Neutrino Masses

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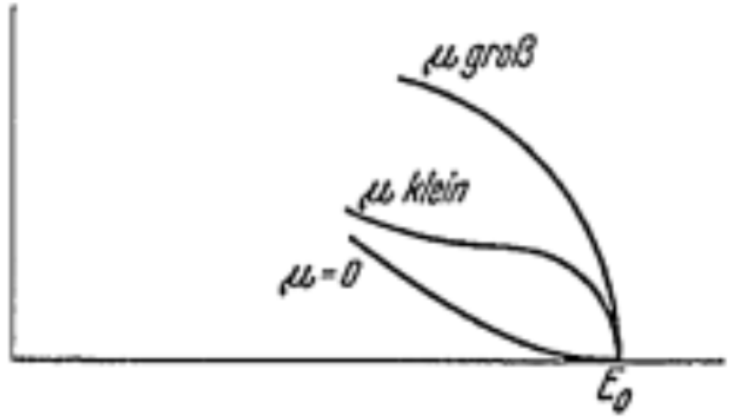
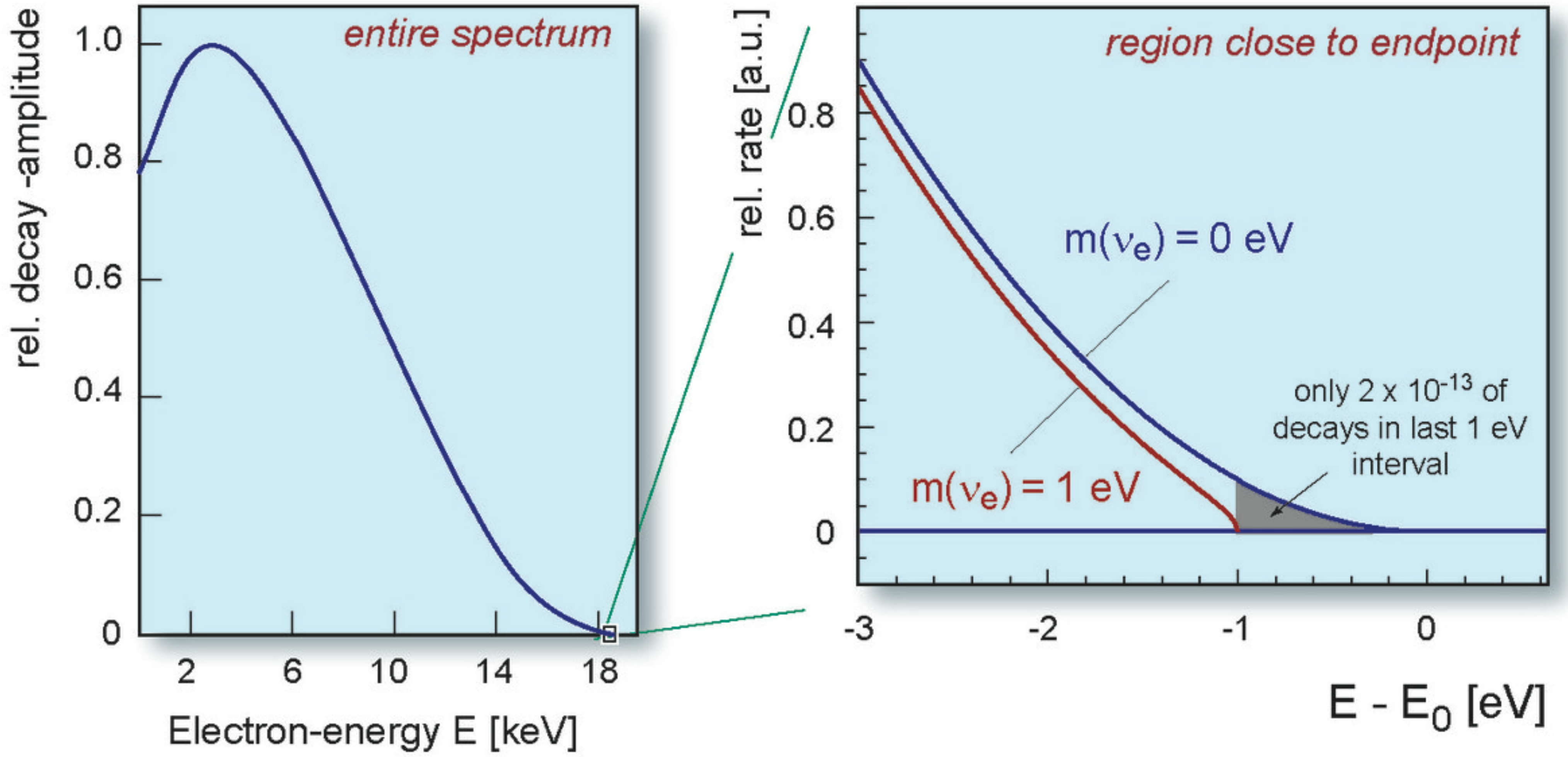
Neutrinos: Discovery & Low Mass

- 1930 by Pauli
  - "I have done a terrible thing. I have postulated a particle that cannot be detected."
  - Original mass estimate: $0.01 * m_p$



Fermi's sketch of the endpoint of the tritium β-decay spectrum (1934)

