Supersymmetry Highlights
Kevin Hambleton
Outline

- SHO Example
- Why SUSY?
- SUSY Fields
- Superspace Lagrangians
- SUSY QED
- MSSM
Warm Up
A Hint Of SUSY...

• Remember Quantum Simple Harmonic Oscillator...

• Canonical Coordinates w/Commutation Relation

\[ [x, p] = i \]

• Input the standard Hamiltonian

\[ H = \frac{1}{2} p^2 + \frac{1}{2} x^2 \]
\[ H = \frac{1}{2} p^2 + \frac{1}{2} x^2 \]

\[ = \frac{1}{2} (p^2 + x^2) \]

\[ = \frac{1}{2} (x - ip)(x + ip) - \frac{i}{2} [x, p] \]

\[ = a^\dagger a + \frac{1}{2} \]
Fermions?

- Anticommutator instead of commutator
  \[ \{x, p\} = i \]
- No terms like $x^2$ or $p^2$ allowed in $H$
- Try...
  \[ H = ixp \]
Fermion Creation
\Annihilation Ops

\[ H = i x p \]
\[ \begin{align*}
&= \frac{i}{2} (x p - p x) + \frac{i}{2} \{x, p\} \\
&= \frac{1}{2} (x - ip)(x + ip) + \frac{i}{2} \{x, p\} \\
&= b^\dagger b - \frac{1}{2}
\end{align*} \]
SUSY Transformations

• Boson (x,p) Fermion (ψ,π) SHOs
\[ H = \frac{1}{2}x^2 + \frac{1}{2}p^2 + i\psi\pi = a^\dagger a + b^\dagger b \]

• Construct operators which commute with H
\[ Q = iab^\dagger, \quad Q^\dagger = -iba^\dagger \]

• Bosonic Modes -> Fermionic Modes, And Visa Versa [1]

• Generators of SUSY Transformations
SUSY Transformations

\[ Q = i a b^\dagger, \quad Q^\dagger = -i b a^\dagger \]

\[ N_F = \{0, 1\} \]

\[ Q |N_B, N_F\rangle = \sqrt{(N_B)(N_F + 1)} |N_B - 1, N_F + 1\rangle \]

\[ Q^\dagger |N_B, N_F\rangle = \sqrt{(N_B + 1)(N_F)} |N_B + 1, N_F - 1\rangle \]

\[ (QQ^\dagger + Q^\dagger Q) |N_B, N_F\rangle = (N_B + N_F) |N_B, N_F\rangle \]

Use \( N_F = \{0, 1\} \)
More On $Q$

- From Previous Slide
  \[ H = \{Q, Q^\dagger\} \]

- The Vacuum Satisfies
  \[ Q|0\rangle = Q^\dagger|0\rangle = 0 \]

- Therefore The Vacuum Energy Is
  \[ \langle 0|H|0\rangle = \langle 0|{Q, Q^\dagger}|0\rangle = 0 \]
Principle Of SUSY

• Equal Number Bosonic And Fermionic Degrees Of Freedom (DoFs)
• Find Conserved Charges Which Generate SUSY Transformations
• Expect Nice Cancellations
  • e.g. Vacuum Energy In SHO Example
Why SUSY?
Why?

- Fermion Loops Come With Extra Minus Signs
- This Can Cause Important Cancellations
- Similar To Ghosts In Non-Abelian Gauge Theory
Quintessential Example: Higgs Mass

- Fermion Loops $\Rightarrow$ Quadratic Divergences
- New Scalar $\Rightarrow$ Same Divergence With The Opposite Sign!
- Dirac fermion (4 DoFs) $\Rightarrow$ Two Complex Scalars
Explicit Loop Calculation

\[ \mathcal{L} \supset -\lambda \bar{H} f f \]

\[ -(-i\lambda)^2 \text{tr} \int \frac{d^4 k}{(2\pi)^4} \frac{i(k + m)}{k^2 - m^2} \frac{i(k + \phi + m)}{(k + p)^2 - m^2} \rightarrow -4\lambda^2 \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \]

