The Vacuum structure of the N-Higgs doublet model

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Introduction

In the standard model, we have one higgs doublet which transforms under $SU_L(2) \times U_Y(1)$

$$\Phi = \begin{pmatrix} \phi^u \\ \phi^d \end{pmatrix} \quad \phi^u, \phi^d \text{ complex fields}$$

In the N-Higgs doublet model (NHDM), we have N such doublets, denoted

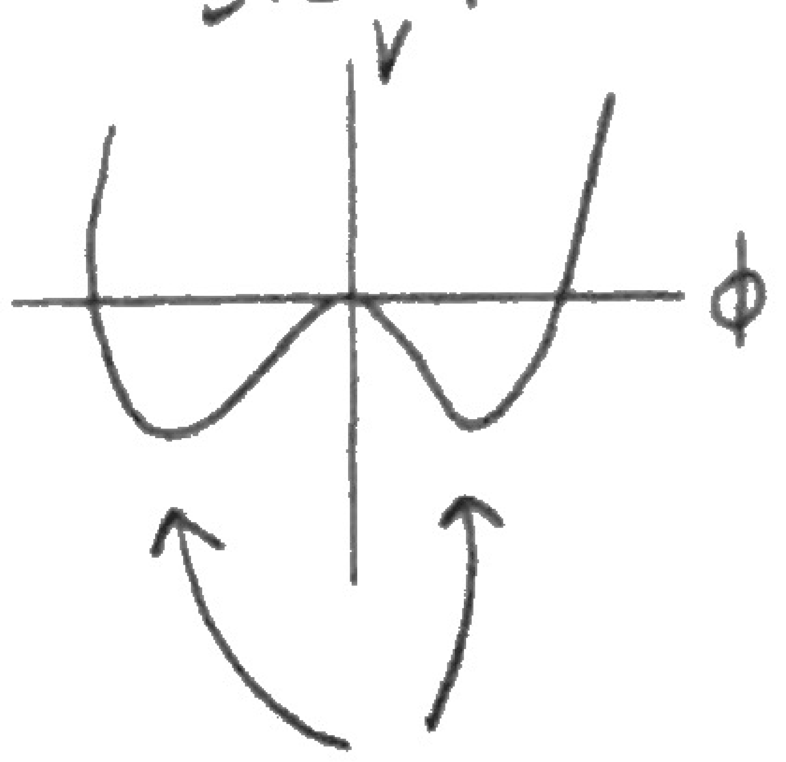
$$\Phi_i = \begin{pmatrix} \phi^u_i \\ \phi^d_i \end{pmatrix} \quad i \in \{1, \ldots, N\}$$

Such models are interesting because

1. They are required by supersymmetry. (MSSM requires $N=2$)
2. They can introduce CP violation, explaining baryogenesis

The aim of this document is to analyze the possible vacua* arising from the more complicated scalar potential. In the Standard model, only one type of vacum is premitted:

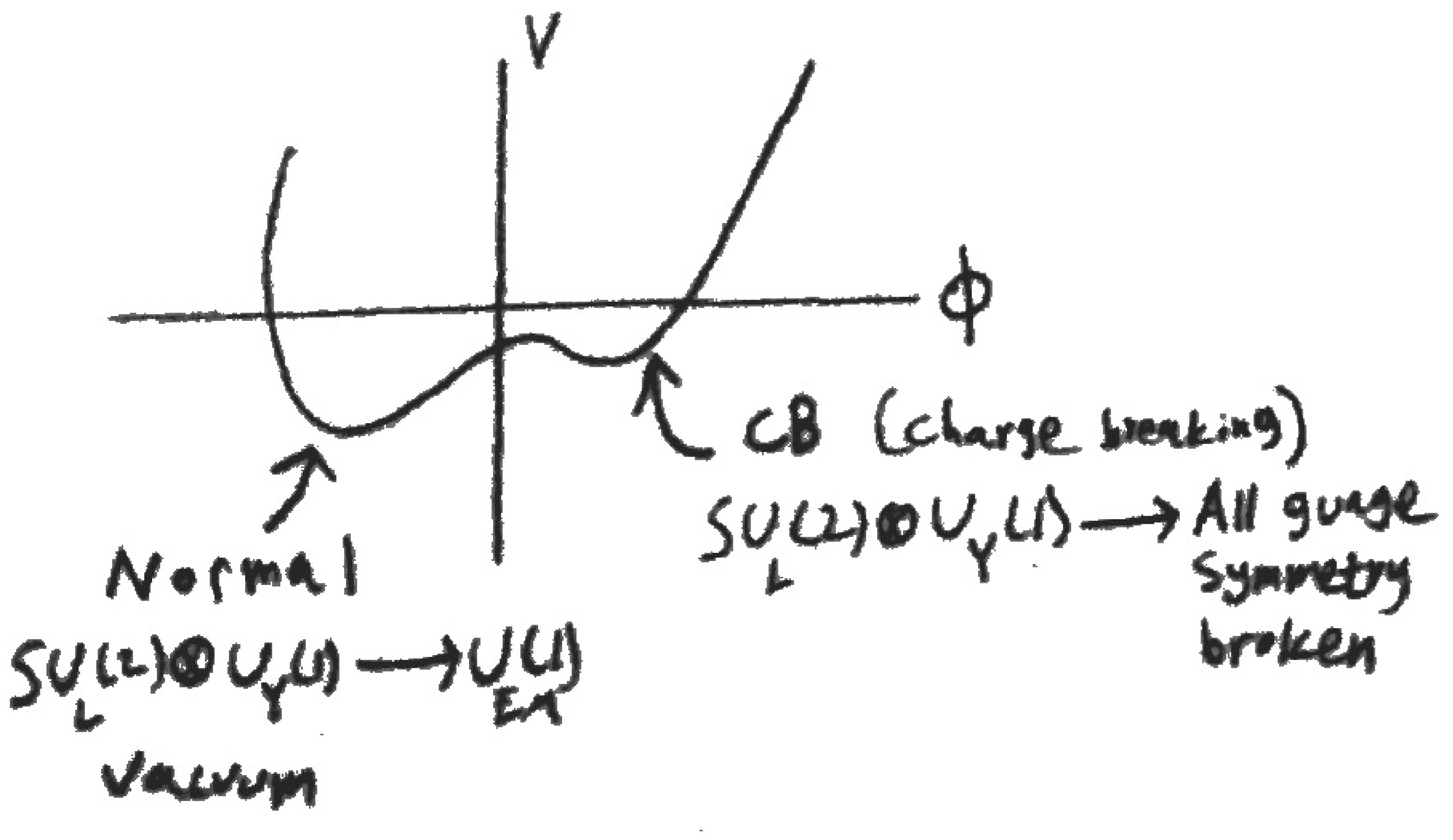
*Vacuum $\longleftrightarrow$ local minimum of the potential



SM Potential

these vacua break $SU_L(2) \otimes U_Y(1)$, leaving $U_{EM}(1)$.

In the NHDM, Various Vacua could break different $SU_L(2) \otimes U_Y(1)$ differently. Heuristically,



It is imperative that a model admits a Normal vacuum, since we live in such a vacuum. If other types of vacua are also admitted, one of the following must hold:

1. The Normal vacuum is the deepest, or if not
2. The tunneling time from the Normal vacuum to the abnormal vacuum is greater than the life time of the universe.