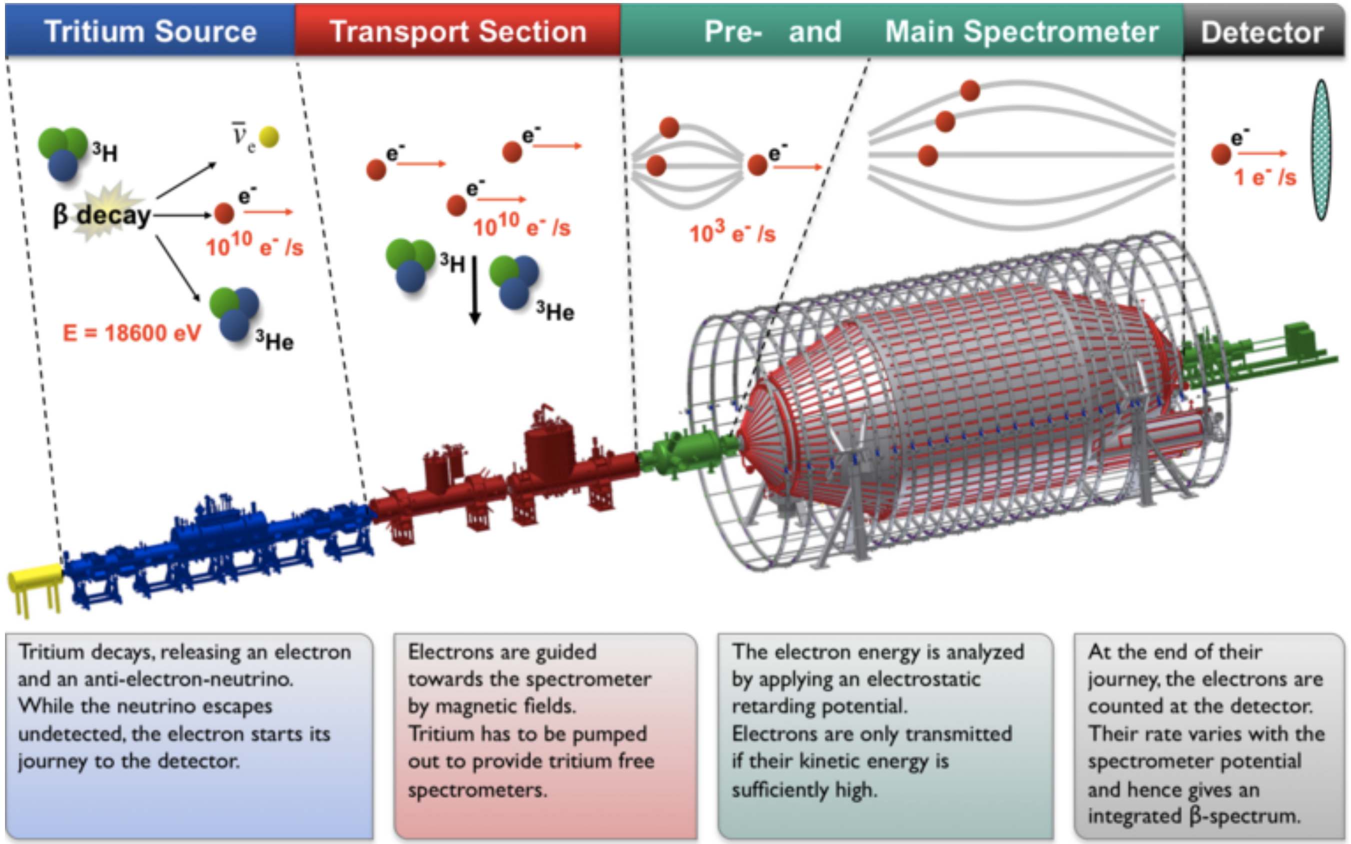
Measuring Neutrino Mass with ^{3}H Beta Decay



Measuring Neutrino Mass

- Mainz et al (2005): $m_\beta < 2.3\ \mathrm{eV}$

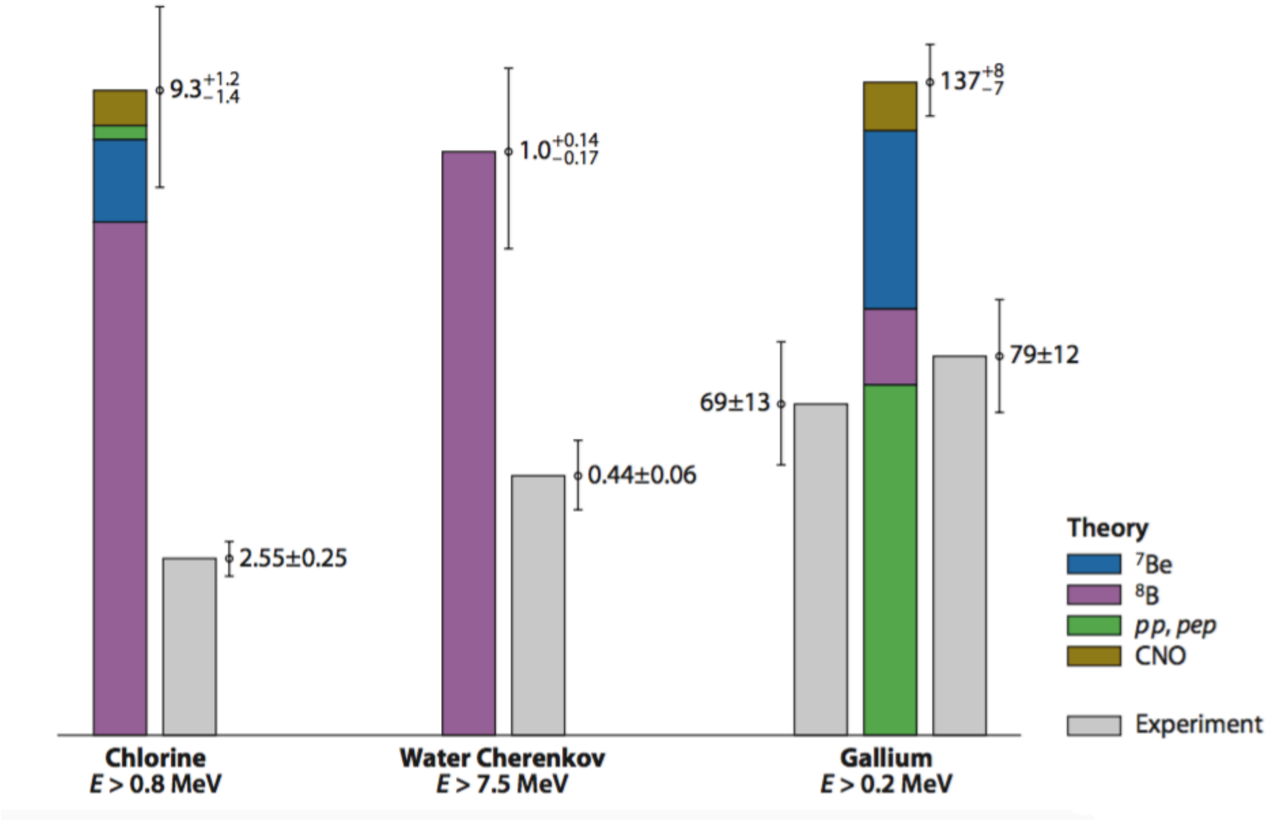
- KATRIN (projection): $m_\beta < 0.2\ \mathrm{eV}$