2 Complex Scalars

\[ \mathcal{L} \supset -\lambda^2 H^2 s^\dagger s \]

\[ 2 \times (-2i\lambda^2) \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m^2} \rightarrow +4\lambda^2 \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \]
Chiral Superfield

- Complex Scalar (sfermion)
- One Weyl Fermion (fermion)
  - Bosonic DoF = 2
  - Fermionic DoF = 2
Lagrangian

\[ L = \partial^\mu \phi^\dagger \partial_\mu \phi + \chi i \bar{\sigma}^\mu \partial_\mu \chi \]

\[ \delta \phi = \epsilon \chi \quad \delta \phi^\dagger = \epsilon^\dagger \chi^\dagger \]

\[ \delta \chi_\alpha = -i (\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi \quad \delta \chi^\dagger_\dot{\alpha} = i (\epsilon \sigma^\mu)_{\dot{\alpha}} \partial_\mu \phi^\dagger \]

\[ \delta L = \text{total divergence} \]
Vector Superfields

- Gauge Invariant Real Vector (Gauge Boson)
- Weyl Fermion (Gaugino)
  - Bosonic DoF = 2
  - Fermionic DoF = 2
\[ \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i \lambda^\dagger \bar{\sigma}^\mu \partial_\mu \lambda \]

\[ \delta A^\mu = \epsilon \sigma^\mu \lambda^\dagger - \epsilon^\dagger \bar{\sigma}^\mu \lambda \]

\[ \delta \lambda_\alpha = \frac{i}{2} (\sigma^\mu \bar{\sigma}^\nu \epsilon)_\alpha F_{\mu\nu} \]

\[ \delta \lambda^\dagger_\dot{\alpha} = \frac{i}{2} (\epsilon^\dagger \bar{\sigma}^\mu \sigma^\nu)_\dot{\alpha} F_{\mu\nu} \]

\[ \delta \mathcal{L} = \text{total divergence} \]
Superspace
Superspace

- SUSY Transformations Can Be Unified
- Define Superspace Coordinates
  - $(x, \theta^{a}, \theta^{\dagger}_{\dot{a}}); a=1,2$
- The $\theta$ coordinates are Grassmann Numbers
Superfields

- Superfields Are Taylor Expansions In \( \theta, \theta^\dagger \)
- General Superfield:

\[
S(x, \theta, \theta^\dagger) = a + \theta \xi + \theta^\dagger \chi^\dagger + \theta \theta b + \theta^\dagger \theta^\dagger c + \theta^\dagger \bar{\sigma}^\mu \theta \nu_\mu + \theta^\dagger \theta^\dagger \theta \eta + \theta \theta \theta^\dagger \zeta^\dagger + \theta \theta \theta^\dagger \theta^\dagger d
\]
Supertransformations

• Combine All Supertransformations [2]

\[ \delta_\epsilon S = -i(\epsilon Q + \epsilon^\dagger Q^\dagger)S \]

\[
= \left( \epsilon^\alpha \frac{\partial}{\partial \theta^\alpha} + \epsilon^\dagger_{\dot{\alpha}} \frac{\partial}{\partial \theta^{\dagger}_{\dot{\alpha}}} + i[\epsilon \sigma^\mu \theta^\dagger + \epsilon^\dagger \bar{\sigma}^\mu \theta] \partial_{\mu} \right) S 
\]

\[
= S(x^\mu + i\epsilon \sigma^\mu \theta^\dagger + i\epsilon^\dagger \bar{\sigma}^\mu \theta, \theta + \epsilon, \theta^{\dagger} + \epsilon^\dagger) - S(x^\mu \theta, \theta^{\dagger})
\]

• Supertransformations -> Translation In Superspace
Vacuum Energy

• Generically, We Can Write The Hamiltonian As [2]

\[\langle \Omega | H | \Omega \rangle \propto \langle \Omega | \{Q_\alpha, Q^\dagger_\beta \} | \Omega \rangle\]

• In Unbroken SUSY, The Vacuum Satisfies

\[Q_\alpha | \Omega \rangle = Q^\dagger_\beta | \Omega \rangle = 0\]