The Scalar Potential

The most general scalar potential of the NHDM can be written as

$V_H(\phi) = \mu_{ij}\,\phi_i^\dagger\phi_j + \lambda_{ij,kl}(\phi_i^\dagger\phi_j)(\phi_k^\dagger\phi_l)$ where repeated indices are implicitly summed

Hermicity implies

$\mu_{ij} = \mu_{ji}^\dagger, \quad \lambda_{ij,kl} = \lambda_{jl,ik}^\dagger$

and we choose

$\lambda_{ij,kl} = \lambda_{kl,ij}.$

If we so desired, we could preform basis transformations in "higgs flavor" space, ie

$\phi_i \rightarrow U_{ij}\phi_j$ where U is a unitary matrix.

We could also write this as

$$\phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_N \end{pmatrix} \rightarrow U\phi = U\begin{pmatrix} \phi_1 \\ \vdots \\ \phi_N \end{pmatrix}$$; where ϕ is a vector of doublets.

The coefficients μ, λ change under these basis transformations. For example,

$$\mu_{ij}\phi_i^\dagger\phi_j = \phi^\dagger\mu\phi \rightarrow \phi'^\dagger \underbrace{U\mu U^\dagger}_{\mu'}\phi'$$

The λ's transform similary. If one were anbitious, perhaps they would write this transformation as

$$\lambda \longrightarrow \lambda' = \underset{U^\dagger}{\overset{U}{U}}\lambda U^\dagger$$

Stationary Points

If V is a stationary point of the potential, then

$$\phi_i = \tilde{\phi}_i + V_i \quad , \quad V_i = \begin{pmatrix} V_i^u \\ V_i^d \end{pmatrix} \qquad V_i^u, V_i^d \in \mathbb{C}$$

and the potential becomes

$$V_H = \mu_{ij} (\phi_i^\dagger + V_i^\dagger)(\phi_j + V_j) + \lambda_{ij,kl} [(\phi_i^\dagger + V_i^\dagger)(\phi_j + V_j)(\phi_k^\dagger + V_k^\dagger)(\phi_l^\dagger + V_l)]$$

$$= \text{const} + \phi_i^\dagger [\mu_{ij} + 2\lambda_{ij,kl} V_k^\dagger V_l] V_j + \text{h.c.} + \text{non-linear terms}$$

where i used $\lambda_{ij,kl} = \lambda_{kl,ij}$.

but if V is a stationary point, then linear terms in $\phi^\dagger, \phi$ must vanish.

$$\longrightarrow \boxed{[\mu_{ij} + 2\lambda_{ij,kl} V_k^\dagger V_l] V_j = 0} \qquad \text{Stationary Point Conditions}$$

dotting with $V_i^\dagger$, we get

$$\lambda_{ij,kl} V_k^\dagger V_l V_i^\dagger V_j = -\frac{1}{2} \mu_{ij} V_i^\dagger V_j$$

which is useful for finding the depth of a stationary point:

$$V_H^{S.P} = \mu_{ij} V_i^\dagger V_j + \lambda_{ij,kl} V_i^\dagger V_j V_k^\dagger V_l$$

$$\longrightarrow \boxed{V_H^{S.P} = \frac{1}{2} \mu_{ij} V_i^\dagger V_j}$$

Simplifying vacua to study charge breaking

let $V = \begin{pmatrix} V_1 \\ \vdots \\ V_N \end{pmatrix} = \begin{pmatrix} V_1^u & V_1^d \\ \vdots & \vdots \\ V_N^u & V_N^d \end{pmatrix}$ be a vacuum of the model.

under a basis transformation

$$V \to UV = \left(U \begin{pmatrix} V_1^u \\ \vdots \\ V_N^u \end{pmatrix} \quad U \begin{pmatrix} V_1^d \\ \vdots \\ V_N^d \end{pmatrix} \right)$$

Pick U' such that the norm of $\begin{pmatrix} V_1^u \\ \vdots \\ V_1^u \end{pmatrix}$ is the $\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ basis vector. Then

$$U'V = \begin{pmatrix} V_1'^u & V_1'^d \\ 0 & V_2'^d \\ \vdots & \vdots \\ 0 & V_N'^d \end{pmatrix}$$ with $V_1'^u \in \mathbb{R}$, and the primes denoting that the $V_i^{u/d}$ have changed.