http://www.katrin.kit.edu/img/KATRIN-Beamline-2011-Slide.png

Solar Neutrino Problem

- Neutrinos are produced in copious numbers ($6.4*10^{10}$ cm^{-2} s^{-1}) due to various nuclear reactions in the Sun:
  - $p + p \rightarrow d + e + \bar{\nu}_e$
  - $e^- + {}^7\mathrm{Be} \rightarrow {}^7\mathrm{Li} + \nu_e$
  - ${}^8\mathrm{B} \rightarrow {}^8\mathrm{Be} + e^+ + \nu_e$
- Early (<1998) experiments only detected a fraction of the SSM prediction
- Neutrino oscillations proposed by Pontecorvo in 1967 as a solution



QM Treatment of Neutrino Oscillations

- Consider just two species of neutrino, ν_e and ν_μ
- Suppose neither neutrino is a mass eigenstate, but the admixture is:

$$\nu_1 = \nu_\mu \cos\theta - \nu_e \sin\theta \qquad\qquad \nu_2 = \nu_\mu \sin\theta + \nu_e \cos\theta$$

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- Then the states ν_1 and ν_2 will propagate forward in time as usual:

$$\nu_1(t) \ = e^{-iE_1 t}\nu_1(0) \qquad \nu_2(t) \ = e^{-iE_2 t}\nu_2(0)$$

- Suppose a electron neutrino is produced ($\nu_e \ = 1$):

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- We are interested in the probability of detecting a flavor eigenstate at some later time t. Solving for ν_e and ν_μ :

$$\nu_e = \nu_2\cos\theta - \nu_1\sin\theta \qquad \nu_\mu = \nu_1\cos\theta + \nu_2\sin\theta$$

- Plugging in:

$$\nu_e = e^{-iE_2 t}\cos^2\theta + e^{-iE_1 t}\sin^2\theta \qquad \nu_\mu = (e^{-iE_2 t} - e^{-iE_1 t})\cos\theta\sin\theta$$

QM Treatment of Neutrino Oscillations

- The probability of detecting a flavor eigenstate is just the square of the amplitude:

$$|\langle\nu_\mu|\nu_\mu\rangle|^2 = (e^{-iE_2t} - e^{-iE_1t})(e^{iE_2t} - e^{iE_1t})\ \sin^2\theta\cos^2\theta$$

$$|\langle\nu_\mu|\nu_\mu\rangle|^2 = (2 - e^{-i(E_2-E_1)t} - e^{i(E_2-E_1)t})\,\frac{\sin^2(2\theta)}{4}$$

$$|\langle\nu_\mu|\nu_\mu\rangle|^2 = (2 - 2\cos[(E_2 - E_1)t])\,\frac{\sin^2(2\theta)}{4} = \sin^2(2\theta)\sin^2\frac{(E_2-E_1)t}{2}$$

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- We can relate this probability to the neutrino mass by using kinematics:

$$E2 - E1 \cong \frac{{m_2}^2-{m_1}^2}{2E}$$

- Then the transition probability is simply:

$$P = \sin^2(2\theta)\sin^2\frac{({m_2}^2-{m_1}^2)t}{4E}$$

3-Flavor Neutrino Mixing

- With three neutrinos, we have a mixing matrix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 1} & U_{\mu 1} \\ U_{\tau 1} & U_{\tau 1} & U_{\tau 1} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

- Where the matrix U (PMNS matrix) can be decomposed in terms of its three mixing angles:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- If neutrinos are Majorana fermions, there are an additional two phases are present:

$$U \to U * \mathrm{diag}\left(1, e^{i\phi_1}, e^{i\phi_2}\right)$$

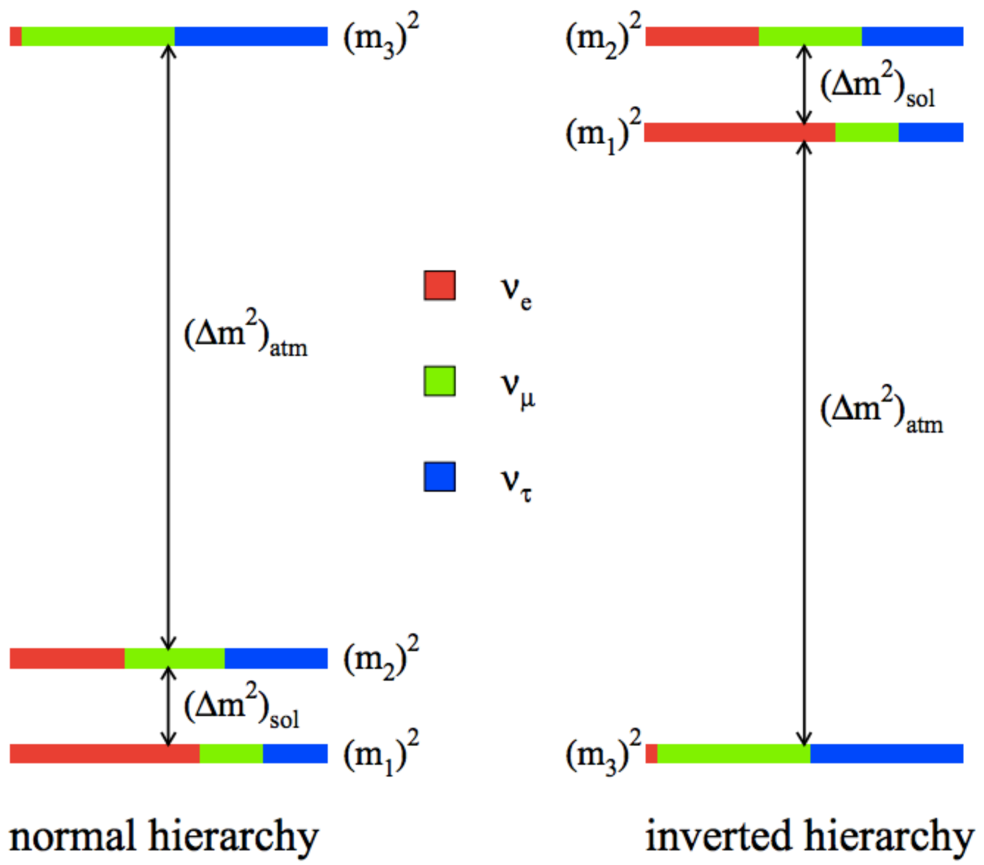
  - Extra phases cancel when computing the Hamiltonian
  - Dirac vs Majorana nature cannot be probed by oscillation experiments