• Therefore We Have

\[\langle \Omega | H | \Omega \rangle = 0\]
Chiral Superfield

• Useful To Use New Coordinates

\[ y = x^\mu + i\theta^\dagger \sigma^\mu \theta \]

• We Can Write Chiral Superfield As

\[ \Phi = \phi(y) + \sqrt{2}\theta \psi(y) + \theta\theta F(y) \]

• F Is An Auxiliary Field -> No On Shell DoFs
Vector Superfield

- Impose That Superfield $S$ Be Real

\[ V = a + \theta \xi + \theta^\dagger \xi^\dagger + \theta \theta b + \theta^\dagger \theta^\dagger b^\dagger + \theta^\dagger \bar{\sigma}^\mu \theta v_\mu + \theta^\dagger \theta^\dagger \theta \eta + \theta \theta \theta^\dagger \eta^\dagger + \theta \theta \theta^\dagger \theta^\dagger d \]

- Super Gauge Transformations

\[ V \rightarrow V + i(\Omega - \Omega^\dagger) \]

- Wess-Zumino Gauge

\[ V_{WZ} = \theta^\dagger \bar{\sigma}^\mu \theta A_\mu + \theta^\dagger \theta^\dagger \theta \lambda + \theta \theta \theta^\dagger \lambda^\dagger + \frac{1}{2} \theta \theta \theta^\dagger \theta^\dagger D \]

- Retains Ordinary Gauge Freedom
Lagrangians In Superspace

• Superspace -> Automatic SUSY Invariance
• $S = \text{Superfield, } \Phi = \text{Chiral Superfield}$
• The Following Are SUSY Invariant

$$\int d^2\theta d^2\theta^\dagger S \quad \text{“D” Term}$$

$$\int d^2\theta \Phi \quad \int d^2\theta^\dagger \Phi^\dagger \quad \text{“F” Term}$$
D-Term Example

- D-Term Only Transforms By A Total Div.

\[ S(x, \theta, \theta^\dagger) = a + \theta \xi + \theta^\dagger \chi^\dagger + \theta \theta b + \theta^\dagger \theta^\dagger c \]

- \[ + \theta^\dagger \bar{\sigma}^\mu \theta v_\mu + \theta^\dagger \theta^\dagger \theta \eta + \theta \theta \theta^\dagger \zeta^\dagger + \theta \theta \theta^\dagger \theta^\dagger d \]

- \[ \left( \epsilon^\alpha \frac{\partial}{\partial \theta^\alpha} + \epsilon^\dagger_{\dot{\alpha}} \frac{\partial}{\partial \theta^\dagger_{\dot{\alpha}}} + i[\epsilon \sigma^\mu \theta^\dagger + \epsilon^\dagger \bar{\sigma}^\mu \theta] \partial_\mu \right) S \]

- \[ \Rightarrow \delta d \propto \epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \eta + \epsilon \sigma^\mu \partial_\mu \zeta^\dagger \]
Super Potential

- Product Of Chiral Superfields = Chiral Superfield

- Construct Super Potential $W$
  \[ W = \frac{1}{2} M_{ij} \Phi_i \Phi_j + \frac{1}{6} y_{ijk} \Phi_i \Phi_j \Phi_k \]

- Use F Term In Lagrangian Plus h.c
  \[ \mathcal{L} \supset [W]_F + \text{h.c} \]
SUSY QED
SUSY QED

- Two Chiral Superfields, Opposite Charge

\[ \mathcal{L}_{SQED} = [\Phi_1^\dagger e^V \Phi_1]_D + [\Phi_2^\dagger e^{-V} \Phi_2]_D \]

\[ + M[\Phi_1 \Phi_2]_F + M[\Phi_1^\dagger \Phi_2^\dagger]_F \]

- D-Terms = Super-Gauge Invariant Kinetic Terms

\[ \Phi_1 \rightarrow e^{i\Omega} \Phi_1, \quad \Phi_2 \rightarrow e^{-i\Omega} \Phi_2, \quad V \rightarrow V - i(\Omega - \Omega^\dagger) \]