Next, pick U'' such that

$$U'' = \begin{pmatrix} 1 & 0 \cdots 0 \\ 0 & \\ \vdots & U''' \\ 0 & \end{pmatrix}$$ with U''' a $n-1 \times n-1$ unitary matrix.

and with the norm of $\begin{pmatrix} V_2'^d \\ \vdots \\ V_N'^d \end{pmatrix}$ is the $\left.\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}\right] N-1$ basis vector.

Then

$$U''U'V = \begin{pmatrix} V_1'^u & V_1'^d \\ U'''\begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} & U'''\begin{pmatrix} V_2'^d \\ \vdots \\ V_N'^d \end{pmatrix} \end{pmatrix} = \begin{pmatrix} V_1'^u & V_1'^d \\ 0 & V_2''^d \\ \vdots & 0 \\ & \vdots \\ 0 & 0 \end{pmatrix}$$ $V_1'^u, V_2''^d \in \mathbb{R}$, $V_1'^d \in \mathbb{C}$

We have thus show that through a series of basis transformations, any vacuum can be written as

$$V: \begin{pmatrix} \alpha \\ V_1 e^{-i\delta} \end{pmatrix}, \begin{pmatrix} 0 \\ V_2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \alpha, V_1, V_2 \in \mathbb{R}$$

What kind of vacuum is V?

clearly, if $\alpha, V_2 \neq 0$, no linear combination of generators can annihilate this vacuum, and it must therefor be charge breaking.

If $\alpha = 0$ & ($V_1 \neq 0$ or $V_2 \neq 0$), then the charge operator $Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ annihilates this vacuum, while no linear combination of the remaining generators can

$\longrightarrow$ Normal vacuum.

Model with CB and Normal Vacuum

Consider an NHDM which admits both a CB and a normal vacuum. Working in the basis where the CB vacuum is reduced, we have

$$C_1 = \begin{pmatrix} \alpha \\ V_1 e^{-i\delta} \end{pmatrix} \quad C_2 = \begin{pmatrix} 0 \\ V_2 \end{pmatrix} \quad C_{i\geq 3} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad C \text{ is the CB vacuum}$$

$$n_1 = \begin{pmatrix} 0 \\ n_1^d \end{pmatrix} \quad n_2 = \begin{pmatrix} 0 \\ n_2^d \end{pmatrix} \quad n_3 = \begin{pmatrix} 0 \\ n_3^d \end{pmatrix} \cdots\cdots n_N = \begin{pmatrix} 0 \\ n_N^d \end{pmatrix} \qquad n \text{ is the normal vacuum}$$

we can preform the same kind of basis transformation to reduce n, provided we don't touch the first two doublets. This just makes n_3^d real and $n_{i\geq 3}^d = 0$

So we have

$$n_1 = \begin{pmatrix} 0 \\ n_1^d \end{pmatrix} \quad n_2 = \begin{pmatrix} 0 \\ n_2^d \end{pmatrix} \quad n_3 = \begin{pmatrix} 0 \\ n_3^d \end{pmatrix} \quad n_{i\geq 4} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad n_1^d, n_2^d \in \mathbb{C}, \quad n_3^d \in \mathbb{R}$$

If we expand the potential about the normal vacuum, we find that upper components of the fields only mix amongst them selves in quadratic terms:

$$V_4(\tilde{\phi}_i + n_i) = \mu_{ij}\tilde{\phi}_i^\dagger\tilde{\phi}_j + \lambda_{ijkl}\Big[2\phi_i^\dagger\phi_j n_k^\dagger n_l + \underbrace{(\phi_i^\dagger n_j)(\phi_k^\dagger n_l) + (n_i^\dagger\phi_j)(n_k^\dagger\phi_l)}_{\text{no } \phi_i^U}\Big] + \text{non-quadratic}$$

$$= \left(\mu_{ij} + 2\lambda_{ijkl} n_k^\dagger n_l\right)\phi_i^{U*}\phi_j^U + \phi^d \text{ quad terms} + \text{non-quadratic terms.}$$