Current Experimental Values & Limits

Parameter	Value
$\delta m^2 = m_2^2 - m_1^2$	$7.53 \pm 0.18 * 10^{-5}$ eV
$\Delta m^2 = \left\lvert m_3^2 - \frac{m_2^2 + m_1^2}{2} \right\rvert$	$2.44 \pm 0.06 * 10^{-3}$ eV
$(\sin \theta_{12})^2$	0.304 ± 0.014
$(\sin \theta_{23})^2$	0.511 ± 0.055
$(\sin \theta_{13})^2$	$2.19 \pm 0.12 * 10^{-2}$

Normal vs Inverted Hierarchy

- Difficult to probe via neutrino oscillations in vacuum, which are sensitive to Δm^2
  - Could in principle be tested by oscillations in matter

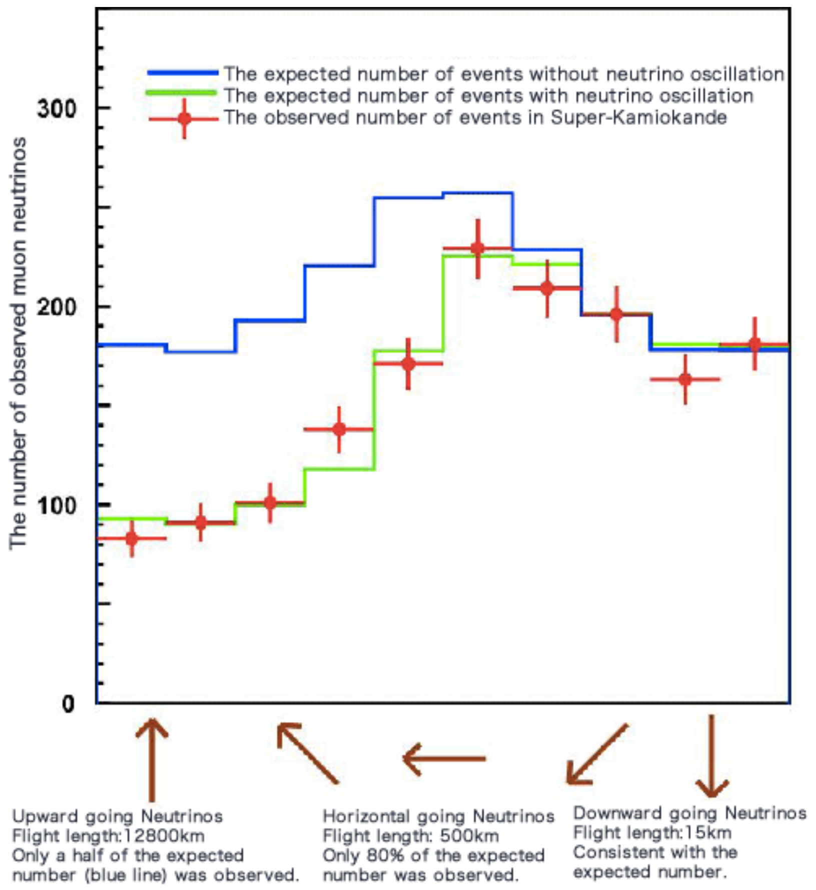


Super-Kamiokande Experiment (~1998)

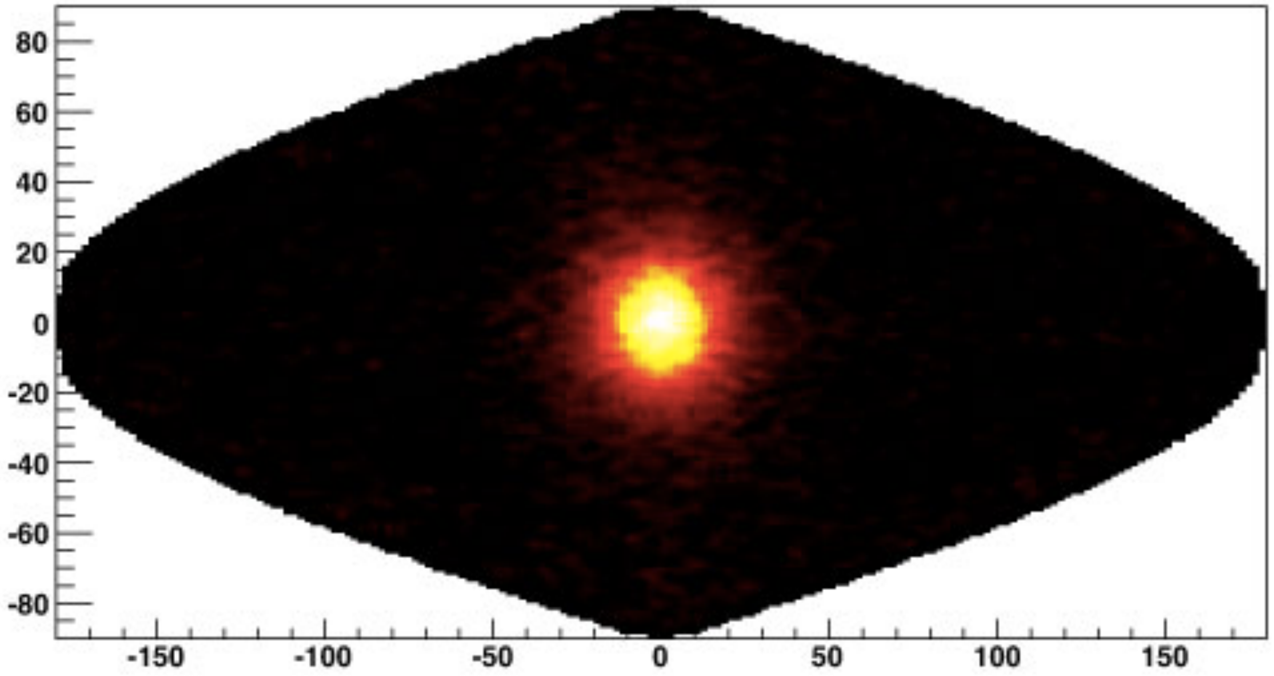
- 50 kt water Cerenkov telescope
  - Sensitive to all neutrinos, but mostly e-type
- Searched for atmospheric neutrinos
  - $\pi^+ \rightarrow \mu^+ + \nu_{\ \mu} \rightarrow e^+ + \nu_\mu + \nu_e + \bar{\nu}_\mu$
  - Robust prediction of ${}^{\Phi(\nu_\mu+\nu_\mu)}/_{\Phi(\nu_e+\nu_e)} \cong 2$
  - Instead found ${}^{\Phi(\nu_\mu+\nu_\mu)}/_{\Phi(\nu_e+\nu_e)} = 1.32$