- F-Terms = Mass Terms
\[ L_{\text{SQED}} = \]

\[- \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} i \bar{\Lambda} \gamma^\mu \partial_\mu \Lambda + \bar{\Psi} (i \gamma^\mu \partial_\mu - M) \Psi \]

\[- \phi_1^\dagger (\partial^2 + M^2) \phi_1 - \phi_2^\dagger (\partial^2 + M^2) \phi_2 \]

\[ + g \bar{\Psi} \gamma^\mu \Psi A_\mu \]

\[ + g i (\phi_1^\dagger \partial_\mu \phi_1 - \phi_1 \partial_\mu \phi_1^\dagger) A^\mu + g^2 \phi_1^\dagger \phi_1 A_\mu A^\mu \]

\[- g i (\phi_2^\dagger \partial_\mu \phi_2 - \phi_2 \partial_\mu \phi_2^\dagger) A^\mu + g^2 \phi_2^\dagger \phi_2 A_\mu A^\mu \]

\[- \sqrt{2} g (\phi_1^\dagger \bar{\Lambda} P_L \Psi + \phi_1 \bar{\Psi} P_R \Lambda - \phi_2^\dagger \bar{\Psi} P_L \Lambda - \phi_2 \bar{\Lambda} P_R \Psi) \]

\[- \frac{1}{2} g^2 (\phi_1 \phi_1 - \phi_2 \phi_2)^2 \]
## MSSM Chiral Superfields [2]

<table>
<thead>
<tr>
<th>Names</th>
<th>spin 0</th>
<th>spin 1/2</th>
<th>$SU(3)_C, SU(2)_L, U(1)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>squarks, quarks (×3 families)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td>$(\tilde{u}_L ; \tilde{d}_L)$</td>
<td>$(u_L ; d_L)$</td>
<td>$(3, 2, \frac{1}{6})$</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td></td>
<td>$u_R^\dagger$</td>
<td>$(\bar{3}, 1, -\frac{2}{3})$</td>
</tr>
<tr>
<td>$\bar{d}$</td>
<td></td>
<td>$d_R^\dagger$</td>
<td>$(\bar{3}, 1, \frac{1}{3})$</td>
</tr>
<tr>
<td>sleptons, leptons (×3 families)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>$(\tilde{\nu} ; \tilde{e}_L)$</td>
<td>$(\nu ; e_L)$</td>
<td>$(1, 2, -\frac{1}{2})$</td>
</tr>
<tr>
<td>$\bar{e}$</td>
<td></td>
<td>$e_R^\dagger$</td>
<td>$(1, 1, 1)$</td>
</tr>
<tr>
<td>Higgs, higgsinos</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_u$</td>
<td>$(H_u^+ ; H_u^0)$</td>
<td>$(\tilde{H}_u^+ ; \tilde{H}_u^0)$</td>
<td>$(1, 2, +\frac{1}{2})$</td>
</tr>
<tr>
<td>$H_d$</td>
<td>$(H_d^0 ; H_d^-)$</td>
<td>$(\tilde{H}_d^0 ; \tilde{H}_d^-)$</td>
<td>$(1, 2, -\frac{1}{2})$</td>
</tr>
</tbody>
</table>
# MSSM Vector Superfields [2]

<table>
<thead>
<tr>
<th>Names</th>
<th>spin 1/2</th>
<th>spin 1</th>
<th>$SU(3)_C$, $SU(2)_L$, $U(1)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>gluino, gluon</td>
<td>$\tilde{g}$</td>
<td>$g$</td>
<td>$(8, 1, 0)$</td>
</tr>
<tr>
<td>winos, W bosons</td>
<td>$\tilde{W}^\pm$, $\tilde{W}^0$</td>
<td>$W^\pm$, $W^0$</td>
<td>$(1, 3, 0)$</td>
</tr>
<tr>
<td>bino, B boson</td>
<td>$\tilde{B}^0$</td>
<td>$B^0$</td>
<td>$(1, 1, 0)$</td>
</tr>
</tbody>
</table>
Why 2HD?

