Neutrino Masses

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Neutrinos: Discovery & Low Mass

• 1930 by Pauli
  • “I have done a terrible thing. I have postulated a particle that cannot be detected.”
  • Original mass estimate: $0.01 \times m_p$

Fermi’s sketch of the endpoint of the tritium $\beta$-decay spectrum (1934)
Measuring Neutrino Mass with $^3\text{H}$ Beta Decay
Measuring Neutrino Mass

- Mainz et al (2005): $m_\beta < 2.3$ eV
- KATRIN (projection): $m_\beta < 0.2$ eV
Solar Neutrino Problem

- Neutrinos are produced in copious numbers \((6.4 \times 10^{10} \text{ cm}^{-2} \text{ s}^{-1})\) due to various nuclear reactions in the Sun:
  - \(p + p \rightarrow d + e + \bar{\nu}_e\)
  - \(e^- + ^7\text{Be} \rightarrow ^7\text{Li} + \nu_e\)
  - \(^8\text{B} \rightarrow ^8\text{Be} + e^+ + \nu_e\)
- Early (<1998) experiments only detected a fraction of the SSM prediction
- Neutrino oscillations proposed by Pontecorvo in 1967 as a solution
QM Treatment of Neutrino Oscillations

• Consider just two species of neutrino, $\nu_e$ and $\nu_\mu$

• Suppose neither neutrino is a mass eigenstate, but the admixture is:

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• Then the states $\nu_1$ and $\nu_2$ will propagate forward in time as usual:

$$\nu_1(t) = e^{-iE_1 t} \nu_1(0)$$
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• Suppose a electron neutrino is produced ($\nu_e = 1$):

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• We are interested in the probability of detecting a flavor eigenstate at some later time $t$. Solving for $\nu_e$ and $\nu_\mu$:

$$\nu_e = \nu_2 \cos \theta - \nu_1 \sin \theta \quad \quad \nu_\mu = \nu_1 \cos \theta + \nu_2 \sin \theta$$

• Plugging in:

$$\nu_e = e^{-iE_2 t} \cos^2 \theta + e^{-iE_1 t} \sin^2 \theta \quad \quad \nu_\mu = (e^{-iE_2 t} - e^{-iE_1 t}) \cos \theta \sin \theta$$
QM Treatment of Neutrino Oscillations

• The probability of detecting a flavor eigenstate is just the square of the amplitude:

\[
|\langle \nu_\mu | \nu_\mu \rangle|^2 = (e^{-iE_2t} - e^{-iE_1t})(e^{iE_2t} - e^{iE_1t}) \sin^2 \theta \cos^2 \theta
\]

\[
|\langle \nu_\mu | \nu_\mu \rangle|^2 = (2 - e^{-i(E_2-E_1)t} - e^{i(E_2-E_1)t}) \frac{\sin^2(2\theta)}{4}
\]

\[
|\langle \nu_\mu | \nu_\mu \rangle|^2 = (2 - 2 \cos[(E_2 - E_1)t]) \frac{\sin^2(2\theta)}{4} = \sin^2(2\theta) \sin^2 \frac{(E_2-E_1)t}{2}
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• We can relate this probability to the neutrino mass by using kinematics:

\[ E2 - E1 \cong \frac{m_2^2 - m_1^2}{2E} \]
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• We can relate this probability to the neutrino mass by using kinematics:

\[E_2 - E_1 \cong \frac{m_2^2 - m_1^2}{2E}\]

• Then the transition probability is simply:

\[P = \sin^2(2\theta) \sin^2 \frac{(m_2^2 - m_1^2)t}{4E}\]
3-Flavor Neutrino Mixing

• With three neutrinos, we have a mixing matrix:

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 1} & U_{\mu 1} \\
U_{\tau 1} & U_{\tau 1} & U_{\tau 1}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

• Where the matrix U (PMNS matrix) can be decomposed in terms of its three mixing angles:

\[
U = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

• If neutrinos are Majorana fermions, there are an additional two phases are present:

\[
U \rightarrow U \ast \text{diag}(1, e^{i\phi_1}, e^{i\phi_2})
\]

• Extra phases cancel when computing the Hamiltonian

• Dirac vs Majorana nature cannot be probed by oscillation experiments
## Current Experimental Values & Limits

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta m^2 = m_2^2 - m_1^2$</td>
<td>$7.53 \pm 0.18 \times 10^{-5}$ eV</td>
</tr>
<tr>
<td>$\Delta m^2 = \left</td>
<td>m_3^2 - \frac{m_2^2 + m_1^2}{2} \right</td>
</tr>
<tr>
<td>$(\sin \theta_{12})^2$</td>
<td>$0.304 \pm 0.014$</td>
</tr>
<tr>
<td>$(\sin \theta_{23})^2$</td>
<td>$0.511 \pm 0.055$</td>
</tr>
<tr>
<td>$(\sin \theta_{13})^2$</td>
<td>$2.19 \pm 0.12 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
Normal vs Inverted Hierarchy

- Difficult to probe via neutrino oscillations in vacuum, which are sensitive to $\Delta m^2$
- Could in principle be tested by oscillations in matter
Super-Kamiokande Experiment (~1998)

- 50 kt water Cerenkov telescope
  - Sensitive to all neutrinos, but mostly e-type
- Searched for atmospheric neutrinos
  - \( \pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_\mu + \nu_e + \bar{\nu}_\mu \)
  - Robust prediction of \( \frac{\Phi(\nu_\mu + \nu_\mu)}{\Phi(\nu_e + \nu_e)} \approx 2 \)
  - Instead found \( \frac{\Phi(\nu_\mu + \nu_\mu)}{\Phi(\nu_e + \nu_e)} = 1.32 \)