Neil DeGrasse Tyson goes boating in Super-K

Super-Kamiokande Experiment (~1998)



Deficit of upward-going μ-type neutrinos

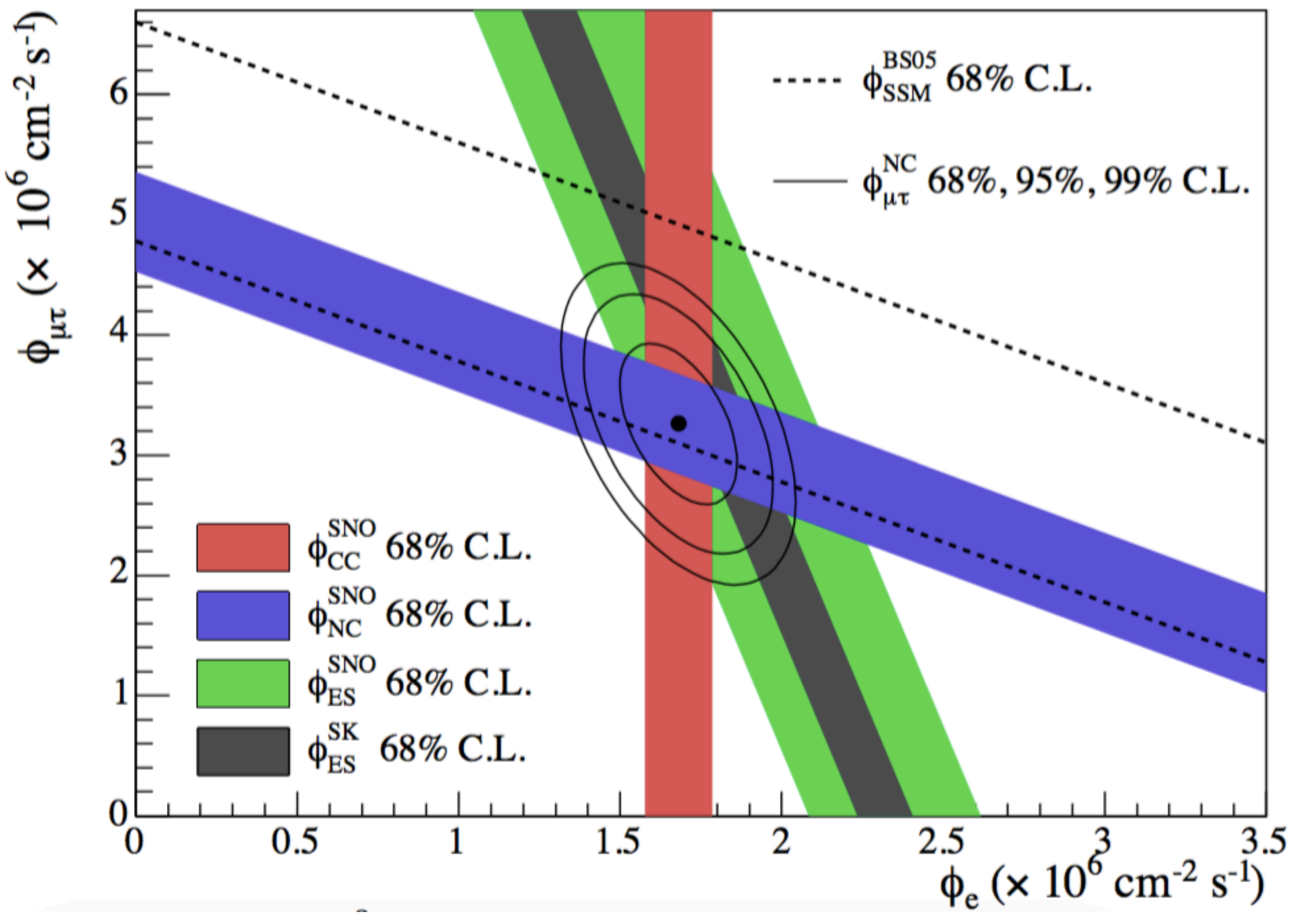


Super-K view of the Sun. Super-K detected about 45% of the expected solar neutrino flux

SNO Experiment (2001)

- Used 1kt heavy water (\$3*10^8) to determine separately the e-type flux and total flux
  - $v_e + d \rightarrow p + p + e^-$
  - $v_x + d \rightarrow p + n + v_x$
- 2 tons of NaCl added in May 2001 to increase the NC signal

SNO Results (2001)



