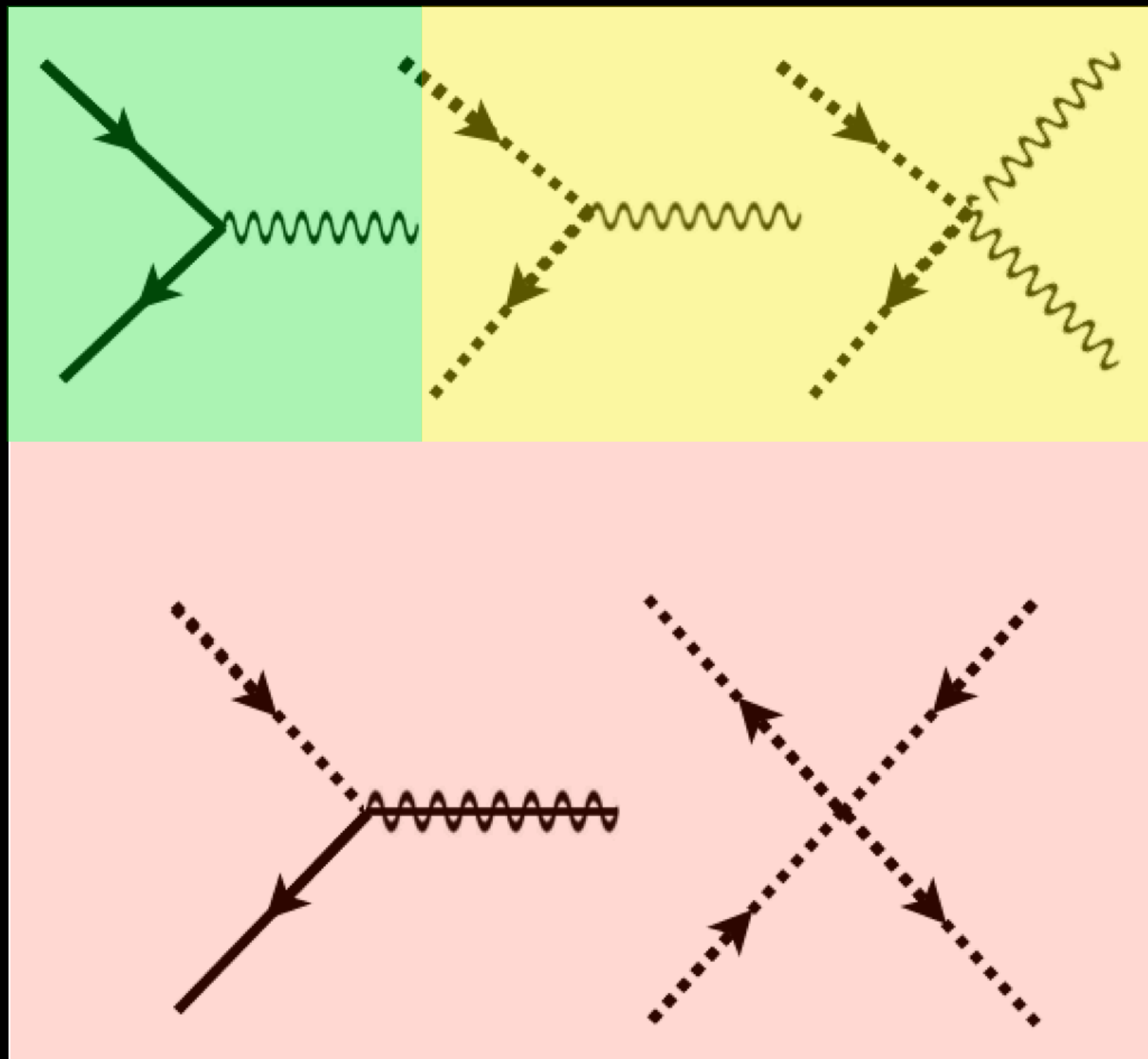
$$+ g\bar{\Psi}\gamma^{\mu}\Psi A_{\mu}$$