Thus we can write down an upper component or "charged" mass matrix

$$\mathcal{M}^{2\pm}_{ij} = \mu_{ij} + 2\lambda_{ijkl} n_k^\dagger n_l$$

if we dot this with $c_i^\dagger c_j$ (and switch indices), we get

$$\mathcal{M}^{2\pm}_{kl} c_k^\dagger c_l = \underbrace{\mu_{kl} c_k^\dagger c_l}_{2V_H^{CB}} + 2\lambda_{ijkl} n_i^\dagger n_j c_k^\dagger c_l \quad \text{using } V_H^{SP} = \tfrac{1}{2}\mu_{ij} v_i^\dagger v_k$$

$$\rightarrow V_H^{CB} = \tfrac{1}{2}\mathcal{M}^{2\pm}_{kl} c_k^\dagger c_l - \lambda_{ij,kl} n_i^\dagger n_j c_k^\dagger c_l \qquad (1)$$

Now, recall the stationary point conditions

$$\left[\mu_{ij} + 2\lambda_{ij,kl} v_k^\dagger v_l\right] v_j = 0$$

Plugging in the CB vacuum, we get

$$\left[\mu_{i1} + 2\lambda_{i1,kl} c_k^\dagger c_l\right]\alpha = 0$$

$$\left[\mu_{i1} + 2\lambda_{i1,kl} c_k^\dagger c_l\right] v_1 e^{-i\delta} + \left[\mu_{i2} + 2\lambda_{i2,kl} c_k^\dagger c_l\right] v_2 = 0$$

Since $\alpha, v_2 \neq 0$ for a CB vacuum, we have

$$\mu_{ij} + 2\lambda_{ij,kl} c_k^\dagger c_l = 0 \quad \text{for } j = 1,2$$

dotting with $n_i^\dagger n_j$, letting $j = 1,2,\dots,N$, and subtracting the extra term, we get

$$\underbrace{\mu_{ij} n_i^\dagger n_j}_{2V_H^N} + 2\lambda_{ij,kl} n_i^\dagger n_j c_k^\dagger c_l - n_i^\dagger\left[\mu_{i3} + 2\lambda_{i3,kl} c_k^\dagger c_l\right] n_3 = 0$$

$$\rightarrow V_H^N = \tfrac{1}{2} n_i^\dagger\left[\mu_{i3} + 2\lambda_{i3,kl} c_k^\dagger c_l\right] n_3 - \lambda_{ij,kl} n_i^\dagger n_j c_k^\dagger c_l \qquad (2)$$

Subtracting: (1) − (2) gives

$$\boxed{V_H^{CB} - V_H^{N} = \frac{1}{2} \mu^{2\pm}_{k\ell}\, C_k^\dagger C_\ell - \frac{1}{2} n_i^{d*} n_3^{d} \left(\mu_{i3} + 2\lambda_{i3,k\ell} \left(C_k^\dagger C_\ell \right) \right)}$$

Notice for the 2HDM

$$V_H^{CB} - V_H^{N} = \frac{1}{2} M^{2\pm}_{k\ell}\, C_k^\dagger C_\ell \xrightarrow{\text{diag}} \frac{1}{2}\left(m_1^2 |C_1|^2 + m_2^2 |C_2|^2 \right) > 0$$ if n is a local minima

thus you are always safe in 2HDM's that admit a CB and a Normal Vacuum.

In $N \geq 3$ DM, the second term may over power the positive contributions of the first term depending on μ, λ. This may be used to eliminate the parameter point.

References

A. Barroso, P. M. Ferreira, R. Santos, Joao P. Silva, Stability of the normal vacuum in multi-Higgs-doublet models, arXiv: hep-ph/0608282v2