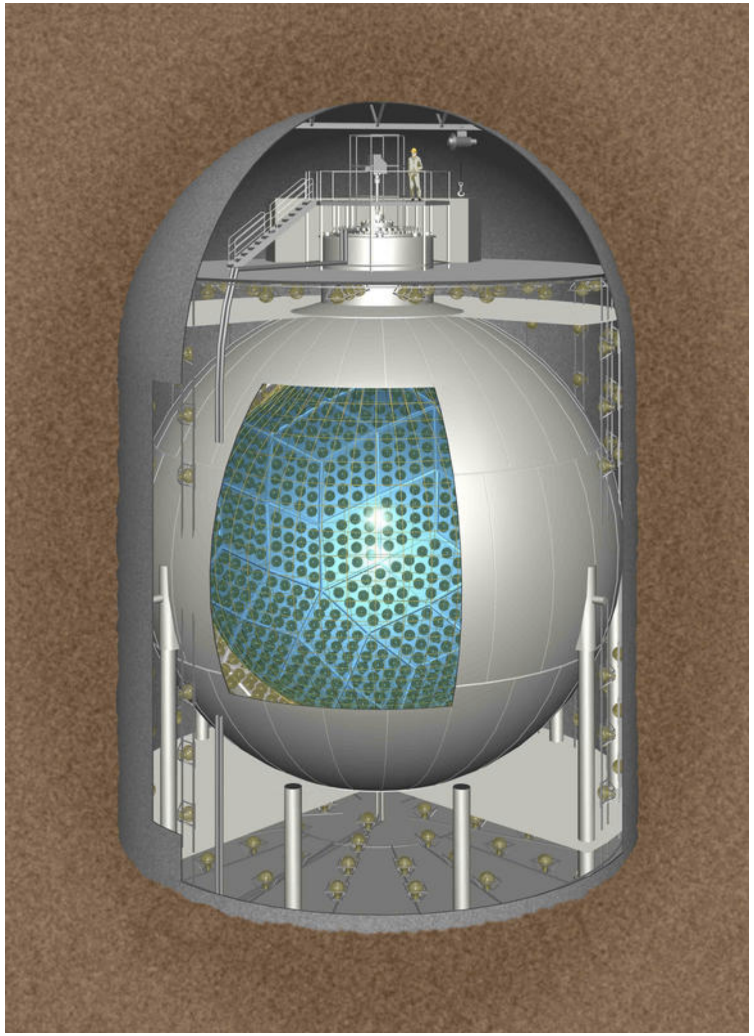
- 35% of solar neutrinos were e-type, 65% were μ-type or τ–type
- 2015 Nobel Prize awarded to Takaaki Kajita of Super-K and Arthur B. McDonald of SNOLAB

KamLAND Experiment (~2002)

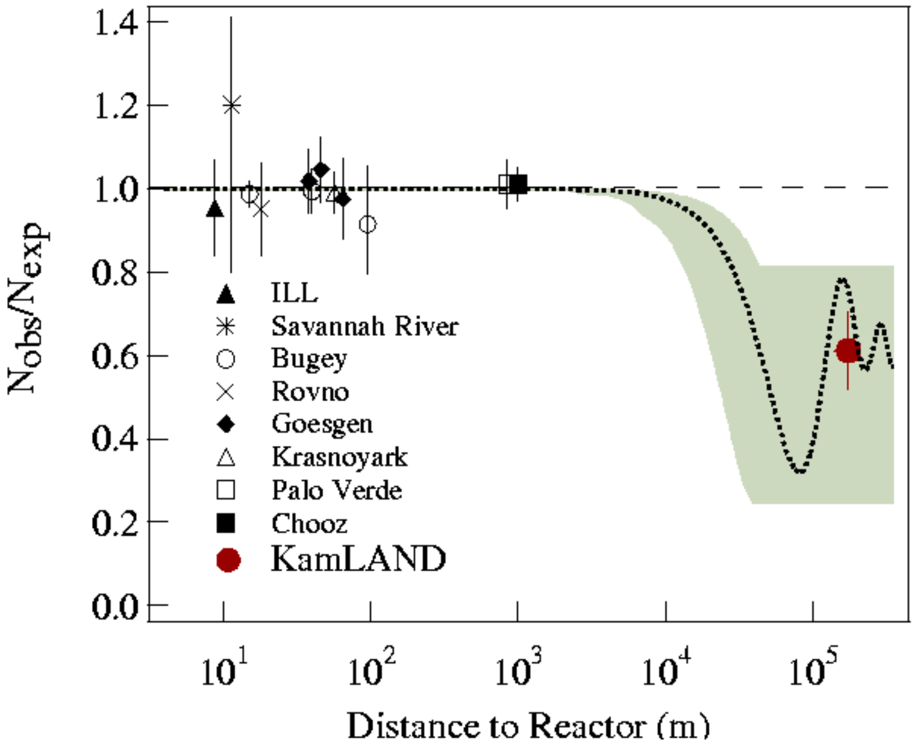
- 1 kt mineral oil scintillator detector
- Detects reactor antineutrinos via:

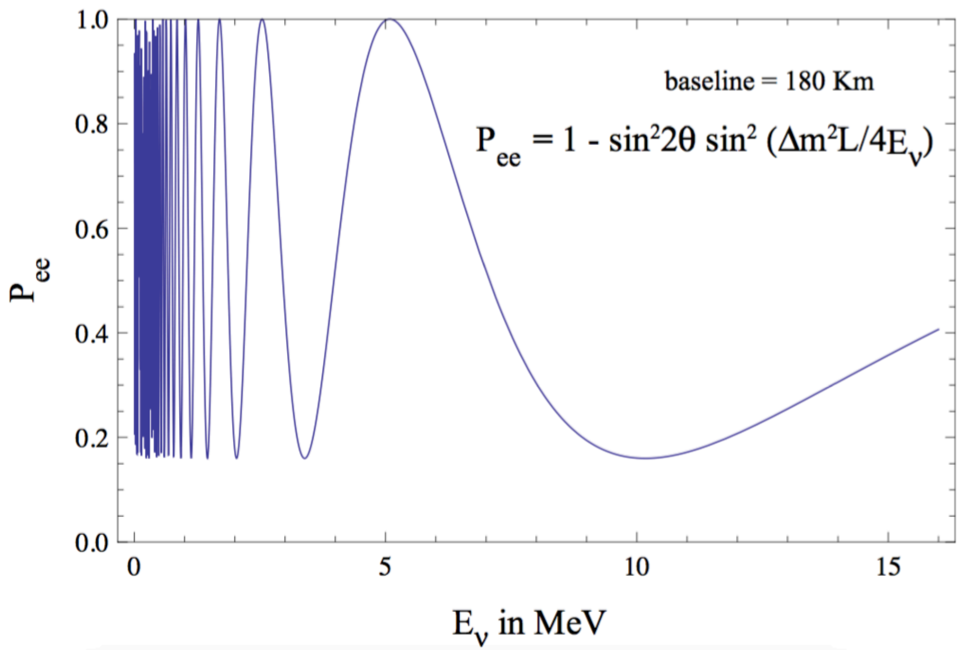
$$\bar{\nu}_e + p \rightarrow n + e^+$$
$$n + p \rightarrow d + \gamma \text{ (2.2 MeV)}$$