$$+ gi(\phi_1^{\dagger}\partial_{\mu}\phi_1 - \phi_1\partial_{\mu}\phi_1^{\dagger})A^{\mu} + g^2\phi_1^{\dagger}\phi_1 A_{\mu}A^{\mu}$$

$$- gi(\phi_2^{\dagger}\partial_{\mu}\phi_2 - \phi_2\partial_{\mu}\phi_2^{\dagger})A^{\mu} + g^2\phi_2^{\dagger}\phi_2 A_{\mu}A^{\mu}$$

$$- \sqrt{2}g(\phi_1^{\dagger}\bar{\Lambda}P_L\Psi + \phi_1\bar{\Psi}P_R\Lambda - \phi_2^{\dagger}\bar{\Psi}P_L\Lambda - \phi_2\bar{\Lambda}P_R\Psi)$$

$$- \frac{1}{2}g^2(\phi_1^{\dagger}\phi_1 - \phi_2^{\dagger}\phi_2)^2$$



MSSM

MSSM Chiral Superfields [2]

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L\ \ \widetilde{d}_L)$	$(u_L\ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
(×3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu}\ \widetilde{e}_L)$	$(\nu\ \ e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H^+_u\ \ H^0_u)$	$(\widetilde{H}^+_u\ \ \widetilde{H}^0_u)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H^0_d\ \ H^-_d)$	$(\widetilde{H}^0_d\ \ \widetilde{H}^-_d)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

MSSM Vector Superfields [2]

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\,\mathbf{8},\ \mathbf{1}\,,\ 0)$
winos, W bosons	$\widetilde{W}^{\pm}\quad \widetilde{W}^{0}$	$W^{\pm}\quad W^{0}$	$(\,\mathbf{1},\ \mathbf{3}\,,\ 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\,\mathbf{1},\ \mathbf{1}\,,\ 0)$

Why 2HD?

- We Would Like To Write

$$H^\dagger H$$

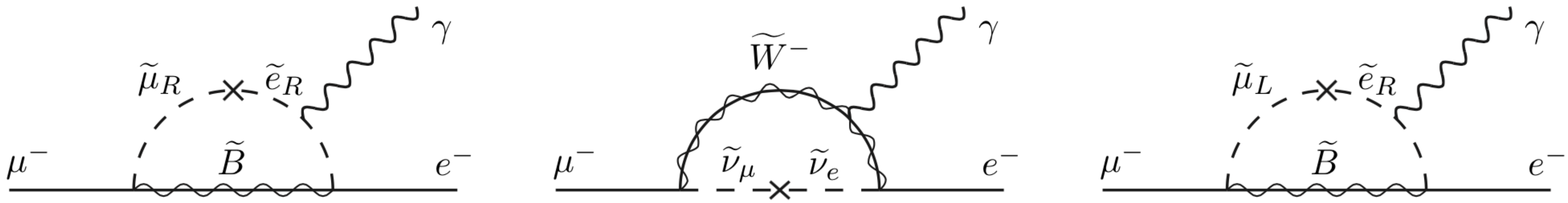
- However This Is Not Allowed In W
  - We Require A Second Higgs Doublet

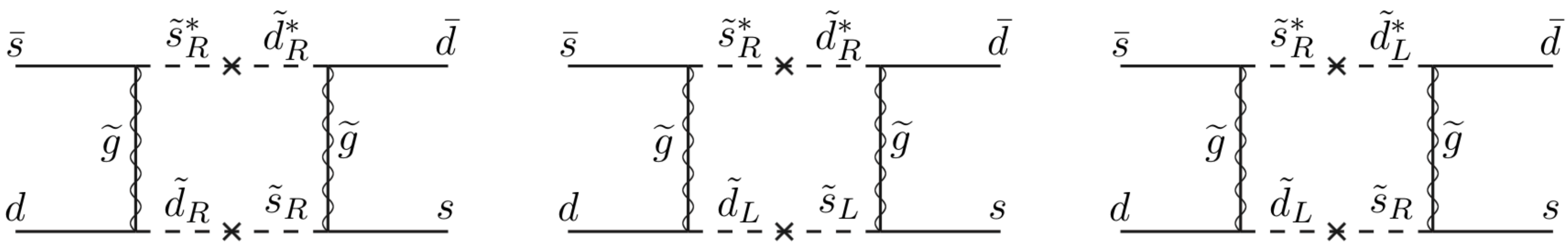
$$W_{\mathrm{MSSM}} = \bar{u}\mathbf{y}_u Q H_u + \bar{d}\mathbf{y}_d Q H_d + \bar{e}\mathbf{y}_e L H_d + \mu H_u H_d$$