- We Would Like To Write

\[ H^+ H \]

- However This Is Not Allowed In W

- We Require A Second Higgs Doublet

\[ W_{\text{MSSM}} = \bar{u}y_u QH_u + \bar{d}y_d QH_d + \bar{e}y_e LH_d + \mu H_u H_d \]
Soft SUSY Breaking

- “Soft” = Positive Mass Dimension
- Maintains EW/Plank Scale Hierarchy [2]
- Gaugino Mass \(-\frac{1}{2} m_a \lambda^a \lambda^a\)
- Holomorphic Scalar Mass \(-\frac{1}{2} b_{ij} \phi_i \phi_j\)
- Non-Holomorphic Scalar Mass \(-m_{ij} \phi_i^\dagger \phi_j\)
- Tri-Scalar Coupling \(-\frac{1}{6} c_{ijk} \phi_i \phi_j \phi_k\)
\[ \mathcal{L}_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2} \left( M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + \text{c.c.} \right) \\
- \left( \tilde{u} a_u \tilde{Q} H_u - \tilde{d} a_d \tilde{Q} H_d - \tilde{e} a_e \tilde{L} H_d + \text{c.c.} \right) \\
- \tilde{Q}^\dagger \begin{pmatrix} m_Q^2 & \tilde{Q} \\ \tilde{L}^\dagger & m_L^2 \end{pmatrix} \begin{pmatrix} \tilde{L} \\ \tilde{u} \end{pmatrix} \begin{pmatrix} \tilde{u}^\dagger \\ \tilde{d} \end{pmatrix} \begin{pmatrix} \tilde{d} \\ \tilde{e} \end{pmatrix} \begin{pmatrix} \tilde{e}^\dagger \end{pmatrix} \\
- m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u H_d + \text{c.c.}). \]
MSSM 124 Parameters [3]

- SUSY Conserving
  - Gauge Couplings
  - Higgs Mass Parameter $\mu$
  - Yukawa Couplings

- SUSY Violating
  - Gaugino/Sfermion/Higgs Masses
  - Higgs-Sfermion-Fermion Couplings
  - $\tan\beta = v_u/v_d$
  - (Note that $v_u^2 + v_d^2$ Fixed By W Mass)
R-Symmetry

• Postulate New U(1) Symmetry [2]

\[ \theta \rightarrow e^{i\alpha} \theta \quad \theta^\dagger \rightarrow e^{-i\alpha} \theta^\dagger \]

• Superfield Components Transform Differently

• Define A Supercharge \( r_s \)

\[ S(x, \theta, \theta^\dagger) \rightarrow e^{ir_s \alpha} S(x, e^{i\alpha} \theta, e^{-i\alpha} \theta^\dagger) \]
R-Parity In MSSM aka Matter Parity

• Discrete Version Of R-Symmetry
• Ordinary Particles Have $P_M = +1$
• Super Partners (Sparticles) Have $P_M = -1$
• Implies Each Vertex Must Have Even Number of Sparticles

• $\Rightarrow$ Lightest Sparticle Is Stable! (LSP)
Matter Parity Cont.

• Consider What Would Happen Without Matter Parity...

• Note that $L$ and $H_d$ Have The Same Quantum Numbers

• In $W_{\text{MSSM}}$ We Could Write $\bar{d}QL$

• Would Violate Lepton Number
LSP As Dark Matter [4]

- Neutral Higgsinos, Winos, and Binos Are Not Mass Eigenstates
- 4 Mass Eigenstates Are Called Neutralinos
- Majorana Fermions
- Thought To Be A Likely Candidate For DM
What Did We Learn?

- You Can Construct SUSY In Ordinary QM
- SUSY Gives Cancellations
- SUSY Fields Chiral & Vector
- Superspace + Superfields => SUSY Automatic
- SUSY QED Lagrangian And Feynman Diagrams
- MSSM: 2HDM & SUSY Breaking & LSP DM
Thank You!
References

• [1] An Introduction to Superstmmmetry in Quantum Mechanical Systems by T. Wellman
• [2] A Super Symmetry Primer
  • arXiv:hep-ph/9709356
• [3] PDG SUSY Review
• [4] LSP As A Candidate For Dark Matter
  • arXiv:hep-ph/0607301v2