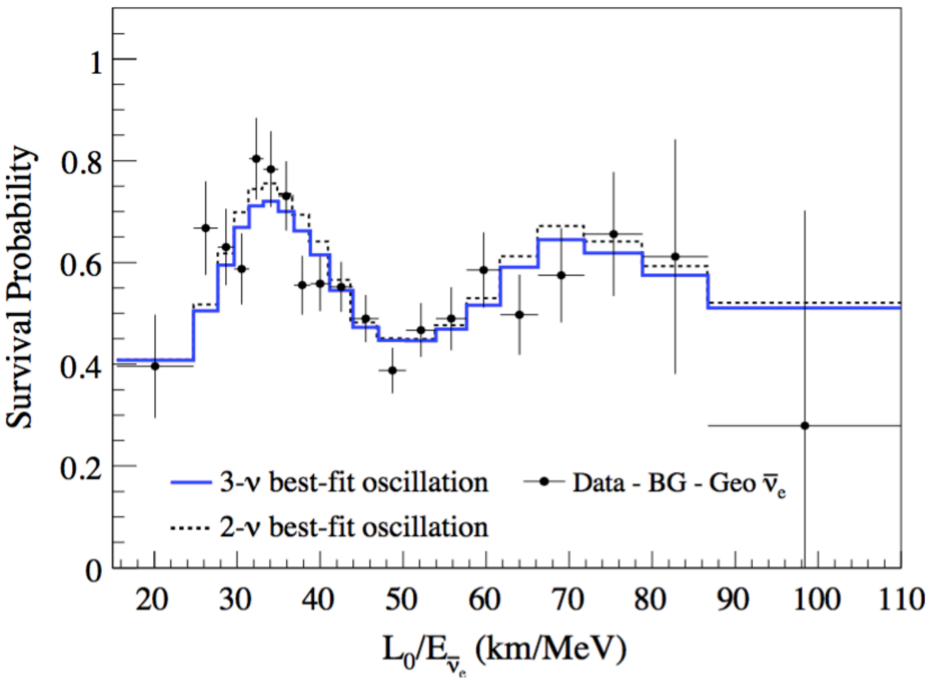
  - Detect prompt coincidence between positron annihilation and gamma signal
  - 51 nuclear reactors in Japan in 2002
- First experiment to show neutrino disappearance



KamLAND Results







Review of Fermion Masses

- For (e.g.) electrons
  - $\mathcal{L}$ could include the gauge-invariant term $-g_e \bar{l}_{eR} \phi e_R + \mathrm{h.c.}$
    - l_{eR} is the electron-type doublet, ϕ is the Higgs doublet
  - After symmetry breaking, we get a term like $-g_e \frac{v}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L)$
    - Precisely a Dirac mass term with mass $m = \frac{g_e v}{\sqrt{2}}$
- Same mechanism works for neutrinos
  - Expermentally, then, $g_e >> g_\nu$ which seems unnatural

See Saw Mechanism

- If right-handed neutrinos exist with Majorana masses and Yukawa-type couplings, we could have a term in the Lagrangian like:

$$\mathcal{L} \in \left(\lambda_{ij}\overline{N_{i,R}}(x)\Phi^{\dagger}(x)\psi_{lL}(x) + \mathrm{h.c.}\right) - \frac{1}{2}M_i\overline{N_i}(x)N_i(x)$$

  - Here λ_{ij} is a matrix of Yukawa couplings and $\psi_{lL}^T = (\nu_{lL}^T, l_L^T)$
- After electroweak symmetry breaking, there are new Dirac mass terms generated:

$$m_{il}^D\,(\bar{N}_{iR}(x)\nu_{lL}(x) + \mathrm{h.c.})$$

  - Can be combined with the Majorana mass term above into a sigle term:

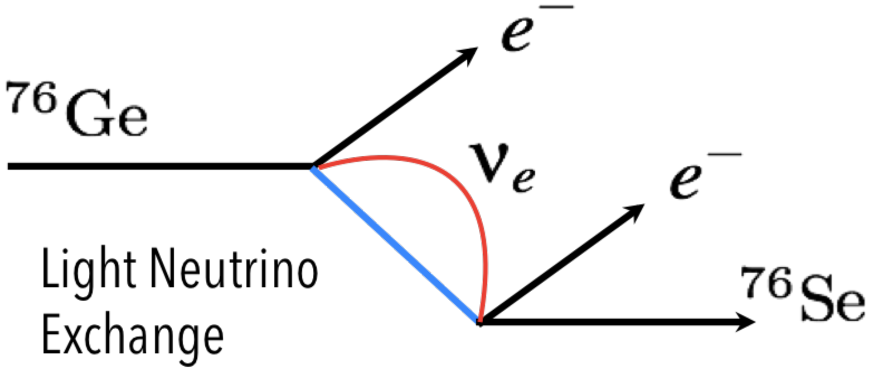
$$-\frac{1}{2}(\bar{N}_{L\boldsymbol{c}}MN_L + \bar{N}_L MN_{L\boldsymbol{c}})$$