Neil DeGrasse Tyson goes boating in Super-K
Super-Kamiokande Experiment (~1998)

Deficit of upward-going $\mu$-type neutrinos

Super-K view of the Sun. Super-K detected about 45% of the expected solar neutrino flux
SNO Experiment (2001)

- Used 1kt heavy water ($3 \times 10^8$) to determine separately the e-type flux and total flux
  - $\nu_e + d \rightarrow p + p + e^-$
  - $\nu_x + d \rightarrow p + n + \nu_x$
- 2 tons of NaCl added in May 2001 to increase the NC signal
SNO Results (2001)

- 35% of solar neutrinos were e-type, 65% were μ-type or τ-type
- 2015 Nobel Prize awarded to Takaaki Kajita of Super-K and Arthur B. McDonald of SNOLAB
KamLAND Experiment (~2002)

• 1 kt mineral oil scintillator detector

• Detects reactor antineutrinos via:
  \[ \bar{\nu}_e + p \rightarrow n + e^+ \]
  \[ n + p \rightarrow d + \gamma (2.2 \text{ MeV}) \]
  • Detect prompt coincidence between positron annihilation and gamma signal
  • 51 nuclear reactors in Japan in 2002

• First experiment to show neutrino disappearance
KamLAND Results

![Graph showing Kamioka Liquid Scintillator Antineutrino Detection (KamLAND) results.](image)

- **Equation**: \( P_{ee} = 1 - \sin^2 2\theta \sin^2 (\Delta m^2 L / 4E_{\nu}) \)

- **Plot**:
  - Base line = 180 km
  - Survival Probability as a function of energy
  - Survival Probability as a function of \( L_{\nu} / E_{\nu} \)

- **Data Points**:
  - ILL
  - Savannah River
  - Bugey
  - Rovno
  - Goessgen
  - Krasnoyark
  - Palo Verde
  - Chooz
  - KamLAND
Review of Fermion Masses

• For (e.g.) electrons
  • $\mathcal{L}$ could include the gauge-invariant term $-g_e \bar{l}_e R \phi e_R + \text{h.c.}$
    • $l_e R$ is the electron-type doublet, $\phi$ is the Higgs doublet
  • After symmetry breaking, we get a term like $-g_e \frac{v}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L)$
    • Precisely a Dirac mass term with mass $m = \frac{g_e v}{\sqrt{2}}$

• Same mechanism works for neutrinos
  • Experimentally, then, $g_e >> g_\nu$ which seems unnatural
See Saw Mechanism

• If right-handed neutrinos exist with Majorana masses and Yukawa-type couplings, we could have a term in the Lagrangian like:

\[ \mathcal{L} \in \left( \lambda_{ij} \overline{N}_{iR}(x) \Phi^+(x) \psi_{iL}(x) + \text{h. c.} \right) - \frac{1}{2} M_i \overline{N}_i(x) N_i(x) \]

• Here \( \lambda_{ij} \) is a matrix of Yukawa couplings and \( \psi_{iL}^T = (\nu_{iL}^T, l_L^T) \)

• After electroweak symmetry breaking, there are new Dirac mass terms generated:

\[ m_{il}^D (\overline{N}_{iR}(x) \nu_{iL}(x) + \text{h. c.}) \]

• Can be combined with the Majorana mass term above into a single term:

\[ -\frac{1}{2} (\overline{N}_{LC} M N_L + \overline{N}_L M N_{LC}) \]
See-Saw Mechanism

• Then the mass matrix is: \( \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \)

• In one scenario \( m_L = 0 \), and then the eigenvalues are \( m_1 \approx m_R \) and \( m_2 \approx \frac{m_D^2}{m_R} \)

• If \( m_R \) is large (~\(10^{16} \) GeV) and \( m_D \) is ~100 GeV, the small neutrino mass is explained

• Also attractive because the Yukawa coupling is not CP-conserving
  
  • \( \Gamma(N_j \to l^+ + \Phi^{(-)}) \neq \Gamma(N_j \to l^- + \Phi^{(+)}) \)
  
  • Sphaleron processes that conserve B-L but not B+L can then produce baryogenesis via leptogenesis
Looking Forward: Searching for $0\nu\beta\beta$

- In some even-even nuclei ($^{76}\text{Ge}$, $^{48}\text{Ca}$, $^{136}\text{Xe}$, etc), $\beta$-decay is energetically forbidden but double $\beta$-decay ($[A,Z] \rightarrow [A,Z+2] + 2e^- + 2\bar{\nu}_e$) is allowed.
- If neutrinos are Majorana fermions, the decay $[A,Z] \rightarrow [A,Z+2] + 2e^-$ should also be allowed.
- Half life is: $T_{1/2}^{0\nu\beta\beta} = \frac{1}{G_{0\nu}|M_{0\nu}|^2 m_{\nu}^2}$
  - Multiple experiments needed to pin down absolute mass scale.
  - Many experiments planned.
MAJORANA

- Needed to test the result from Kleingrothaus et al.
  - Claim of discovery with $T_{1/2}^{0\nu\beta\beta} = 10^{25}$ years

- Uses 30 kg of enriched $^{76}$Ge as simultaneous detector and source
- Plan to scale up to 1 ton of Germanium
- Goal of 1 background event/year in 4 keV window around Q value

Heidelberg-Moscow experiment results-claim of $4\sigma$ excess at 2039 keV
Questions?
References

1. K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014) and 2015 update
7. H. V. Kleingrothaus et al, Foundations of Physics, Vol. 32, No. 8, August 2002