Soft SUSY Breaking

- "Soft" = Positive Mass Dimension
  - Maintains EW/Plank Scale Hierarchy [2]
- Gaugino Mass $-\frac{1}{2}m_a\lambda^a\lambda^a$
- Holomorphic Scalar Mass $-\frac{1}{2}b_{ij}\phi_i\phi_j$
- Non-Holomorphic Scalar Mass $-m_{ij}\phi_i^\dagger\phi_j$
- Tri-Scalar Coupling $-\frac{1}{6}c_{ijk}\phi_i\phi_j\phi_k$

$$
\begin{aligned}
\mathcal{L}_{\rm soft}^{\rm MSSM} \;=\;& -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{c.c.}\right) \\
& - \left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + \text{c.c.}\right) \\
& -\widetilde{Q}^\dagger\,\mathbf{m^2_Q}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m^2_L}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m^2_{\overline{u}}}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m^2_{\overline{d}}}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m^2_{\overline{e}}}\,\widetilde{\overline{e}}^\dagger \\
& - m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + \text{c.c.})\,.
\end{aligned}
$$





MSSM 124 Parameters [3]

- SUSY Conserving
  - Gauge Couplings
  - Higgs Mass Parameter μ
  - Yukawa Couplings

- SUSY Violating
  - Gaugino/Sfermion/ Higgs Masses
  - Higgs-Sfermion-Fermion Couplings
  - $\tan\beta = v_u/v_d$
    - (Note that $v_u^2 + v_d^2$ Fixed By W Mass)

R-Symmetry

- Postulate New U(1) Symmetry [2]

$$\theta \rightarrow e^{i\alpha}\theta \quad \theta^\dagger \rightarrow e^{-i\alpha}\theta^\dagger$$

- Superfield Components Transform Differently
- Define A Supercharge r_s

$$S(x,\theta,\theta^\dagger) \rightarrow e^{ir_s\alpha}S(x,e^{i\alpha}\theta,e^{-i\alpha}\theta^\dagger)$$

R-Parity In MSSM aka Matter Parity

- Discrete Version Of R-Symmetry
- Ordinary Particles Have $P_M = +1$
- Super Partners (Sparticles) Have $P_M = -1$
- Implies Each Vertex Must Have Even Number of Sparticles
  - => Lightest Sparticle Is Stable! (LSP)

Matter Parity Cont.

- Consider What Would Happen Without Matter Parity...
- Note that L and H_d Have The Same Quantum Numbers
- In W_{MSSM} We Could Write $\bar{d}QL$
  - Would Violate Lepton Number

LSP As Dark Matter [4]

- Neutral Higgsinos, Winos, and Binos Are Not Mass Eigenstates
- 4 Mass Eigenstates Are Called Neutralinos
  - Majorana Fermions
- Thought To Be A Likely Candidate For DM

What Did We Learn?

- You Can Construct SUSY In Ordinary QM
- SUSY Gives Cancellations
- SUSY Fields Chiral & Vector
- Superspace + Superfields => SUSY Automatic
- SUSY QED Lagrangian And Feynman Diagrams
- MSSM: 2HDM & SUSY Breaking & LSP DM

Thank You!

References

- [1] An Introduction to Superstmmetry in Quantum Mechanical Systems by T. Wellman
- [2] A Super Symmetry Primer
  - arXiv:hep-ph/9709356
- [3] PDG SUSY Review
  - http://pdg.lbl.gov/2015/reviews/rpp2015-rev-susy-1-theory.pdf
- [4] LSP As A Candidate For Dark Matter
  - arXiv:hep-ph/0607301v2