See-Saw Mechanism

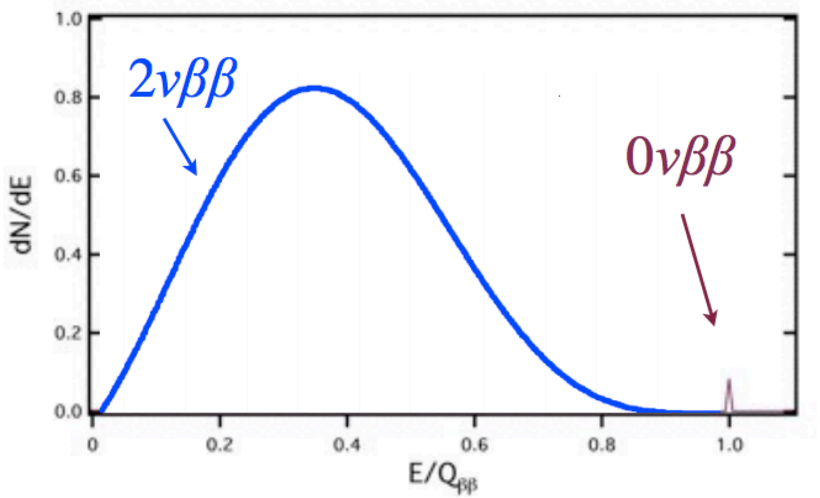
- Then the mass matrix is: $\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$
- In one scenario $m_L = 0$, and then the eigenvalues are $m_1 \approx m_R$ and $m_2 \approx \frac{{m_D}^2}{m_R}$
- If m_R is large (~10^{16} GeV) and m_D is ~100 GeV, the small neutrino mass is explained
- Also attractive because the Yukawa coupling is not CP-conserving
  - $\Gamma\left(N_j \rightarrow l^+ + \Phi^{(-)}\right) \neq \Gamma(N_j \rightarrow l^- + \Phi^{(+)})$
  - Sphaleron processes that conserve B-L but not B+L can then produce baryogenesis via leptogenesis

Looking Forward: Searching for $0\nu\beta\beta$

- In some even-even nuclei (^{76}Ge, ^{48}Ca, ^{136}Xe, etc), β-decay is energetically forbidden but double β-decay ([A,Z] → [A,Z+2] + $2e^{-}+2\bar{\nu}_e$) is allowed
- If neutrinos are Majorana fermions, the decay [A,Z] → [A,Z+2] + $2e^{-}$ should also be allowed
- Half life is: $T_{1/2}^{0\nu\beta\beta} = \frac{1}{G_{0\nu}|M_{0\nu}|^2 m_{\beta\beta}^2}$
  - Multiple experiments needed to pin down absolute mass scale
- Many experiments planned



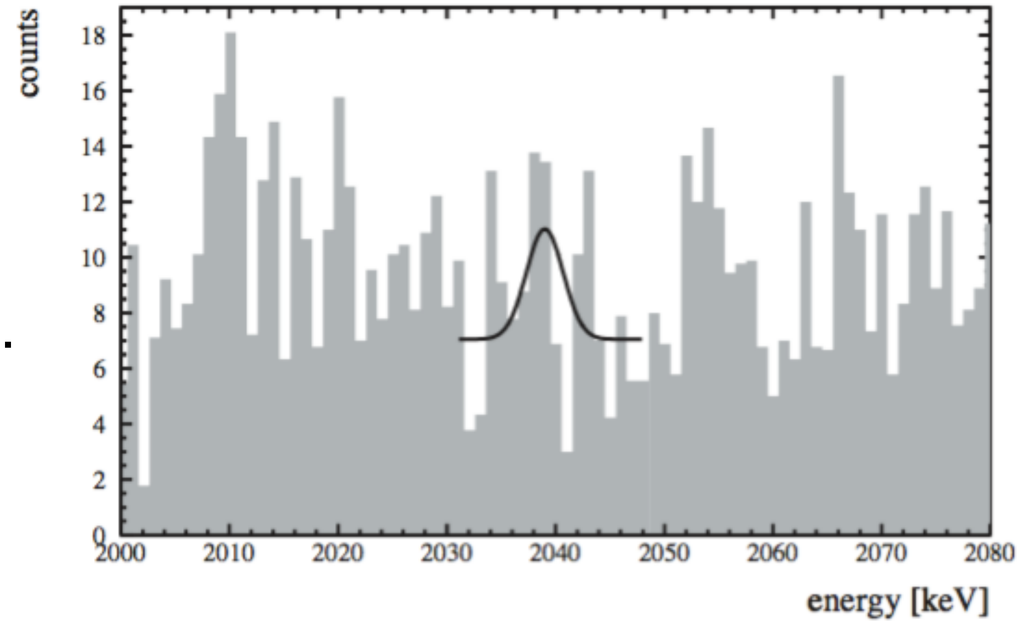
Example Feynman diagram of $0\nu\beta\beta$ in ^{76}Ge



Cartoon spectrum from $0\nu\beta\beta$

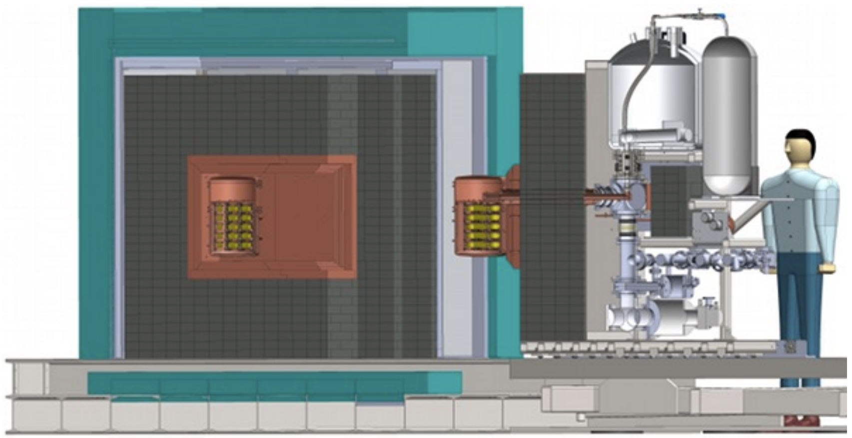
MAJORANA

- Needed to test the result from Kleingrothaus et al.
  - Claim of discovery with $T^{0\nu\beta\beta}_{1/2} = 10^{25}$years



- Uses 30 kg of enriched $^{76}\mathrm{Ge}$ as simultaneous detector and source
- Plan to scale up to 1 ton of Germanium
- Goal of 1 background event/year in 4 keV window around Q value

Heidelberg-Moscow experiment results- claim of 4σ excess at 2039 keV



Questions?

References

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